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Research Article

Distribution of Ratios of Generalized Order Statistics From Pareto Distribution and Inference

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Abstract

The aim of this paper is to study distribution of ratios of generalized order statistics from pareto distribution. parameter estimation of Pareto distribution based on generalized order statistics and ratios of them have been obtained. Inferences using method of moments and unbiased estimator have been obtained to develop point estimations. Consistency of unbiased estimator has been illustrated. To compare the performances of the employed methods, numerical results have been computed. Illustrative example using real data is also given.

Keywords : Generalized Order Statisitcs; Pareto Distribution; Parameter Estimation.

1 Introduction

KAmps [5, 6] discussed generalized order statistics (GOS) as a unified form of different models of ordered random variables(r.v.'s). Let F be a comulative distribution function (CDF) with probability density function (PDF) of f, with constants and parameters, $\widetilde{m} = (m_1, m_2, \ldots, m_{n-1})$, $k \geq 1$, $n \in \mathbb{N}$, $n \geq 2$ and γ_r is given for all $1 \leq r \leq n-1$, $m_1, m_2, \ldots, m_{n-1} \in \mathbb{R}$, where $\gamma_r = k + n - r + M_r \geq 1$, and $M_r = \sum_{j=r}^{n-1} m_j$. Suppose X=$(1, \ldots, n)$ denote n GOS, then their joint PDF, $f^{X_{(1,n,\widetilde{m},k)},\ldots,X_{(n,n,\widetilde{m},k)}}(x_1, \ldots, x_n)$, can be written as:

$$f^{X_{(1,n,\widetilde{m},k)},\ldots,X_{(n,n,\widetilde{m},k)}}(x_1, \ldots, x_n) = k(\prod_{j=1}^{n-1} \gamma_j)[\prod_{i=1}^{n-1} \overline{F}^{m_i}(x_i) f(x_i)] \times \overline{F}^{k-1}(x_n) f(x_n) \quad (1.1)$$

on the cone of

$$F^{-1}(0) < x_1 \leq \ldots \leq x_n < F^{-1}(1),$$

where $c_{r-1} = \prod_{i=1}^{r} \gamma_i$, $r = 1, \ldots, n$, and $\overline{F} = 1 - F$. is the survival function.
Marginal PDF of r is

$$f^r(x_r) = \frac{c_{r-1}}{(m+1)^r \Gamma(r)} \times \overline{F}^{\gamma_r - 1}(x_r)[1 - \overline{F}^{m+1}(x_r)]^{r-1} f(x_r) \quad (1.2)$$

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also the joint PDF of $(X_r\ X_s)$, where $r<s$, is

$$f^{X_{(r,n,m,k)},X_{(s,n,m,k)}}(x_r,x_s)=\frac{c_{s-1}}{(m+1)^{s-2}\Gamma(r)\Gamma(s-r)}\overline{F}^{m}(x_r)\times[1-\overline{F}^{m+1}(x_r)]^{r-1}[\overline{F}^{m+1}(x_r)-\overline{F}^{m+1}(x_s)]\times\overline{F}^{\gamma_s-1}(x_s)f(x_r)f(x_s). \quad (1.3)$$

[9] had studied a model of personal income exceeded given level and he showed that it can be approximated by Pareto law. The PDF and CDF of Pareto law or exactly Pareto distribution (PD) are as follows:

$$f(x)=\alpha\beta^{\alpha}x^{-(\alpha+1)}, \quad (1.4)$$

and

$$F(x)=1-\overline{F}(x)=1-(\frac{\beta}{x})^{\alpha}, \quad x>\beta. \quad (1.5)$$

For detailed information one can refer to [4]. Several methods have been proposed in the literature for estimating the PD parameters. [11] introduced uniformly minimum variance unbiased estimatior for unknown parameters of Pareto PD. [10] studied consistency of some estimators, and he suggested different popular techniques for inference. [4] discussed best linear unbiased estimates, [3] estimated PD unknown parameters using generalized median estimator. Estimation of parameters from PD through order statistics and some other ordered random variables illustrated by [12] and [13].
Distribution of sum, product or ratios of random variables is an important issues of reliability and distribution theory which considered by many authors. [2] studied distribution of ratios of generalized life distribution, [7] derived distribution of ratio of two normal random variables. Also, Kotz type distributions was studied by [8] about ratios. The present paper has focused on point estimations based on GOS from PD. Ratios of GOS has been studied and related distributions has obtained. Moment estimator based on ratios and Pareto random variables and Unbiased estimator have been employed in the present paper. To investigate consistency of the unbiased estimator, single and product moments for GOS of PD have been calculated. Numerical results for comparison of the estimators have been computed. Illustrative example based on real data from iranian rural household income is also given. The rest of paper is organized as follows: Section 2 includes ratio distribution of GOS from PD. inferences about parameters of PD have been discussed in Section 3. Sections 4 and 5 include numerical results and conclusions, respectively.

2 Distribution of ratios of GOS from PD

Theorem 2.1 *Let r and s denote r^{th} and s^{th} GOS having Pareto as underlying distribution. Then $\left(\frac{r}{s}\right)^{(\alpha(m+1))}$ distributed as beta with parameters $\frac{\gamma_s}{m+1}$ and $s-r$, where $r<s$.*

Proof. For simplicity, define $x_r=r$ and $x-s=s$. Considering $R^*=\frac{x_r}{x_s}$ and $Q=x_s$, it has been obtained, $X_r=QR^*$ and $X_s=Q$, so Jacobian has been computed

$$|J|=\begin{vmatrix}1 & 0\\ R^* & Q\end{vmatrix}=Q.$$

Using 1.3 joint distribution of $Q=q$ and $R^*=R$ can be written as

$$f_{Q,R^*}(q,R)=qf_{X_r,X_s}(qR,q)=\frac{qC_{s-1}}{(m+1)^{s-2}\Gamma(r)\Gamma(s-r)}\overline{F}^{m}(qR)\times\left[1-\overline{F}^{m+1}(qR)\right]^{r-1}\times\left[\overline{F}^{m+1}(qR)-\overline{F}^{m+1}(q)\right]^{s-r-1}\times\overline{F}^{\gamma_s-1}(q)f(qR)f(q). \quad (2.6)$$

Using 1.4 and 1.5, above relation can be rewritten as

$$f_{Q,R^*}(q,R)=(-1)^{s-r-1}\frac{\alpha^2\beta^{2\alpha}C_{s-1}}{(m+1)^{s-2}\Gamma(r)\Gamma(s-r)}\times q^{-2\alpha-1}\left(\frac{\beta}{q}\right)^{\alpha(\gamma_r-2)}R^{-\alpha(m+1)-1}\times\left[1-R^{-\alpha m+1}\right]^{s-r-1}\times\left[1-\left(\frac{\beta}{q}\right)^{\alpha(m+1)}R^{-\alpha(m+1)}\right]^{r-1}. \quad (2.7)$$

In order to compute the distribution of R^*, integration from 2.7 with respect to q is needed. Integral limits can be calculated, since $X_r<X_s$

we have $R < 1$ so it can be obtaied $\frac{b}{R} > b$, furthermore, it is known that $b < Q$, so it is been concluded $b < \frac{b}{R} < Q$, so taking integral as follows,

$$f_{R^*}(R) = g(R) \int_{\frac{\beta}{R}}^{\infty} q^{-2\alpha-1} \left(\frac{\beta}{q}\right)^{\alpha(\gamma_r-2)} \times \left[1 - \left(\frac{\beta}{q}\right)^{\alpha(m+1)} R^{-\alpha(m+1)}\right]^{r-1} q, \quad (2.8)$$

where,

$$g(R) = (-1)^{s-r-1} \frac{\alpha^2 \beta^{2\alpha} C_{s-1}}{(m+1)^{s-2}\Gamma(r)\Gamma(s-r)} \times R^{-\alpha(m+1)-1} \left[1 - R^{-\alpha m+1}\right]^{s-r-1}.$$

To get the solution of 2.8, following change of variable has been considered

$$z = 1 - \left(\frac{\beta}{q}\right)^{\alpha(m+1)} R^{-\alpha(m+1)},$$

and it can be written

$$f_{R^*}(R) = g(R)h(R) \int_0^1 z^{r-1}(1-z)^{\frac{\gamma_r}{m+1}-1} z, \quad (2.9)$$

where $h(R) = \frac{R^{\alpha\gamma_r}}{\alpha\beta^{2\alpha}(m+1)}$, so we have

$$f_{R^*}(R) = g(R)h(R) Bet\left(r, \frac{\gamma_r}{m+1}\right),$$

where, $Bet(a, b) = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$ is beta function. According to the fact, relation 1.2 is a density function, and integration based on it over the support of the random variable X equals to 1, therefore, $Bet\left(r, \frac{\gamma_r}{m+1}\right) \frac{C_{r-1}}{\Gamma(r)(m+1)^r} = 1$, so we have

$$\frac{C_{r-1}}{\Gamma(r)(m+1)^r} = Bet\left(r, \frac{\gamma_r}{m+1}\right). \quad (2.10)$$

Using recent equlity and relations $\gamma_r - \gamma_{r+1} = m+1$ and $\alpha\gamma_{r+1} - 1 - \alpha(m+1)(s-r-1) = \alpha\gamma_s - 1$, DF of R^* could be written as

$$f_{R^*}(R) = \frac{\alpha C_{s-1} Bet\left(r, \frac{\gamma_r}{m+1}\right)}{(m+1)^{s-1}\Gamma(r)\Gamma(s-r)} \times R^{\alpha\gamma_s-1} \left[1 - R^{\alpha(m+1)}\right]^{s-r-1}. \quad (2.11)$$

By change of variable $W = R^{\alpha(m+1)}$, and using equality

$$\frac{\Gamma\left(s - r + \frac{\gamma_s}{m+1}\right)}{\Gamma\left(\frac{\gamma_s}{m+1}\right)} = \frac{\prod_j = r + 1^s \gamma_j}{(m+1)^{s-r}},$$

we get to

$$\frac{C_{s-1} Bet\left(r, \frac{\gamma_r}{m+1}\right)}{(m+1)^s \Gamma(r)\Gamma(s-r)} = \frac{1}{Bet\left(\frac{\gamma_s}{m+1}, s-r\right)},$$

which completes the proof.

Theorem 2.2 *Let r denotes r^{th} GOS based on Pareto distribution with parameters α and β. Then $\gamma_r \frac{r}{r-1}$ has Pareto distribution with parameters $\alpha\gamma_r$ and γ_r.*

Proof. Using 1.3, 1.4 and 1.5 and transformations $U = \gamma_r \frac{r}{r-1}$ and $z = r - 1$, it can be written

$$f^{r,s}(u, z) = \frac{z C_{r-1}}{\gamma_r (m+1)^{r-2}\Gamma(r-1)} \times \left(\frac{\beta}{z}\right)^{\alpha m} \left[1 - \left(\frac{\beta}{z}\right)^{\alpha(m+1)}\right]^{r-2} \times \alpha^2 \beta^{2\alpha} z^{-(\alpha+1)} \left(\frac{\beta\gamma_r}{uz}\right)^{\alpha(\gamma_r-1)} \left(\frac{uz}{\gamma_r}\right)^{-(\alpha+1)}.$$

Following transformation has been assumed

$$h = \left(\frac{\beta}{z}\right)^{\alpha(m+1)},$$

after some calculation it been obtained

$$f(u) = \frac{\alpha^2 \beta^{\alpha m + 2\alpha} C_{r-1}}{\gamma_r (m+1)^{r-2}\Gamma(r-1)} \times \frac{(\beta\gamma_r)^{\alpha(\gamma_r-1)} u^{-(\alpha+1)}}{u^{\alpha(\gamma_r-1)} \gamma_r^{-(\alpha+1)}} \times \frac{\beta}{\alpha(m+1)} \beta^{-\alpha\gamma_r - m\alpha - alpha + 1} \times \int_0^1 h^{\frac{\gamma_r+m+1}{m+1}-1} (1-h)^{r-2} h,$$

which gives the proof of theorem,

$$f_U(u) = \alpha\gamma_r \gamma_r^{\alpha\gamma_r} u^{-(\alpha\gamma_r+1)}, \qquad u > \gamma_r. \quad (2.12)$$

Table 1: Moment Estimations.

	Estimations		MSE	
n	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\alpha}$	$\hat{\beta}$
10	4.137	3.4718	2.1811	0.8243
50	3.1574	3.5103	0.8615	0.4187
100	2.9899	3.5143	0.6452	0.3333

Table 2: Estimation based on GOS.

			Method of Moments		Unbiased		
n	k	m	$\hat{\alpha}$	MSE for $\hat{\alpha}$	$\hat{\beta}$	MSE for $\hat{\beta}$	$Var\left(\hat{\beta}\right)$
90.5cm10	30.5cm1	0	2.3403	0.1612	3.7277	0.77	0.3316
		2	2.2948	0.2053	3.4276	1.0923	0.0301
		5	2.283	0.217	3.3536	0.6728	0.0071
	30.5cm2	0	2.3773	0.1245	3.3259	1.0643	0.1865
		2	2.3089	0.1912	3.3469	0.1484	0.0255
		5	2.2905	0.2095	3.3766	0.8139	0.0068
	30.5cm6	0	2.4957	0.0211	3.511	0.1104	0.0761
		2	2.3645	0.1356	3.5225	0.1244	0.0185
		5	2.3205	0.1795	3.4371	0.376	0.0056
90.5cm50	30.5cm1	0	2.5029	0.006	3.4264	0.1716	0.0491
		2	2.4824	0.0177	3.5455	1.2964	0.0051
		5	2.4772	0.0229	3.4539	1.0198	0.0012
	30.5cm2	0	2.524	0.0245	3.5729	1.2654	0.049
		2	2.4895	0.0106	3.3726	0.6766	0.0045
		5	2.4894	0.0107	3.4726	0.0314	0.0047
	30.5cm6	0	2.6048	0.105	3.3868	0.0855	0.0321
		2	2.5177	0.0178	3.4417	0.2955	0.0042
		5	2.495	0.0051	3.559	0.0203	0.0012
90.5cm100	30.5cm1	0	2.6353	0.1353	3.4362	0.9898	0.022
		2	2.6207	0.1207	3.4738	0.6066	0.0022
		5	2.6171	0.1117	3.5148	0.6436	0.00053
	30.5cm2	0	2.6501	0.1501	3.7037	06445	0.0244
		2	2.6258	0.1258	3.3992	0.4732	0.0022
		5	2.6196	0.1196	3.5093	0.0806	0.00052
	30.5cm6	0	2.7099	0.21	3.6924	0.9148	0.0203
		2	2.6459	0.1459	3.4613	0.4729	0.002
		5	2.6298	0.1298	3.4964	0.2604	0.0005

Table 3: Real data.

41187	5796	1133	167	21	32403	1666	42627
23606	2612	15921	488	55237	37012	18225	

2.1 Real DATA

Real data survey are also included. 15 data from census results of 2014 rural household income in islamic republic of Iran are taken and illustrative example has constructed. Based on proposed methods, unknown parameters of population has been estimated.

Table 4: Estimation results.

Method of Moments		GOS Moment	Unbiased
$\hat{\alpha}$	$\hat{\beta}$	$\hat{\alpha}$	$\hat{\beta}$
2.424	11124	2.3157	11226
$k=1, m=0$			

3 Inference

3.1 GOS moments of Pareto distribution

It is well known that the expectation value and variance of PD 1.4 are respectively,

$$E(X) = \frac{\alpha\beta}{\alpha - 1} \quad \alpha > 1 \tag{3.13}$$

$$Var(X) = \frac{\alpha\beta^2}{(\alpha-1)^2(\alpha-2)} \quad \alpha > 1, \tag{3.14}$$

second moment of PD can be derived by 3.13 and 3.14 as follows:

$$E(X^2) = \frac{\alpha\beta^2}{\alpha - 2} \quad \alpha > 2 \tag{3.15}$$

In order to compute GOS moments of PD, it is necessary to obtain distribution of GOS from PD. Using 1.2, 1.4 and 1.5, PDF of rth GOS from PD can be presented as:

$$f^r(x) = \frac{\alpha\beta^{\alpha\gamma_r}C_{r-1}}{\Gamma(r)(m+1)^{r-1}} \qquad x > \beta. \tag{3.16}$$

Based on 3.16, t^{th} moment is given by

$$E\left(r\right)^t = \int_\beta^\infty x^t f^r(x)x,$$

taking integral leads to the following relation,

$$E\left(r^t\right) = \beta^t \frac{C_{r-1}}{\Gamma(r)(m+1)^r} \times Bet\left(r, \frac{\alpha\gamma_r - t}{(m+1)\alpha}\right). \tag{3.17}$$

Setting t=0 leads to 2.10. Using 2.10, relation 3.17 can be rewritten as:

$$E\left(r^t\right) = \beta^t \frac{Bet\left(r, \frac{\alpha\gamma_r - t}{(m+1)\alpha}\right)}{Bet\left(r, \frac{\gamma_r}{(m+1)}\right)}. \tag{3.18}$$

First and second moments can be obtained by setting $t = 1, 2$ in the relation 3.18, therefore we get to:

$$E\left(r\right) = \beta \frac{Bet\left(r, \frac{\alpha\gamma_r - 1}{(m+1)\alpha}\right)}{Bet\left(r, \frac{\gamma_r}{(m+1)}\right)}, \tag{3.19}$$

and

$$E\left(r\right)^2 = \beta^2 \frac{Bet\left(r, \frac{\alpha\gamma_r - 2}{(m+1)\alpha}\right)}{Bet\left(r, \frac{\gamma_r}{(m+1)}\right)}. \tag{3.20}$$

Variance of GOS from PD can be calculated as

$$Var\left(r\right) = \beta^2 [\frac{Bet\left(r, \frac{\alpha\gamma_r - 2}{(m+1)\alpha}\right)}{Bet\left(r, \frac{\gamma_r}{(m+1)}\right)} - \left(\frac{Bet\left(r, \frac{\alpha\gamma_r - 1}{(m+1)\alpha}\right)}{Bet\left(r, \frac{\gamma_r}{(m+1)}\right)}\right)^2]. \tag{3.21}$$

3.2 Method of moments

In order to get the moment estimators of unknown parameters of PD, we have to consider following relations,

$$E(X) = \overline{X}, \;\; E(X^2) = \overline{X^2}, \tag{3.22}$$

where $\overline{X} = \frac{1}{n}\sum_{i=1}^n x_i$ and $\overline{X^2} = \frac{1}{n}\sum_{i=1}^n x_i^2$.

3.2.1 Method of moments using PD

Using 3.13, 3.15 and 3.22, it can be obtained

$$\beta = \frac{\alpha - 1}{\alpha}\overline{x}, \quad \beta^2 = \frac{\alpha - 2}{\alpha}\overline{x^2}. \tag{3.23}$$

Using 3.23 and simple calculation moment estimators of α and β could be achieved respectively:

$$\hat{\alpha}_m = 1 + \sqrt{\frac{\overline{x^2}}{\overline{x^2} - \overline{x}^2}}, \tag{3.24}$$

and

$$\hat{\beta}_m = \frac{\hat{\alpha}_m - 1}{\hat{\alpha}_m}\overline{x}. \tag{3.25}$$

3.2.2 Method of moments using GOS from PD

Using Theorem 2.2 and similar to the 3.2.1, moment estimator of α could be obtained based on GOS from PD. According to relations 3.22, 3.13 and 3.15, clearly it can be written

$$E\left(\gamma_r \frac{X_r}{X_{r-1}}\right) = \frac{\alpha\gamma_r^2}{\alpha\gamma_r - 1} = \overline{Y}, \tag{3.26}$$

and

$$E\left(\gamma_r \frac{X_r}{X_{r-1}}\right)^2 = \frac{\alpha\gamma_r^3}{\alpha\gamma_r - 2} = \overline{Y^2}, \quad \alpha\gamma_r > 2, \tag{3.27}$$

where

$$n\overline{Y} = \gamma_1 Y_1 + \sum_{i=2}^{n}\left(\gamma_i \frac{X_i}{X_{i-1}}\right),$$

and

$$n\overline{Y^2} = (\gamma_1 Y_1)^2 + \sum_{i=2}^{n}\left(\gamma_i \frac{X_i}{X_{i-1}}\right)^2,$$

furthermore, Y_i, $i = 1, 2, \cdots, n$ are GOS from PD. Therefore, moment estimator for α based on GOS can be obtained as:

$$\hat{\alpha}_{gm} = 1 + \sqrt{\frac{\overline{Y^2}}{\overline{Y^2} - \overline{Y}^2}}.$$

3.3 Unbiased estimation

At this subsection β has been estimated through unbiased estimator when α has been assumed to be known. Considering 3.19, it can be written

$$E\left(\frac{r}{G(\alpha, r)}\right) = \beta, \tag{3.28}$$

where, $G(\alpha, r) = \frac{Bet\left(r, \frac{\alpha\gamma_r - 1}{(m+1)\alpha}\right)}{Bet\left(r, \frac{\gamma_r}{(m+1)}\right)}$, therefore, it can be concluded $\hat{\beta}_{Unb} = \frac{r}{G(\alpha,r)}$ is the unbiased estimator of β. In order to evaluate consistency of the estimator, variance can be computed. Using 3.21, variance of unbiased estimator can be obtained,

$$Var\left(\hat{\beta}_{Unb}\right) = \frac{1}{G^2(\alpha, r)} var(r).$$

4 Numerical study

4.1 Simulated DATA

For comparison performances of methods and evaluation of estimators in different circumstances numerical study are considered. different samples with different sample sizes ($n = 10, 50, 100$), and values k=1,2,6 m=0,2,5 and also with parameter values of $\alpha = 2.5$, and $\beta = 3.5$ are derived based on algorithm discussed in [1].

5 Conclusion

In this paper distribution of ratios of GOS from PD were obtained. Single moments of were derived and based on them moment estimators for PD unknown parameters were constructed. Using ratio distribution moment inference was done. Unbiased estimator based on moments of PD through GOS was derived, and consistency of estimator was studied. To compare of methods and different parameter values the numerical studies were presented.
Based on numerical results following conclusions are obtained:

- Table 1 shows that when sample size increases, the MSE of both unknown parameters of PD estimated based on method of moments is low, so the estimators give better performances.
- Table 1 and 2 showed that the estimators based on GOS are better than the others.
- Based on GOS method of moments when n and m increase simultaneously estimator gives better performances.
- Table 2 shows that when n increases and k decreases, estimator slightly performs good.
- Unbiased estimator gives better results when n and k increase.
- Table 2 shows that unbiased estimator of β gets close to real parameter value, while n is growing, and variance of estimator is decreasing, so unbiased estimator is asymptotically consistent.

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