Multi-Objective Location-Routing Model With Multiple Transportation Modes For Designing Relief Networks Under Uncertainty

E. Haghi

Abstract

Natural disasters and crisis are inevitable and each year impose destructive effects on human and damage to property. In natural disasters and after the outbreak of the crisis, demand for logistical goods and services increase. Effective distribution of emergency aid could have a significant role in minimizing the damage and fatal accidents. In this study, a three-level relief chain including a number of suppliers in fixed locations, candidate distribution centers and affected areas at certain points are considered. For this purpose, a mixed integer nonlinear programming model is proposed for open transportation location routing problem by considering split delivery of demand. In order to solve a realistic problem, foregoing parameters are considered as fuzzy in our proposed mode. The objectives of the proposed model include total cost minimization, minimization of the maximum travel time of vehicles and minimization of unmet demands. In order to solve the problem of the proposed model, fuzzy multi-objective planning is used. For efficiency and effectiveness of the proposed model and solution approach, several numerical examples are studied. Computational results show the effectiveness and efficiency of the model and the proposed approach.

Keywords: Emergency logistics; location-routing problem; Split delivery; Fuzzy possibilistic programming.

1 Introduction

Unexpected events and natural disasters (floods, earthquakes, hurricanes, etc.) and their consequences require the current societies to plan for assistance in such crisis. Doing this is faced with challenges such as damage to infrastructure, transportation, limited time and resources, difficulties in coordination between different factors and so on. So, compared to conventional logistics, providing assistance in crisis and emergency logistics is complicated and challenging [19, 24]. In the incidence of natural disasters at the time of critical condition, demand for logistic goods and services increase and quick distribution of essential facilities can be effective in minimizing the damage and fatal accidents. Therefore, the affected areas shall be supported with various emergency items such as tents, water, etc., which are needed quickly in times of crisis. Emergency aid process includes the transfer of the needed goods from different suppliers (Red Crescent, airports, local suppliers, etc.) by local distribution centers to the damaged areas. So, one of the important logistic strategies to improve performance and reduce latency is the location and establishing distribution centers near
the affected areas. If distribution centers are at appropriate locations from the network, which could cover demand created in these conditions appropriately, it would be very important in the successful rescue operation. In all cases listed, poor selection of suitable locations will increase the probability of capital loss and ultimately will lead to many human losses. Of other logistic activities that are of great importance and affect the optimal distribution network, is the planning to transport essential items and required goods to the affected areas. Therefore, deciding on the number of vehicles allocated to each distribution center, each transportation route for the delivery of goods to critical areas, as well as the allocation of critical areas to created distribution centers can affect the decision to choose distribution centers.

Aid distribution features after the crises are crucial for decision-making. One of the features that can be considered for vehicle routing problems in times of crisis which brings the problem closer to a real emergency situation is so that vehicles do not return to distribution center after serving the last part of the route. In other words, the path is open for vehicles, because in the real world vehicles that distribute aid to affected areas mostly belong to volunteer forces or they are rented from transportation companies and do not need to come back to distribution centers after the completion of the work. The open location-routing problem is a new issue in the literature and the first study on it was performed by [10]. Another problem despite high demand for assistance in affected areas after the crisis is that when demand is greater than the capacity of the server vehicle, the critical area can receive more than one service. In the literature, this is called split delivery. To allow split delivery leads to substantial savings in the costs which have been shown experimentally in the research by [1]. Thirdly, the dynamic and complex nature of disaster aid chain imposes a high degree of uncertainty in aid logistics planning decisions and highly affects the performance of the chain. As a result, taking into account the uncertainty for key parameters helps the quality of the decisions made at the strategic, tactical and operational level of aid chain, [7].

According to the considered issues in this study, it is tried to propose a multi-objective, multi-product model for three-level aid chain under uncertainty to transport aid supplies from suppliers (Red Crescent, airports, local suppliers) to created distribution centers, the allocation of critical areas and vehicles to distribution centers and designing directions from distribution centers to critical areas by considering split delivery. Moreover, the study considers all routes for vehicles as open routes. Objectives considered in the model include (1) minimizing the cost of the entire system, (2) minimizing the maximum travel time on the track, and (3) minimizing unmet demand. To get closer the model to reality, using fuzzy possibilistic programming the model expands in the non-deterministic state. In addition, in order to solve the proposed model, a fuzzy multi-objective programming will be used.

The rest of the article is as follows: Section 2 is a brief review of the literature which discusses the logistics of aid. Section 3 includes the statement of the problem and the proposed model. The suggested solution is provided in section 4. Computational results to validate the model are expressed in section 5. Finally, in section 6 conclusions and recommendations for future studies are offered.

2 Literature Review

In recent years, emergency logistical problems have attracted the attention of many researchers. Below the published studies on the topic are briefly studied. Fiedrich [9], investigated casualties after a disaster and calculated the casualties and related losses and tried to provide a model to minimize these losses. As mentioned, this study only examines the transport of the injured and there has been no discussion about relief supplies. They used Tabu search and simulated annealing to solve their models. Saydam [18], proposed a multiple-period location-covering method for dispatching ambulances. The model was designed to improve the performance of emergency medical services System, especially to respond to demands in events and disasters. However, routing vehicles has not been considered. Tzeng [23], provided a definitive multi-criteria model for the distribution of emergency goods to the affected areas considering the cost, response time, and satisfaction of customers, and they solved it by fuzzy multi-objective programming. Also Sheu [23], investigated a combined fuzzy clustering approach
for the optimization of multi-objective dynamic programming. The weighting method to convert the distribution cost minimization and maximization of demand coverage rate to one objective is applied.

Ozdamar and Yi [23], provided a multi-period, multi-product and randomized model. In their model, emergency repair facilities were considered. They used non-dominated sorting genetic algorithm (NSGA-II) to solve the routing problem for distribution after the earthquake. Their model is based on the assumption that disaster information does not depend on time and routing the vehicles. Wang [28], provided a multi-objective model for open locating-routing problem for distribution after the earthquake. The considered disaster network in their study included distribution centers and affected areas. In the presented model, emergency repair of roads and damaged communication channels were not considered. They used non-dominated sorting genetic algorithm (NSGA-II) to solve the model.

Bozorgi-Amiri [7], developed a multi-objective robust stochastic programming model for relief logistic in the conditions of uncertainty. In this research, not only demand was considered, but also supply and purchase and transportation costs were considered as uncertain parameters. Their model includes two stages. The first stage is concerned with determining the distribution center locations and required inventory of any relief items under storage, and the second stage is concerned with determining the level of goods transferred from relief distribution centers to affected areas. Their model is based on the assumption that disaster information does not depend on time and routing the vehicles. Eshghi and Najafi [8], In order logistical management of relief items and injured people, they proposed a multi-objective, multi-product, multi-period, and randomized model. Disaster network explained in their research included affected centers, hospitals, and transfer centers of relief items. Objectives of the model included a minimizing total number of non-serviced people, the total number of unmet demands, and the total number of transportation vehicles required. In their investigation, uncertainty for the sent items, the number of affected persons, and the capacity of suppliers and hospitals are considered. For this purpose, a robust approach was developed in the model to face with uncertainty, and a solution was proposed based on hierarchical objective functions.

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relief goods is programmed for suppliers of relief before the disaster, while routing of vehicles and allocation of relievers are programmed after the disaster. Martínez-Salazar [14], provided a single period, two objective model for transportation - location - routing for three-level supply chain and used two algorithms including multi-objective genetic algorithm using non-dominated sorting and scatter multi-objective search algorithm for solving the problem. Their problem objectives include minimizing the total cost and producing a balanced set of routes for vehicles.

Talarico [20], considered a routing problem for ambulances in the scenario of response to natural disasters. The ambulances are used for carrying medical personnel and patients. They considered two groups of patients: People who have less injury and can receive the relief, and people who are severely injured and should be taken to the hospital. Since ambulances indicate a source of scarce in critical conditions, efficient use of them is important. Two mathematical formulas have been provided to get the route programs that are the least relief delivery total time. Bozorgi Amiri and Khorsi [6] provided a multi-objective dynamic location-routing model for mid-planning and short-term regarding relief with uncertain conditions in demand, travel time, and cost parameters. Their model objectives include minimizing the maximum deficit among the affected areas in all periods, travel time, and total cost. The proposed model is solved by using the -constraint method. In their model, the intended route was considered close and split delivery of demand was not considered. Tofighi [6], introduced a two-stage possibilistic-stochastic approach based on the scenario to design a relief logistic network in Tehran. In the first stage, central warehouse and local distribution centers with a predetermined amount of relief supplies are determined. In the second stage, a relief distribution program for different disaster scenarios is presented. In addition, this study extended a meta-heuristic algorithm for obtaining a practical and convenient solution in the appropriate time for the Tehran case. Vahdani [26], a multi-objective, multi period mathematical model is proposed to locate distribution centers for distributing relief after earthquake by considering emergency roadway repair operations. Moreover, they utilized two multi-objective meta-heuristic algorithms, namely NSGAII and MOPSO for solving the proposed model. Zokaee [32], presented a robust scheduling model for three-level relief chain consisting of suppliers, relief distribution center and damaged areas in the uncertainty conditions. The goals of their model were to minimize total cost of relief operation costs, while it maximizes the satisfaction of victim people simultaneously by minimizing relief goods shortage. For model efficiency, they conducted a case study in Alborz area of Iran. The proposed model in this study is consistent with characteristics that are rarely considered in previous studies. In this study a nonlinear integer multi-objective open transportation location routing model is presented for three levels relief chain. The distribution of aid to the affected areas is along with split delivery of demand. In addition, an important point that must be considered in planning the logistical response to the crisis, is dynamic and uncertain nature of the information which is not considered in many studies. Such cases have led to their reduced efficiency of implementation. So, to evaluate uncertainty, this study is based on fuzzy possibilistic programming and fuzzy multi-objective programming provided by Torabi and Hassini [22], is used to solve the model.

3 Definition of the Problem and Mathematical Model

In the aftermath of natural disasters, emergency response to victims and providing services to the injured people is essential. In this study, a network of relief after a disaster is considered. For this purpose a nonlinear mixed integer programming model is presented for multi-commodity and multi-objective open transportation location routing problem in three level relief chain. This chain consists of suppliers (the gathering centers of relief goods) in fixed locations, some candidate distribution centers and finally a set of affected areas with different demands of each type of goods. Relief goods are transferred from suppliers to distribution centers in affected areas. Thus, the location of distribution centers in appropriate places in the network that can cover the affected areas appropriately is important to carry out a successful rescue operation. In the proposed problem, the process of providing relief is so that first subsets of the distribution centers are spec-
Table 1: Sets and indices

<table>
<thead>
<tr>
<th>Set</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H$</td>
<td>Set of suppliers ${1, \ldots, h}$</td>
</tr>
<tr>
<td>$N$</td>
<td>Set of disaster areas ${1, \ldots, n}$</td>
</tr>
<tr>
<td>$M$</td>
<td>Set of candidate DCs ${n + 1, \ldots, n + m}$</td>
</tr>
<tr>
<td>$V$</td>
<td>Set of vehicles ${1, \ldots, k}$</td>
</tr>
<tr>
<td>$L$</td>
<td>Set of relief ${1, \ldots, l}$</td>
</tr>
<tr>
<td>$E$</td>
<td>Set of available traffic links ${(i, j), i, j \in v, i \neq j}$</td>
</tr>
<tr>
<td>$i, j$</td>
<td>Indices to nodes $i, j \in v$</td>
</tr>
<tr>
<td>$l$</td>
<td>Indices to relief</td>
</tr>
<tr>
<td>$k$</td>
<td>Indices to vehicles</td>
</tr>
</tbody>
</table>

Table 2: Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_i$</td>
<td>Fixed cost of establishing the DC $i$, $\forall i \in M$</td>
</tr>
<tr>
<td>$e_{ij}$</td>
<td>Distance of link $(i, j)$, $\forall (i, j) \in E$</td>
</tr>
<tr>
<td>$\bar{S}_{hl}$</td>
<td>Transportation cost per unit of relief $l$ from supplier $h$ to distribution $i$</td>
</tr>
<tr>
<td>$\bar{D}_{il}$</td>
<td>Quantity of relief $l$ demanded by disaster area $i$</td>
</tr>
<tr>
<td>$s_{vl}$</td>
<td>Unit volume of relief $l$, $\forall l \in L$</td>
</tr>
<tr>
<td>$\bar{O}_{hl}$</td>
<td>Amount of relief $l$ available in supplier $h$</td>
</tr>
<tr>
<td>$\bar{Q}_{il}$</td>
<td>Maximum capacity of the distribution center $i$ from relief $l$</td>
</tr>
<tr>
<td>$c_k$</td>
<td>Transportation cost per kilometer of vehicle $k$</td>
</tr>
<tr>
<td>$v_k$</td>
<td>Normal speed of terrestrial vehicle $k$</td>
</tr>
<tr>
<td>$\bar{C}A_k$</td>
<td>Loading capacity of terrestrial vehicle $k$</td>
</tr>
</tbody>
</table>

Table 3: Decision variables

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y_i$</td>
<td>1, if candidate DC $i$ is opened, 0, else, $\forall i \in M$</td>
</tr>
<tr>
<td>$x_{ijk}$</td>
<td>1, if $i$ precedes $j$ in route of vehicle $k$, 0, else</td>
</tr>
<tr>
<td>$R_{ijk}$</td>
<td>1, if $i$ is on route of vehicle $k$, 0, else</td>
</tr>
<tr>
<td>$P_{lk}$</td>
<td>1, if the last demand point serviced by vehicle $k$ is node $i \in N$; 0, else</td>
</tr>
<tr>
<td>$W_{hil}$</td>
<td>Quantity of relief $l$ transported from supplier $h$ to distribution center $i$</td>
</tr>
<tr>
<td>$dev_{il}$</td>
<td>Amount of unsatisfied demand relief type $l$ at node $i$ at the end of the operation</td>
</tr>
<tr>
<td>$q_{ijk}$</td>
<td>Quantity of relief $l$ distributed by $k$ to demand point $i$</td>
</tr>
</tbody>
</table>

Table 4: Dimensions of the Problem

<table>
<thead>
<tr>
<th>Test problems</th>
<th>(h)</th>
<th>(M)</th>
<th>(N)</th>
<th>(K)</th>
<th>(L)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>11</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>4</td>
<td>10</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>2</td>
<td>8</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>3</td>
<td>6</td>
<td>3</td>
<td>2</td>
</tr>
</tbody>
</table>

ified for reopening. The suppliers transfer their aid in large-scale through different transportation networks to created distribution centers and at a later stage of distribution, vehicles and critical areas will be allocated to distribution centers and track of vehicles from distribution centers to critical areas are designed for rapid distribution of the emergency aid. Split delivery of demand is required as demand in the critical area is larger than the capacity of the vehicle, and each critical area can be served more than once and by different vehicles. Heterogeneous vehicles are considered with different speed and capacities. It should be noted that any vehicle is allowed to transport multiple types of assistance to each allocation, various types of aid are allowed at the same time
Table 5: Model Parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_i$</td>
<td>$\sim$uniform(10000,30000)</td>
</tr>
<tr>
<td>$e_{ij}$</td>
<td>$\sim$uniform(60,250)</td>
</tr>
<tr>
<td>$O_{hl}$</td>
<td>$\sim$uniform(14000,24000)</td>
</tr>
<tr>
<td>$Q_{il}$</td>
<td>$\sim$uniform(9000,12000)</td>
</tr>
<tr>
<td>$D_{jl}$</td>
<td>$\sim$uniform(800,2500)</td>
</tr>
<tr>
<td>$V_k$</td>
<td>$\sim$uniform(70,90)</td>
</tr>
<tr>
<td>$CA_k$</td>
<td>$\sim$uniform(23,39)</td>
</tr>
<tr>
<td>$w_l$</td>
<td>$\sim$uniform(0.0123, 0.028)</td>
</tr>
<tr>
<td>$c_k$</td>
<td>$\sim$uniform(3,5)</td>
</tr>
<tr>
<td>$s_{hil}$</td>
<td>$\sim$uniform(8,10)</td>
</tr>
</tbody>
</table>

Table 6: Results of sensitivity analysis for problems based on $\varphi = 0.4$ and $\alpha = 0.3$

<table>
<thead>
<tr>
<th>Test problem</th>
<th>Deterministic</th>
<th>Fuzzy possibilistic</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$(z_1, \mu_1)$</td>
<td>$(z_2, \mu_2)$</td>
</tr>
<tr>
<td>1</td>
<td>(346923, 0.94)</td>
<td>(12.70, 0.77)</td>
</tr>
<tr>
<td>2</td>
<td>(512639.7, 0.79)</td>
<td>(8.74, 0.86)</td>
</tr>
<tr>
<td>3</td>
<td>(287040, 0.82)</td>
<td>(6.82, 0.91)</td>
</tr>
<tr>
<td>4</td>
<td>(183124.7, 0.93)</td>
<td>(3.56, 0.85)</td>
</tr>
</tbody>
</table>

Table 7: Results of sensitivity analysis for problems based on $\varphi = 0.4$ and $\alpha = 0.3$

<table>
<thead>
<tr>
<th>Test problem</th>
<th>Fuzzy possibilistic</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$(z_1, \mu_1)$</td>
<td>$(z_2, \mu_2)$</td>
</tr>
<tr>
<td>1</td>
<td>(375467.5, 0.81)</td>
<td>(12.92, 0.75)</td>
</tr>
<tr>
<td>2</td>
<td>(426795.16, 0.90)</td>
<td>(10.34, 0.49)</td>
</tr>
<tr>
<td>3</td>
<td>(692864.83, 0.71)</td>
<td>(6.96, 0.85)</td>
</tr>
<tr>
<td>4</td>
<td>(198211.09, 0.76)</td>
<td>(3.96, 0.64)</td>
</tr>
</tbody>
</table>

in one vehicle load. In addition, after the completion of operation when the vehicles serve the last node of the route, they do not need to return to their origin. So, the route for the vehicles is considered open. Intended objectives in the problem are: the minimization of total cost including fixed cost of creating distribution centers, travel cost of the vehicle and costs of goods transport from suppliers to distribution centers; the second objective is the minimization of maximum travel
time on the route (Maximum travel time means the latest completion time of the service among all critical areas) and the third objective is the minimization of unmet demand.

\[ z_3 = \min \sum_{j \in N} \sum_{i \in L} \sum_{k \in K} (\tilde{D}_{il} - q_{ijk})R_{ijk} \] 
\[ \text{S.t.} \]
\[ y_i \geq x_{ijk}, \quad \forall i \in M, (i,j) \in E, k \in K : i \neq j \] 
\[ y_i \geq R_{ik}, \quad \forall i \in M, (i,j) \in E, k \in K \] 
\[ R_{ik} \geq x_{ijk}, \quad \forall i \in V, (i,j) \in E, k \in K : i \neq j \] 
\[ R_{ik} \geq P_{ik}, \quad \forall i \in V, k \in K \]
\[ \sum_{i \in V} P_{ik} = 1, \quad \forall k \in K \]
\[ \sum_{k \in K} x_{ijk} \leq 1, \quad \forall (i,j) \in E : i \neq j \]

### 3.1 Mathematical model

\[
\begin{align*}
\sum_i M_{ijk} = & \min \left( \sum_{i \in M} i y_i + \sum_{k \in K} \sum_{(i,j) \in E} c_{jk} d_{ijk} x_{ijk} + \sum_{i \in M} \sum_{j \in E} \tilde{S}_{ijl} W_{ijl} \right) \\
\text{Min } z_2 = & \max \left\{ \sum_{(i,j) \in E} \frac{e_{ij} x_{ijk}}{v_k}, \quad k \in K \right\}
\end{align*}
\]

#### Table 8: Sensitivity analysis of (α) based on φ = 0.4

<table>
<thead>
<tr>
<th>Test problem</th>
<th>(θ₁, θ₂, θ₃)</th>
<th>(z₁, μ₁)</th>
<th>(z₂, μ₂)</th>
<th>(z₃, μ₃)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(0.3, 0.3, 0.4)</td>
<td>(346923, 0.94)</td>
<td>(12.7, 0.77)</td>
<td>(3404.7, 0.78)</td>
</tr>
<tr>
<td>2</td>
<td>(0.3, 0.4, 0.3)</td>
<td>(399438, 0.86)</td>
<td>(11.8, 0.89)</td>
<td>(3779.2, 0.74)</td>
</tr>
<tr>
<td>3</td>
<td>(0.3, 0.3, 0.4)</td>
<td>(318152, 0.87)</td>
<td>(10.3, 0.78)</td>
<td>(3408, 0.72)</td>
</tr>
</tbody>
</table>

#### Table 9: Sensitivity analysis of (α) based on φ = 0.4

<table>
<thead>
<tr>
<th>Test problem</th>
<th>(θ₁, θ₂, θ₃)</th>
<th>(z₁, μ₁)</th>
<th>(z₂, μ₂)</th>
<th>(z₃, μ₃)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(0.3, 0.3, 0.4)</td>
<td>(426795, 0.90)</td>
<td>(13.2, 0.71)</td>
<td>(4290.26, 0.88)</td>
</tr>
<tr>
<td>2</td>
<td>(0.3, 0.4, 0.3)</td>
<td>(454297, 0.74)</td>
<td>(13.1, 0.72)</td>
<td>(4480.21, 0.66)</td>
</tr>
<tr>
<td>3</td>
<td>(0.3, 0.3, 0.4)</td>
<td>(36172, 0.85)</td>
<td>(10.7, 0.79)</td>
<td>(3941.61, 0.84)</td>
</tr>
<tr>
<td>4</td>
<td>(0.3, 0.4, 0.3)</td>
<td>(263669, 0.92)</td>
<td>(6.96, 0.70)</td>
<td>(3295.42, 0.92)</td>
</tr>
</tbody>
</table>

\[ z_3 = \min \sum_{j \in N} \sum_{i \in L} \sum_{k \in K} (\tilde{D}_{il} - q_{ijk})R_{ijk} \] 
\[ \text{S.t.} \]
\[ y_i \geq x_{ijk}, \quad \forall i \in M, (i,j) \in E, k \in K : i \neq j \]
\[ y_i \geq R_{ik}, \quad \forall i \in M, (i,j) \in E, k \in K \]
\[ R_{ik} \geq x_{ijk}, \quad \forall i \in V, (i,j) \in E, k \in K : i \neq j \]
\[ R_{ik} \geq P_{ik}, \quad \forall i \in V, k \in K \]
\[ \sum_{i \in V} P_{ik} = 1, \quad \forall k \in K \]
\[ \sum_{k \in K} x_{ijk} \leq 1, \quad \forall (i,j) \in E : i \neq j \]
Table 10: Results of sensitivity analysis on $\varphi$-value for problems based on the $\alpha = 0.3$ and $\theta = (0.3, 0.3, 0.4)$

<table>
<thead>
<tr>
<th>Test problem</th>
<th>Deterministic</th>
<th>Fuzzy possibilistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varphi$</td>
<td>$(z_1, \mu_1)$</td>
<td>$(z_2, \mu_2)$</td>
</tr>
<tr>
<td>0.1</td>
<td>(325031.5,0.95)</td>
<td>(9.42, 0.82)</td>
</tr>
<tr>
<td>0.2-0.4</td>
<td>(346923.2 ,0.94)</td>
<td>(12.70,0.77)</td>
</tr>
<tr>
<td>1</td>
<td>(377635.1,0.87)</td>
<td>(13.61,0.69)</td>
</tr>
<tr>
<td>0.5-0.7</td>
<td>(418583.5,0.75)</td>
<td>(14.27,0.62)</td>
</tr>
<tr>
<td>0.8-0.9</td>
<td>(499309.2,0.85)</td>
<td>(6.07,0.94)</td>
</tr>
<tr>
<td>0.1-0.3</td>
<td>(512639.7,0.79)</td>
<td>(2142.38,0.85)</td>
</tr>
<tr>
<td>0.4-0.6</td>
<td>(547687.5,0.72)</td>
<td>(9.80,0.75)</td>
</tr>
<tr>
<td>2</td>
<td>(590369.2,0.64)</td>
<td>(11.26,0.62)</td>
</tr>
</tbody>
</table>

Table 11: Results of sensitivity analysis on $\varphi$-value for problems based on the $\alpha = 0.3$ and $\theta = (0.3, 0.3, 0.4)$

<table>
<thead>
<tr>
<th>Test problem</th>
<th>Fuzzy possibilistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varphi$</td>
<td>$(z_1, \mu_1)$</td>
</tr>
<tr>
<td>1</td>
<td>(420687.5,0.93)</td>
</tr>
<tr>
<td>0.2-0.4</td>
<td>(426795.2,0.90)</td>
</tr>
<tr>
<td>0.5-0.7</td>
<td>(459456.7,0.82)</td>
</tr>
<tr>
<td>0.8-0.9</td>
<td>(49614.7,0.71)</td>
</tr>
<tr>
<td>0.4-0.6</td>
<td>(589369.3,0.63)</td>
</tr>
<tr>
<td>0.1-0.3</td>
<td>(621713.4,0.58)</td>
</tr>
<tr>
<td>0.4-0.6</td>
<td>(653091.8,0.51)</td>
</tr>
<tr>
<td>0.9</td>
<td>(689769.9,0.46)</td>
</tr>
</tbody>
</table>

\[
\begin{align*}
\sum_{i,j,k}^K x_{ijk} & \leq 1, \quad \forall i \in N, k \in K : i \neq j \quad (3.10) \\
\sum_{j \in M} \sum_{i \in J} x_{ijk} & \leq 1, \quad \forall k \in K \quad (3.11) \\
\sum_{i \in M} W_{hil} \leq \hat{Q}_{hl}, \quad \forall h \in H, l \in L \quad (3.12) \\
\sum_{h \in H} W_{hil} \leq \hat{Q}_{dhl}, \quad \forall i \in M, l \in L \quad (3.13) \\
\sum_{l \in M} \sum_{k \in K} q_{ijk} R_{lk} \leq \hat{Q}_{dl}, \quad \forall i \in M, l \in L \quad (3.14) \\
dev_{jl} = \hat{D}_{jl} - (\sum_{k \in K} q_{ijk}) \geq O, \quad \forall j \in N, l \in L \quad (3.15) \\
\sum_{l \in H} \sum_{i \in N} W_{hil} \geq \sum_{j \in J} \hat{D}_{jl}, \quad \forall l \in L \quad (3.16) \\
\sum_{j \in J} \sum_{k \in K} \sum_{l \in L} s_{ijk} \leq C_{lk}, \quad \forall k \in K \quad (3.17) \\
\sum_{i \in V} \sum_{k \in K} x_{ijk} \geq 1, \quad \forall j \in N \quad (3.18) \\
\left(\sum_{j \in J, i \in I} x_{ijk} - \sum_{j \in J, i \in I} x_{ij} \right) \cdot P_{jk} &= P_{jk}, \\
\forall i \in N, k \in K \quad (3.19) \\
\left(\sum_{j \in J, i \in I} x_{ijk} - \sum_{j \in J, i \in I} x_{ij} \right) \cdot R_{ik} &= -R_{ik}, \\
\forall i \in M, k \in K \quad (3.20) \\
\left(\sum_{j \in J, i \in I} x_{ijk} - \sum_{j \in J, i \in I} x_{ij} \right) \cdot P_{ik} &= 0, \\
\forall i \in M, k \in K \quad (3.21) \\
\left(\sum_{j \in J, i \in I} x_{ijk} - \sum_{j \in J, i \in I} x_{ij} \right) \cdot R_{ik} &= 0, \\
\forall i \in V, k \in K \quad (3.22) \\
\left(\sum_{j \in J, i \in I} x_{ijk} - \sum_{j \in J, i \in I} x_{ij} \right) \cdot R_{ik} &= 0, \\
\forall i \in V, k \in K \quad (3.23) \\
\end{align*}
\]

\[
\begin{align*}
\sum_{i \in M} x_{ijk} &= O, \quad \forall j \in M, k \in K, t \in T \quad (3.24) \\
u_{st} - u_{kj} + n \times x_{ijk} & \leq n - 1, \quad \forall i, j \in N \quad (3.25) \\
k \in K \quad (3.26) \\
g_{ij} \geq 0, \quad \forall (i, j) \in E, l \in L, k \in K \quad (3.27) \\
W_{hil} \geq 0, \quad \forall i \in M, l \in L, h \in H \quad (3.28) \\
y_{j} \in (0,1), \quad \forall i \in M \quad (3.29) \\
x_{ijk} \in (0,1), \quad \forall (i, j) \in E, k \in K \quad (3.30) \\
R_{ik} \in (0,1), \quad \forall i \in V, k \in K \quad (3.31)
\end{align*}
\]
\[ P_{ik} \in (0,1), \ \forall i \in N, k \in K \]  
(3.32)

\[ u_{ik} \in (0,1), \ \forall i \in N, k \in K \]  
(3.33)

Equation (3.1) is the first objective function which minimizes the distribution costs including fixed costs of creating distribution centers, travel expenses of the vehicles and the cost of transporting goods from suppliers to distribution centers. Equation (3.2) as the second objective function minimizes the maximum travel time of the vehicles. The objective function (3.3) minimizes the total unmet demand. Constraints (3.4) and (3.5) specify that just established distribution centers can obtain service. Constraint (3.6) ensures that every vehicle can be travel through connection \((i,j)\), if and only if node \(i\) to be on the route of each vehicle. Constraint (3.7) specifies that the nodes in the end of the route of each vehicle must be serviced by the same vehicle. Equation (3.8) ensures that every vehicle must ultimately remain in a disaster area or distribution center. Constraint (3.9) shows that only one vehicle is selected for each route. Constraint (3.10) ensures that any vehicle serves once at most for any critical area. Constraint (3.11) ensures that any vehicle is sent from one distribution center at most. Constraint (3.12) ensures that the amount of aid transferred by any supplier of any goods to all distribution centers does not exceed the maximum amount. (3.13) and (3.14) are capacity constraints of distribution centers. Constraint (3.15) shows that the amount of relief distributed to each node does not exceed the amount demanded by that node. Constraint (3.16) ensures that there will be no shortage of the goods. Constraint (3.17) ensures that amount of all the relief distributed to disaster areas by a vehicle do not exceed their capacity. Constraint (3.18) ensures that every disaster area can be visited at least once. The assumption of split delivery in this constraint has been well illustrated. Constraints (3.19) - (3.24) are the limits of maintaining the flow which also ensures the openness assumption of the routes (Ensure that each vehicle at any point is dispatched from that point, and at the last node of the route does not return to the distribution center). Constraint (3.25) ensures that distribution centers are not related with each other. It means that goods are not exchanged between distribution centers. Constraint (3.26) is constraints of elimination sub-tours. Constraints (3.27) - (3.33) related to nonnegative values and integer and numbers of zero and one for decision variables. The model was presented given the certainty of parameters section 3.5. In the real world there is uncertainty in many of these parameters. To bring the model closer to real conditions in the future, the model has also been expanded in non-deterministic conditions. To develop the model a robust optimization approach is used.

### 3.2 Uncertainty approach

With respect to the above-mentioned consideration, a mixed integer programming model with fuzzy parameters is proposed. Next, the proposed model, by virtue of a new technique based on the possibilistic method \([11, 16, 22]\), is converted to its commensurate deterministic version.

The commensurate adjuvant crisp model:

Suppose that \(\tilde{c}\) is a triangular fuzzy number (TFN), the Eq. (3.34) as the membership function of \(\tilde{c}\):

\[
\mu_{\tilde{c}}(X) = \begin{cases} 
\frac{x - c^p}{c^m - c^p} & \text{if } c^p \leq x \leq c^m \\
1 & \text{if } x = c^m \\
\frac{c^m - x}{c^m - c^p} & \text{if } c^m \leq x \leq c^p \\
0 & \text{if } x \leq c^p \text{ or } x \geq c^p \nend{cases}
\]

(3.34)

The following FMP model is considered, in which all parameters are defined as TFNs:

\[
\min \quad z = \tilde{c}^T x \\
\text{s.t.} \\
\tilde{a}_i x \geq \tilde{b}_i, \quad i = 1, \ldots, l \\
\tilde{a}_i x = \tilde{b}_i, \quad i = l + 1, \ldots, m \\
x \geq 0
\]

(3.35)

The commensurate crisp \(\alpha\)-parametric model of the model (3.35) is able to be written as bellows (Jimenez[11]):

\[
\min \quad z = EV(\tilde{c})x \\
\text{s.t.} \\
[(1 - a)E_{2i} + aE_{1i}] x \geq aE_{2i} + (1 - a)E_{1i}, \\
i = 1, 2, \ldots, l \\
\left[(1 - a)E_{2i} + aE_{1i}\right] x \geq \frac{a}{2}E_{2i} + (1 - \frac{a}{2})E_{1i}, \\
i = l + 1, \ldots, m \\
\left[\frac{a}{2}E_{2i} + (1 - \frac{a}{2})E_{1i}\right] x \leq (1 - \frac{a}{2})E_{2i} + \frac{a}{2}E_{1i}, \\
i = l + 1, \ldots, m \\
x \geq 0
\]

(3.36)
Where \( EV(\hat{c}) = \frac{c^p + 2c^m + c^o}{4}, E_1^a = \frac{a}{2}(a^p + a^m), E_2^a = \frac{a}{2}(a^m + a^o), E_1^b = \frac{a}{2}(b^p + b^m) \) and \( E_2^b = \frac{a}{2}(b^m + b^o) \). Based on above explanations, the commensurate adjuvant crisp model of the proposed model is as bellows:

\[
\begin{align*}
    z_1 &= \min \sum_{i \in M} \left( \frac{p_i^p + 2f_i^m + f_i^o}{4} \right) y_i + \\
    &+ \sum_{k \in K} \sum_{(i,j) \in E} \left( \frac{e_k^p + 2e_k^m + e_k^o}{4} \right) d_ij x_{ijk} + \sum_{h \in H} \sum_{i \in M} \left( \frac{s_{hkl}^m + s_{hkl}^o}{4} \right) w_{hl} \\
    &+ \sum_{i \in M} \sum_{l \in L} \left( \frac{s_{hkl}^m + s_{hkl}^o}{4} \right) w_{hl} \\

    \text{Min } z_2 &= \max \left\{ \sum_{(i,j) \in E} \frac{e_{ij} x_{ijk}}{v_k}, k \in K \right\} \\
    z_3 &= \min \sum_{j \in V} \sum_{i,j \in E} \left( \frac{D_{ij}^p + 2D_{ij}^m + D_{ij}^o}{4} \right) \eta_{ijk} R_{ijk}
\end{align*}
\]

(3.37)

\[
\begin{align*}
    S.t.: y_i &\geq x_{ijk}, \ \forall i \in M, (i, j) \in E, k \in K \neq j \\
    &\geq R_{ik}, \ \forall i \in M, (i, j) \in E, k \in K
\end{align*}
\]

(3.38)

(3.39)

(3.40)

(3.41)

(3.42)

(3.43)

(3.44)

(3.45)

(3.46)

(3.47)

(3.48)

\[
\begin{align*}
    \sum_{i \in M} w_{hl} &\leq \left[ a \left( \frac{Q_{hl}^p + Q_{hl}^m}{2} \right) + (1 - a) \left( \frac{Q_{hl}^o + Q_{hl}^m}{2} \right) \right] y_i, \\
    &\forall i \in M, l \in L
\end{align*}
\]

(3.49)

\[
\begin{align*}
    \sum_{i \in M} \sum_{k \in K} \eta_{ijk} R_{ik} &\leq \left[ a \left( \frac{Q_{hl}^p + Q_{hl}^m}{2} \right) + (1 - a) \left( \frac{Q_{hl}^o + Q_{hl}^m}{2} \right) \right], \\
    &\forall i \in M, l \in L
\end{align*}
\]

(3.50)

\[
\begin{align*}
    \sum_{k \in K} \eta_{ijk} R_{ik} &\geq \left[ (1 - a) \left( \frac{D_{ij}^p + D_{ij}^m}{2} \right) + a \left( \frac{D_{ij}^o + D_{ij}^m}{2} \right) \right] \\
    &\forall j \in N, l \in L
\end{align*}
\]

(3.51)

\[
\begin{align*}
    \sum_{k \in K} \sum_{i \in M} w_{hl} &\geq \left[ (1 - a) \left( \frac{D_{ij}^p + D_{ij}^m}{2} \right) + a \left( \frac{D_{ij}^o + D_{ij}^m}{2} \right) \right], \\
    &\forall i \in L
\end{align*}
\]

(3.52)

(3.53)

\[
\begin{align*}
    \sum_{j \in N} \left[ (1 - a) \left( \frac{D_{ij}^p + D_{ij}^m}{2} \right) + a \left( \frac{D_{ij}^o + D_{ij}^m}{2} \right) \right], \\
    &\forall k \in K
\end{align*}
\]

(3.54)

\[
\begin{align*}
    \sum_{i \in M} \sum_{j \in N} x_{ijk} &\geq \left[ (1 - a) \left( \frac{C_{ijk}^p + C_{ijk}^m}{2} \right) + a \left( \frac{C_{ijk}^o + C_{ijk}^m}{2} \right) \right], \\
    &\forall j \in N, k \in K
\end{align*}
\]

(3.55)

\[
\begin{align*}
    &\forall j \in N, k \in K
\end{align*}
\]

(3.56)
\[
(\sum_{j/(i,j)\in E} x_{ijk} - \sum_{j/(i,j)\in E} x_{ijk}).R_{ik} =
- R_k, \quad \forall i \in M, k \in K
\]
(3.57)
\[
(\sum_{j/(i,j)\in E} x_{ijk} - \sum_{j/(i,j)\in E} x_{ijk}).P_{ik} = 0,
\forall i \in M, k \in K
\]
(3.58)
\[
(\sum_{j/(i,j)\in E} x_{ijk} - \sum_{j/(i,j)\in E} x_{ijk}).R_{ik} = 0,
\forall i \in M, k \in K
\]
(3.59)
\[
(\sum_{j/(i,j)\in E} x_{ijk} - \sum_{j/(i,j)\in E} x_{ijk}).(1 - P_{ik}) =
P_{ik}(1 - P_{ik}), \quad \forall i \in N, k \in K
\]
(3.60)
\[
(\sum_{j/(i,j)\in E} x_{ijk} - \sum_{j/(i,j)\in E} x_{ijk}).(1 - R_{ik}) =
R_{ik}(1 - R_{ik}), \quad \forall i \in N, k \in K
\]
(3.61)
\[
\sum_{i \in M} x_{ijk} = O, \quad \forall j \in M, k \in K, t \in T
\]
(3.62)
\[
u_{ik} - u_{jk} + n \times x_{ijk} \leq n - 1,
\forall i, j \in N, k \in K i \neq j
\]
(3.63)
\[
q_{ijk} \geq 0, \quad \forall (i,j) \in E, l \in L, k \in K
\]
(3.64)
\[
w_{hil} \geq 0, \quad \forall i \in M, l \in L, h \in H
\]
(3.65)
\[
y_i \in (0,1), \quad \forall i \in M
\]
(3.66)
\[
x_{ijk} \in (0,1), \quad \forall (i,j) \in E, k \in K
\]
(3.67)
\[
R_{ik} \in (0,1), \quad \forall i \in V, k \in K
\]
(3.68)
\[
P_{ik} \in (0,1), \quad \forall i \in N, k \in K
\]
(3.69)
\[
u_{ik} \in (0,1), \quad \forall i \in N, k \in K
\]
(3.70)

4 The Proposed Solution Approach

In this study, to solve the proposed model, a hybrid solution approach, combining the techniques presented in the previous section and the fuzzy solution approach derived from the method of Torabi and Hosseini [22]:

Steps in the proposed hybrid solutions approach are summarized as follows:

**Step 1**: determining the parameters and variables of uncertainty and considering the distribution functions needed to use in the model.

**Step 2**: formulating the proposed model with the parameters defined in the previous step.

**Step 3**: converting the constraints of mixed-integer programming model to constraints of the certain counterpart by applying the approach outlined in the previous section.

**Step 4**: determining the positive ideal and negative ideal solution for every objective function in which it is possible level. To calculate the positive and negative ideal solution i.e. \(W_{1}^{NIS}, x_{1}^{NIS}\) and \(W_{2}^{NIS}, x_{2}^{NIS}\) of each certainty model is separately solved for each of the objective functions and the positive ideal solution is obtained, and then the negative ideal solution is estimated as follows:

\[
W_{1}^{NIS} = W_{1}(x_{2}^{PIS}), W_{2}^{NIS} = W_{2}(x_{1}^{PIS})
\]

**Step 5**: determining a linear membership function for each objective function as follows:

\[
\mu_{1}(x) =
\begin{cases}
1 & \text{if } W_{1} < W_{1}^{PIS} \\
\frac{W_{1}^{NIS} - W_{1}}{W_{1}^{NIS} - W_{1}^{PIS}} & \text{if } W_{1}^{PIS} \leq W_{1} \leq W_{1}^{NIS} \\
0 & \text{if } W_{1} > W_{1}^{NIS}
\end{cases}
\]
(4.71)

\[
\mu_{2}(x) =
\begin{cases}
1 & \text{if } W_{2} < W_{2}^{NIS} \\
\frac{W_{2} - W_{1}^{NIS}}{W_{2}^{PIS} - W_{2}^{NIS}} & \text{if } W_{2}^{NIS} \leq W_{2} \leq W_{2}^{PIS} \\
0 & \text{if } W_{2} > W_{2}^{NIS}
\end{cases}
\]
(4.72)

In fact, \(\mu_{h}(x)\) represents the satisfaction degree of the \(h^{th}\) objective function. It should be noted that \(\mu_{1}(x)\) has been used for minimization objec-
tive functions and $\mu_2(x)$ for maximization objective function.

**Step 6:** converting the certainty mixed integer programming model to a certainty single-objective mixed integer programming model using the integrated function which is calculated as follows:

$$\lambda(x) = \psi \lambda_0 + (1 - \psi) \sum_k \phi_k \mu_k(x)$$

$$s.t$$

$$\lambda_0 \mu(x), \quad h = 1, 2$$

$$x \in F(x), \quad \lambda_0 and \lambda \in [0, 1]$$

**Step 8:** determining the parameters $\theta_h, \rho$ and $\psi$ and solving single-objective models created in the previous step. If the answer is satisfactory for decision makers, it stops; otherwise, in order to achieve new answers, change the values of parameters $\psi$ and $\rho$ and if needed, change the value of $\theta_h$.

5 Computational Results

The computational results of the model are presented in this section. To demonstrate the validity and usefulness of the model and the solution approach, several numerical tests are run and the results are presented in this section. To this end, four different problems with different aspects were considered. The information related to the dimensions is shown in Table 1 and the information about the parameters of the model is shown in Table 2. It should be noted that, to generate the triangular fuzzy parameters according to Lai and Hwang [12], three prominent points are obtained for each imprecise parameter. The most likely ($C^m$) value of each parameter is first provided randomly by utilizing the uniform distributions specified in Table 2. Thus, without loss of generality, two random numbers ($r_1, r_2$) are generated between 0.2 and 0.8 by applying uniform distribution. The most pessimistic ($C^p$) and optimistic ($C^o$) values of a fuzzy number ($\tilde{C}$) are calculated as follows (Vahdani (2014) [25]).

$$C^o = (1 + r_1)C^m$$

$$C^p = (1 - r_2)C^m$$

All problems considered in both deterministic and non-deterministic conditions were solved by software GAMS version 23.6 and Baron Solver. In order to illustrate the precision and accuracy of the model and the solution method, the answer to the problem by GAMS software is depicted in Figure 11 for the first problem in two deterministic and non-deterministic conditions. As you can see in solving the problem in deterministic condition distribution centers 1 and 2 have been opened and the suppliers of collected aid are sent to them and vehicles tailored to the track status and demands of critical areas are assigned to distribution centers. You can see the route is open to all vehicles and they do not return to the distribution center. Also due to high demand critical areas 7 and 8 that are bigger than the remaining capacity of the vehicle 3 in the first stage of service, some of the remaining demand in area 8 is met by vehicle 1 in the next stage, and area 7 will meet again by vehicle 2. As can be seen in non-deterministic condition, centers 1 and 3 have been opened for providing aid and critical area 3 due to high demand had been met in three stages by various vehicles.

The computational results are summarized in Tables 3 and 4 in both deterministic and non-deterministic conditions based on three levels of (0.1, 0.3, 0.5) and various degrees of importance for the objective functions. The value for levels of uncertainty for all model parameters at each stage of the implementation is considered constant and this value is $\rho = 0$ for certain models. In addition to the impact of the penalty coefficient ($\varphi$) on objective functions in both deterministic and non-deterministic conditions, sensitivity analysis was conducted that because of the required time for this analysis, it was conducted only on two problems and the results are shown in Table 5.

According to the computational results presented in Tables 3 and 4, it can be seen that all uncertain problems have answers worse than certain problems. In addition, it can be concluded from the results in Table 4 that TH method acquires unique solutions for every different degree of importance for the objective functions. In general, it can be said that TH is a good and eligible method for planning multi-objective problems, because it can achieve effective and efficient
solutions.

6 Conclusions and Recommendations for Future

In this study, for the first time multi-objective open transportation location routing problem by considering split delivery of the demand for assistance distribution after the crisis in the three-level emergency chain as nonlinear integer programming was modeling. Few studies have focused on the open location-routing problem in emergency logistics, but in reality, vehicles that are responsible for distributing aid to affected areas often work voluntarily by people or are rented from companies, so this means they do not return to distribution centers after the completion of operations. Therefore, in this study the routes for all vehicles were considered open. Furthermore, when providing relief to the affected areas, the demand may not be met in one travel. As a result, in this study, split delivery of demand was raised aiming to get closer to the real-world situations and in order to meet maximum demand and savings in costs. The model objectives include minimizing logistics costs, minimizing the maximum travel time of the vehicles and minimizing unmet demand. Since in time of crisis information is not definite, in order to deal with uncertainty in the model a possibilistic programming approach was used. Furthermore, as the model is multi-objective, fuzzy multi-objective programming approach was used to solve the model. According to the computational results, it is believed that the model and the solution can offer an effective and credible methodology for the management of relief distribution in an uncertain environment. Items that can be considered for future research:

- Integrated emergency chain network design taking into account tactical purposes such as inventory management during the response.
- In the incidence of the crisis many roads and communication routes in affected areas are destructed and blocked. To expedite relief, the repair of damaged roads can be taken into account.
- In addition to ground transportation network to provide relief to remote areas, air transport network can be considered as well.
- Development of meta-heuristic algorithms can be considered to solve the above model in large scale.

References


Elnaz Haghi was born in 1984 and gotten PhD degree in applied mathematics in the field of numerical analysis from Science and Research branch of Islamic Azad University, Tehran in 2013. She has published several papers in national and international journals. Her research topics and interests include linear systems with uncertainty. She is member of academic board of Islamic Azad University Robat Karim branch since 2014.