Available online at http://ijim.srbiau.ac.ir/
Int. J. Industrial Mathematics (ISSN 2008-5621)
Vol. 9, No. 1, 2017 Article ID IJIM-00760, 8 pages
Research Article

Science and Research Branch (IAU)

Mathematical modelling of Sisko fluid flow through a stenosed artery

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Received Date: 2014-11-19 Revised Date: 2015-06-09 Accepted Date: 2016-04-22

Abstract

In the present study, the nonlinear model of non-Newtonian blood flow in cosine-shape stenosed elastic artery is numerically examined. The model is carried out for axisymmetric, two-dimensional and fully developed blood flow. The vessel wall is assumed to be have time-dependent radius that is important factor for study of blood flow. The cosine-shape stenosis convert to rigid artery by using a appropriate coordinate transformation and closed form solutions are discovered. The Sisko non-Newtonian fluid model is used for discribing blood rheology. The Navier-stokes equations of momentom containing pulastic pressure gradient. The resulting explicit of the governing nonlinear equations have been obtained numerically with the help of the finite differece scheme and Matlab program. The key dynamic parametrs similar resistance impedance, velocity profiles and the volumetric flow rate are studied. The influence of non-Newtonian rheological properties of unsteady blood flow and stenosis severity are found and computer modeling and simulation shown graphically.

Keywords : Navier-Stokes equation; Finite difference method; Time-dependent Stenosis; Sisko fluid.

1 Introduction

Recently, since development of many cardiovascular diseases in the world, the study of blood flow is important. one of this diseases is Atheroscléreis that is the problem artery in cardiovascular. Atherosclerois occur when cholesterols and fats growth in the artery and the artery becomes stenotic [1, 2, 3]. A number of authors have been studied the blood flow in vessel. Blood flow in the stenotic vessel is assumed to be newtonian in most articles [4]-[7]. Most experimental and numerical simulation have been done by them [8]-[12]. They agreed that blood have non-Newtonian property for example at high shear rate exhibits of blood is shear-thinning and at low shear rate is viscoelasticity. Singh and Shah [13] discussed the effect of the shape of stenosis on non-Newtonian fluid. They used Power-low fluid model and realized the wall shear strees change with change stenosis size, shape and length. Mandel [14] studied the non-newtonian and axisymmetric blood flow in stenotic vessel. He used the generalised Power-law model for discribe the rheology behavior of blood and solved them with finite difference scheame. Mandel et al [15] developed a mathematical model for this study [14] and re-examined the study for the shear-thickening and shear-thining property of the two-dimensional blood flow and pointed out the effect of body acceleration on non-newtonian blood flow. Haghighi and shahbaziasl [16] studied on the problem of a unsteady and two-dimintional flow of blood in the stenotic vessel. They numerically solved

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the nonlinear equations by using the explicit finite difference. The geometry of the flexible vessel considered to be non-symmetric. Haghighi et al [17] also observed a two-dimintional, axisymmetric and fully developed blood flow in the stenotic vessel. The explicit momentom continuty equation for the unsteady flow solved by appling numerically method. The geometry of the deformable wall vessel considered to be tapered with overlapping stenosis. Branes et al [18] discussed that in the medium shear rate Power-low model fluid is a appropriate but it is not appropriate for high and low shear rate. In this paper, we have encompassed the Sisko model that is a generalized Power-low model [19] and pointed out non-linear flow of blood in a elastic vessel considering the time-dependent stenotic. Sisko fluid model appropriate for medium and high shear rate and it can discribe many property Newtonian and non-Newtonian fluid flow by considered material parametrs[19, 20]. Also, we have encompassed pressure gradient in the partial differential equation. By appling finite difference manner, this nonlinear equations have been solved numerically. In the last section, result have been shown by graphical representations.

2 Mathematical formulation and analysis

2.1 geometry of the stenosis

Bood flow encompassed to be non-Newtonian that characterised by Sisko fluid. Blood flow have been analysied by appling cylindrical polar coordinate system (r, θ, z). The schematic diagram of this research is given in Fig. 1. The geometry elastic arterial for the stenosis under assumeation is written mathematically [21]:

$$R(z,t) = \begin{cases} [r_0 - (\frac{\tau_m}{2})\{1 + cos(\frac{2\pi}{L_1}(z - d \\ -\frac{L_1}{2}))\}]a_1(t) \quad d \leq z \leq d + L_1 \\ r_0 a_1(t) \qquad otherwise \end{cases} \tag{2.1}$$

Where $R(z,t)$ shows the radius of the elastic arterial in the stenosed region. L the length of the elastic arterial assumed. τ_m the maximum height of the stenosis, d the location of the stenosis. $a_1(t) = 1 + bcos(\omega t - \phi)$ [17, 20] is time-variant formula where $\omega = 2\pi f_p$ is angular frequency, b being the amplitude parameter and ϕ is phase angle.

2.2 Governing equations

The laminar and fully development blood flow is considered in a stenotic elastic arterial assumed. Let us pointed out coordinate system (r, θ, z) in the governing equations. Give velocity field for this study is

$$u = [u(r,z,t), 0, w(r,z,t)]$$

Equation of continiuity [20]

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0 \tag{2.2}$$

Nonlinear equation of axial and radial momentum [20]

$$\rho(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z}) =$$

$$-\frac{\partial p}{\partial r} - (\frac{1}{r}\frac{\partial}{\partial r}(rT_{rr}) + \frac{\partial}{\partial z}(T_{rz})) \tag{2.3}$$

$$\rho(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial z}) =$$

$$-\frac{\partial p}{\partial z} - (\frac{1}{r}\frac{\partial}{\partial r}(rT_{rz}) + \frac{\partial}{\partial z}(T_{zz})) \tag{2.4}$$

We can writte the relationship between the shear stress and the shear rate as follows:

$$T_{rr} = -2[\bar{a} + \bar{b}\{|[(\frac{\partial u}{\partial r})^2 + (\frac{u}{r})^2$$

$$+ (\frac{\partial w}{\partial z})^2 + (\frac{\partial u}{\partial z} + \frac{\partial w}{\partial r})^2]^{\frac{1}{2}}|^{n-1}\}](\frac{\partial u}{\partial r}) \tag{2.5}$$

$$T_{zz} = -2[\bar{a} + \bar{b}\{|[(\frac{\partial u}{\partial r})^2 + (\frac{u}{r})^2 + (\frac{\partial w}{\partial z})^2$$

$$+ (\frac{\partial u}{\partial z} + \frac{\partial w}{\partial r})^2]^{\frac{1}{2}}|^{n-1}\}](\frac{\partial w}{\partial z}) \tag{2.6}$$

$$T_{rz} = -[\bar{a} + \bar{b}\{|[(\frac{\partial u}{\partial r})^2 + (\frac{u}{r})^2 + (\frac{\partial w}{\partial z})^2$$

$$+ (\frac{\partial u}{\partial z} + \frac{\partial w}{\partial r})^2]^{\frac{1}{2}}|^{n-1}\}](\frac{\partial w}{\partial r} + \frac{\partial u}{\partial z}) \tag{2.7}$$

Here $T = -pI + S$ is cauchy stress tensor in Sisko model, where p is the pressure, I is the identity tensor and $S = [\bar{a} + \bar{b}(|\sqrt{\pi}|)^{n-1}]A_1$ is the extra stress tensor [19, 20, 22]. In which $\bar{a}$ and $\bar{b}$ are the asymptotic value of viscosity at very high shear rates and the consistency the constant parametr respectively. It shoud be noted that in this study the Sisko fluid is the generalized Power low by considered $\bar{a} = 0$. $w(r, z, t)$ and $u(r, z, t)$ are the axial and the radial velocity respectively, ρ is the density .

The pressure gradient $\frac{\partial p}{\partial z}$ had appearid in Eq 2.4, is written this forme:

$$-\frac{\partial p}{\partial z} = A_0 + A_1 cos\omega t$$

[17, 20, 22]-[25].

Where A_0 is the constant amplitude of the pressure gradient, A_1 is the amplitude of the pulsatile component giving rise to systolic and diastolic pressure and $\omega_b = 2\pi f_p$, where f_p is being the pulse frequency.

The boundary conditions, the shear stres and initial conditions can be written in mathematical form [20, 25]:

$$on\ r = 0: \quad u(r, z, t) = 0, \quad \frac{\partial w(r, z, t)}{\partial r} = 0$$

$$and\ T_{rz} = 0, \tag{2.8}$$

$$on\ r = R(z, t): \quad u(r, z, t) = \frac{\partial R}{\partial t}$$

$$w(r, z, t) = 0, \tag{2.9}$$

$$u(r, z, 0) = 0, \quad w(r, z, 0) = 0, \tag{2.10}$$

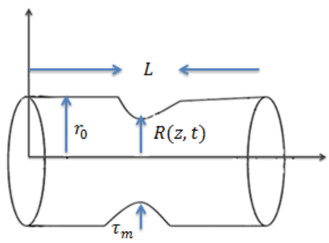

Figure 1: Schematic diagram of a stenosed elastic artery

3 Radial coordinate transformation

We appliyed a radial coordinate transformation $\xi = \frac{r}{R(z,t)}$ [16, 17, 26]-[28]. The radial coordinate transformatione immobilized the elastic vessel wall, so equation 2.3 and 2.4 the boundary

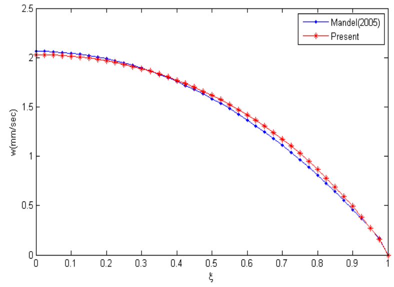

Figure 2: The axial velocity profiles at $(z = 28, t = 0.45, t_m = 0.4a, d = 20mm, l_0 = 16mm)$

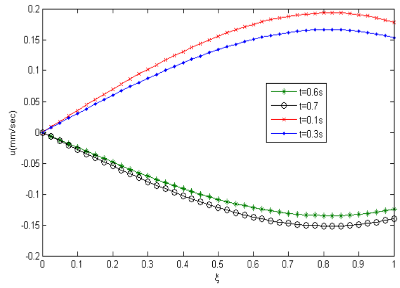

Figure 3: The radial velocity profiles at different time $(z = 28, t_m = 0.4a, d = 20mm, l_0 = 16mm)$

and initial conditions become:

$$\frac{\partial w}{\partial t} = \{\frac{\xi}{R}\frac{\partial R}{\partial t} - \frac{u}{R} + \frac{\xi}{R}\frac{\partial R}{\partial z}w\}\frac{\partial w}{\partial \xi}$$
$$-\frac{1}{\rho}\frac{\partial p}{\partial z} - w\frac{\partial w}{\partial z} - \frac{1}{\rho}\{\frac{1}{\xi R}T_{\xi z} + \frac{1}{R}\frac{\partial T_{\xi z}}{\partial \xi}$$
$$-\frac{\partial T_{zz}}{\partial z} + \frac{\xi}{R}\frac{\partial R}{\partial z}\frac{\partial T_{zz}}{\partial \xi}\} \tag{3.11}$$

$$\frac{1}{R}\frac{\partial u}{\partial \xi} + \frac{u}{\xi R} + \frac{\partial w}{\partial z} - \frac{\xi}{R}\frac{\partial R}{\partial z}\frac{\partial w}{\partial \xi} = 0 \tag{3.12}$$

$$T_{\xi z} = -[\bar{a} + \bar{b}\{|[(\frac{1}{R}\frac{\partial u}{\partial \xi})^2 + (\frac{u}{\xi R})^2 + (\frac{\partial w}{\partial z}$$
$$-\frac{\xi}{R}\frac{\partial R}{\partial z}\frac{\partial w}{\partial \xi})^2 + (\frac{\partial u}{\partial z} - \frac{\xi}{R}\frac{\partial R}{\partial z}\frac{\partial u}{\partial \xi} + \frac{1}{R}\frac{\partial w}{\partial \xi})]^{\frac{1}{2}}|^{n-1}\}]$$
$$\times (\frac{\partial u}{\partial z} - \frac{\xi}{R}\frac{\partial R}{\partial z}\frac{\partial u}{\partial \xi} + \frac{1}{R}\frac{\partial w}{\partial \xi}) \tag{3.13}$$

$$T_{zz} = -2[\bar{a} + \bar{b}\{|[(\frac{1}{R}\frac{\partial u}{\partial \xi})^2 + (\frac{u}{\xi R})^2 + (\frac{\partial w}{\partial z}$$
$$-\frac{\xi}{R}\frac{\partial R}{\partial z}\frac{\partial w}{\partial \xi})^2 + (\frac{\partial u}{\partial z} - \frac{\xi}{R}\frac{\partial R}{\partial z}\frac{\partial u}{\partial \xi} + \frac{1}{R}\frac{\partial w}{\partial \xi})]^{\frac{1}{2}}|^{n-1}\}]$$
$$\times (\frac{\partial w}{\partial z} - \frac{\xi}{R}\frac{\partial R}{\partial z}\frac{\partial w}{\partial \xi}) \tag{3.14}$$

$$T_{\xi\xi} = -2[\bar{a} + \bar{b}\{|[(\frac{1}{R}\frac{\partial u}{\partial \xi})^2 + (\frac{u}{\xi R})^2 + (\frac{\partial w}{\partial z}$$

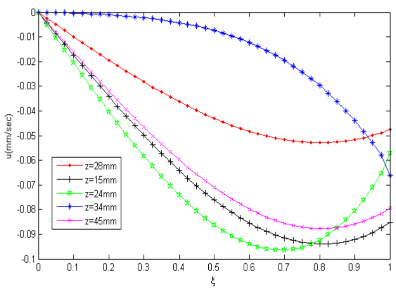

Figure 4: The radial velocity profiles for different locations through artery $(t = 0.45, t_m = 0.4a, d = 20mm, l_0 = 16mm)$

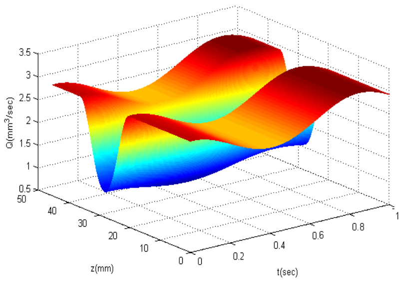

Figure 5: 3-dimentian the volumetric flow rate in the stenosed artery$(z = 28, t = 1, t_m = 0.4a, d = 20mm, l_0 = 16mm)$

$$-\frac{\xi}{R}\frac{\partial R}{\partial z}\frac{\partial w}{\partial \xi})^2 + (\frac{\partial u}{\partial z} - \frac{\xi}{R}\frac{\partial R}{\partial z}\frac{\partial u}{\partial \xi} + \frac{1}{R}\frac{\partial w}{\partial \xi})]^{\frac{1}{2}}|^{n-1}\}] \times (\frac{1}{R}\frac{\partial u}{\partial \xi}) \quad (3.15)$$

$$on\ \xi = 0: \quad u(\xi, z, t) = 0, \quad \frac{\partial w(\xi, z, t)}{\partial \xi} = 0$$

$$and\ T_{\xi z} = 0, \quad (3.16)$$

$$on\ \xi = R(z): \quad u(\xi, z, t) = \frac{\partial R}{\partial t}$$

$$w(\xi, z, t) = 0, \quad (3.17)$$

$$u(\xi, z, 0) = 0, \quad w(\xi, z, 0) = 0, \quad (3.18)$$

4 The velocity profile

4.1 The radial velocity component

Multiplying Eq 2.2 by ξR and integrating with respect to ξ from 0 to ξ. we can obtained the explicit expression for $u(\xi, z, t)$.

$$u(\xi, z, t) = \xi\frac{\partial R}{\partial z}w - \frac{R}{\xi}\int_0^{\xi}\xi\frac{\partial w}{\partial z}d\xi$$

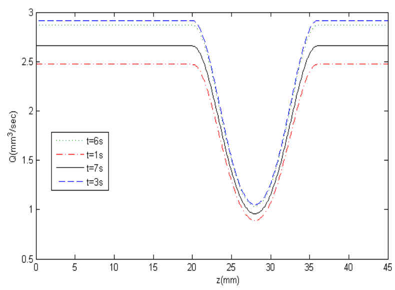

Figure 6: The volumetric flow rateat different time $(z = 28, t_m = 0.4a, d = 20mm, l_0 = 16mm)$

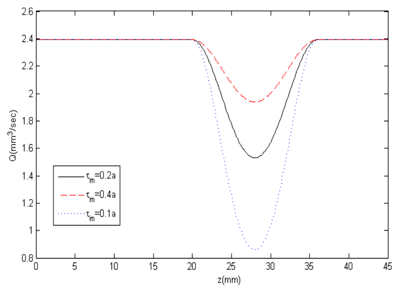

Figure 7: The volumetric flow rate at different stenosis size $(z = 28, t = 0.45, d = 20mm, l_0 = 16mm)$

$$-\frac{2}{\xi}\frac{\partial R}{\partial z}\int_0^{\xi}\xi w d\xi. \quad (4.19)$$

by using the boundary cinditions for $\xi = 1$ Eq 4.19 becomes as follow:

$$-\int_0^1 \xi\frac{\partial w}{\partial z}d\xi = \int_0^1 \xi[\frac{2}{R}\frac{\partial R}{\partial z}w + \frac{1}{R}\frac{\partial R}{\partial t}f(\xi)]d\xi \quad (4.20)$$

We can considered $f(\xi)$ of the form $f(\xi) = -4(\xi^2 - 1)$ satisfying $\int_0^1 \xi f(\xi)d\xi = 1$ By integration of both sides of 4.20 we arrive :

$$\frac{\partial w}{\partial z} - \frac{2}{R}\frac{\partial R}{\partial z}w + \frac{4}{R}(\xi^2 - 1)\frac{\partial R}{\partial t} \quad (4.21)$$

Introducing 4.21 in 4.19 we find:

$$u(\xi, z, t) = \xi[\frac{\partial R}{\partial z}w + \frac{\partial R}{\partial t}(2 - \xi^2)] \quad (4.22)$$

4.2 The axial velocity component

We have pointed out the finite difference method for discretization of governing nonlinear equation by making use central difference approximations

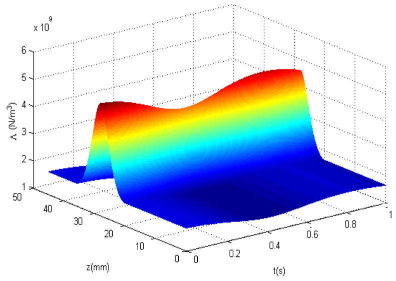

Figure 8: 3-dimentian the resistance impedance along the stenosed artery $(z = 28, t = 1, t_m = 0.4a, d = 20mm, l_0 = 16mm)$

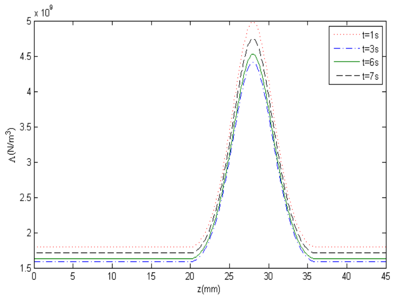

Figure 9: The resistance impedance at different time $(z = 28, t_m = 0.4a, d = 20mm, l_0 = 16mm)$

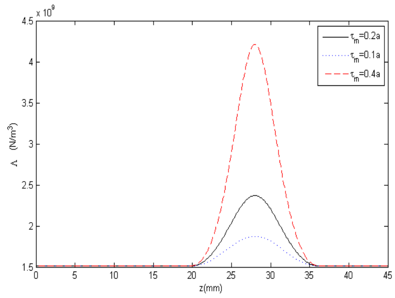

Figure 10: The resistance impedance at different stenosis size $(z = 28, t = 0.45, d = 20mm, l_0 = 16mm)$

for all the spatial derivatives and forward difference approximations for all the time derivatives in the following form:

$$\frac{\partial w}{\partial \xi} = \frac{w_{i,j+1}^{k} - w_{i,j-1}^{k}}{2\Delta\xi} + o(h^2) \qquad (4.23)$$

$$\frac{\partial w}{\partial z} = \frac{w_{i+1,j}^{k} - w_{i-1,j}^{k}}{2\Delta z} + o(h^2) \qquad (4.24)$$

$$\frac{\partial w}{\partial t} = \frac{w_{i,j}^{k+1} - w_{i,j}^{k}}{\Delta t} + o(h) \qquad (4.25)$$

we found discretized expression for the T_{zz} and T_{xz} .

$$\frac{\partial T_{zz}}{\partial \xi} = \frac{T_{i,j+1}^{k} - T_{i,j-1}^{k}}{2\Delta\xi}, \quad \frac{\partial T_{zz}}{\partial z} = \frac{T_{i+1,j}^{k} - T_{i-1,j}^{k}}{2\Delta z},$$

$$\frac{\partial T_{xz}}{\partial \xi} = \frac{T_{i,j+1}^{k} - T_{i,j-1}^{k}}{2\Delta\xi} \qquad (4.26)$$

Here,we define

$$\xi_j = (j-1)\Delta\xi; \quad j = 1, 2, ..., N+1 \;\; where \; \xi_{N+1} = 1$$

$$z_j = (i-1)\Delta z; \quad i = 1, 2, ..., N+1$$

$$t_k = (k-1)\Delta t; \quad k = 1, 2, ..., N+1$$

By appling above discritization method Eqs (3.11-3.15) have been transformed to the following:

$$w_{i,j}^{k+1} = w_{i,j}^{k} + \Delta t[-\frac{1}{\rho}(\frac{\partial p}{\partial z})^{k+1} + \{\frac{\xi_j}{R_i^k}(\frac{\partial R}{\partial t})_i^k - \frac{u_{i,j}^k}{R_i^k} + \frac{\xi_j}{R_i^k}(\frac{\partial R}{\partial z})_i^k w_{i,j}^k\} \times (\frac{\partial w}{\partial \xi})_{i,j}^k - w_{i,j}^k(\frac{\partial w}{\partial z})_{i,j}^k - \frac{1}{\rho}\{\frac{1}{\xi_j R_i^k}(T_{\xi z})_{i,j}^k + \frac{1}{R_i^k}[(T_{\xi z})_{f\xi}]_{i,j}^k - [(T_{zz})_{fz}]_{i,j}^k + \frac{\xi_j}{R_i^k}[(T_{zz})_{f\xi}]_{i,j}^k\}] \qquad (4.27)$$

$$(T_{\xi z})_{i,j}^k = -[\bar{a}+\bar{b}\{|[(\frac{1}{R_i^k}(\frac{\partial u}{\partial \xi})_{i,j}^k)^2+(\frac{u_{i,j}^k}{\xi_j R_i^k})^2+((\frac{\partial w}{\partial z})_{i,j}^k - \frac{\xi_j}{R_i^k}(\frac{\partial R}{\partial z})_{i,j}^k(\frac{\partial w}{\partial \xi})_{i,j}^k)^2 + ((\frac{\partial u}{\partial z})_{i,j}^k - \frac{\xi_j}{R_i^k}(\frac{\partial R}{\partial z})_{i,j}^k (\frac{\partial u}{\partial \xi})_{i,j}^k + \frac{1}{R_i^k}(\frac{\partial w}{\partial \xi})_{i,j}^k)]^{\frac{1}{2}}|^{n-1}\}] \times [((\frac{\partial u}{\partial z})_{i,j}^k - \frac{\xi_j}{R_i^k}(\frac{\partial R}{\partial z})_{i,j}^k(\frac{\partial u}{\partial \xi})_{i,j}^k + \frac{1}{R_i^k}(\frac{\partial w}{\partial \xi})_{i,j}^k)] \qquad (4.28)$$

$$(T_{zz})_{i,j}^k = -2[\bar{a}+\bar{b}\{|[(\frac{1}{R_i^k}(\frac{\partial u}{\partial \xi})_{i,j}^k)^2 + (\frac{u_{i,j}^k}{\xi_j R_i^k})^2 + ((\frac{\partial w}{\partial z})_{i,j}^k - \frac{\xi_j}{R_i^k}(\frac{\partial R}{\partial z})_{i,j}^k(\frac{\partial w}{\partial \xi})_{i,j}^k)^2 + ((\frac{\partial u}{\partial z})_{i,j}^k (-\frac{\xi_j}{R_i^k}(\frac{\partial R}{\partial z})_{i,j}^k \frac{\partial u}{\partial \xi})_{i,j}^k + \frac{1}{R_i^k}(\frac{\partial w}{\partial \xi})_{i,j}^k)]^{\frac{1}{2}}|^{n-1}\}] \times ((\frac{\partial w}{\partial z})_{i,j}^k - \frac{\xi_j}{R_i^k}(\frac{\partial R}{\partial z})_{i,j}^k(\frac{\partial w}{\partial \xi})_{i,j}^k) \qquad (4.29)$$

$$(T_{\xi\xi})_{i,j}^k = -2[\bar{a}+\bar{b}\{|[(\frac{1}{R_i^k}(\frac{\partial u}{\partial \xi})_{i,j}^k)^2 + (\frac{u_{i,j}^k}{\xi_j R_i^k})^2$$

$$+((\frac{\partial w}{\partial z})^k_{i,j} - \frac{\xi_j}{R^k_i}(\frac{\partial R}{\partial z})^k_{i,j}(\frac{\partial w}{\partial \xi})^k_{i,j})^2 + ((\frac{\partial u}{\partial z})^k_{i,j}$$

$$-\frac{\xi_j}{R^k_i}(\frac{\partial R}{\partial z})^k_{i,j}(\frac{\partial u}{\partial \xi})^k_{i,j} + \frac{1}{R^k_i}(\frac{\partial w}{\partial \xi})^k_{i,j})]^{\frac{1}{2}}|^{n-1}\}]$$

$$\times(\frac{1}{R^k_i}(\frac{\partial u}{\partial \xi})^k_{i,j}) \quad (4.30)$$

The discretized boundary condition are given as:

$$on \ \ \xi = 0, \ \ u^k_{i,j} = 0, \ w^k_{i,1} = w^k_{i,2},$$

$$(T_{\xi z})^k_{i,1} = 0 \quad (4.31)$$

$$on \ \ \xi = 1, \ \ w^k_{i,N+1} = 0, \ u^k_{i,1} = (\frac{\partial R}{\partial t})^k_i \quad (4.32)$$

$$u^1_{i,j} = 0, \ w^k_{i,j} = 0 \quad (4.33)$$

The volumetric flow rate Q and the resistiv impedance Λ can be obtained by:

$$Q^k_i = 2\pi(R^k_i)^2 \int_0^1 \xi_j(w)^k_{i,j} d\xi_j \quad (4.34)$$

$$\Lambda^k_i = \frac{|L(\frac{\partial p}{\partial z})^k|}{Q^k_i} \quad (4.35)$$

5 Numerical results and discussion

For the get of numerical simulation of this study the following parametr values are used: [20, 25, 28]
$a = 0.1cm, n = 0639, \rho = 1.06gcm^3, b = 0.1, l = 45, A_0 = 10gcm^{-2}s^{-2}$
$A_1 = 0.2A_0, f_b = 1.2Hz, \tau_m = 0.4a, a_0 = 10gcm^{-2}s^{-2}, f_b = 1.2Hz, \Delta t = 0.0001$

By creating mesh 450×40 and by appling Matlab program, exhibit rheology non-Newtonian of blood and geometry parametrs are abtained through 2-10 figs and discussed.
As noted in section 2 the Power-low model is a special case of Sisko model so, the axial velocity profile of this study in the maximum stenotic positions $z = 28mm, t = 0.3s$ compared with the result given in Ref [29]. It is seen that the outcome are found to be in well agreement.
The results for the radial velocity curves for various time are given in Fig. 3. Due, the pressure gradient be maded by the heart, all curves visualized. It is seen that the radial velocity have negative values during the diastolic phase from 0.5s to 0.7s. But, during the systolic phase from 0.1s to 0.3s ,the profiles give positive values.

The results of the radial velocity at 5 distinct place at $t = 0.45$, $\tau_m = 0.4a$ shown in Fig. 4. $z = 45mm, z = 15mm$ is a non-stenotic region, so, curves of them have the same form. Also, the radial velocity profiles at $z = 28mm$ is shown that is the center of the stenosis location. Fig. 4 shows that all curves are similarin behavior except the curve for $z = 34$. Because the $z = 34$ is the end of stenosis region.

Fig. 5 shows 3-dimentian the volumetric flow rate in the stenosed elastic artery in finite time(t=1) at $\tau_m = 0.4a$. It is seen that the volumetric flow rate is increasing with increas the time.

The results for the volumetric flow rate for various time are gven in Fig. 6. It is seen that the volumetric flow rate increase during the diastolic phase from 0.5s to 0.7s. while the volumetric flow rate decrease during the systolic phase from 0.1 to 0.3s.

Figure 7 shows the influence of stenosis severity on the volumetric flow rate at the same specific location of $z = 28mm$. According to this figure the volumetric flow rate decreses by increasing the size of the stenosis.

Fig. 8 shows 3-dimentian the resistance impedance along the stenosed elastic artery in finit time (t=1) at $\tau_m = 0.4a$. It is seen that the resistance impedance is increasing with decresing the time.

The results for the resistance impedance for various time are given in Fig. 9. It is seen that the resistance impedance decrease during the diastolic phase from 0.5s to 0.7s. while the resistance impedance rate increase during the systolic phase from 0.1 to 0.3s.
Figure 10 shows the effect of stenosis severity on the resistance impedance at the same specific location of $z = 28mm$. According to this figure the resistance impedance increase by decreasing the severity of the stenosis.

6 conclusions

In this research the unsteady pulsatile flow of blood in a time-dependent stenotic elastic artery is studied. The important factor of this study is that the geometry of the elastic arterial to be have time-dependen stenotic. For discribe rheology behavior of blood flow we used Sisko model. We pointed out the explicit finite difference manner for solving equation because of it found be good effective for numerically solving. It is recorded that the parametr such as volumetric flow, velocity profile and resistance impedance are analysed and shown graphically. The radial velocity and resistance imedance decreased during the diastolic phase but the volumetric flow rate increase during the diastolic phase.

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