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Research Article

A new attitude coupled with the basic thinking to ordering for ranking fuzzy numbers

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Abstract

Ranking fuzzy numbers is generalization of the concepts of order, and class, and so have fundamental applications. Moreover, deriving the final efficiency and powerful ranking are helpful to decision makers when solving fuzzy problems. Selecting a good ranking method can apply to choosing a desired criterion in a fuzzy environment. There are numerous methods proposed for the ranking of fuzzy numbers, some of them seem to be good in a particular context but not in general. In this paper, a new attitude coupled with the basic thinking of ordering for ranking of fuzzy numbers is proposed. The properties of the proposed method are discussed in detail. The ranking results show that the proposed method can overcome certain shortcomings that exist in the previous ranking methods.The method also has very easy and simple calculations compared to other methods. Finally, numerical examples are presented to illustrate the advantage of our proposed method, and compare them with other common ranking methods. The future prospect of this paper is a new attitude to fuzzy distance, which will be referred to in the end.

Keywords : Ranking fuzzy numbers; Transmission average (TA); Fuzzy par; Ambiguity rank; Fuzzy partial order.

1 Introduction

Ordering of fuzzy quantities is based on extracting various features from fuzzy sets. These features may be a center of gravity, an area under the membership function, or various intersection points between fuzzy numbers. A particular fuzzy number ranking method extracts a specific feature from fuzzy numbers, and then ranks them (fuzzy numbers) based on that feature. In order to rank fuzzy numbers, one fuzzy number needs to be compared with the others but it is difficult to determine clearly which of them is larger or smaller. The method for ranking was first proposed by Jain [17], since then, numerous methods have been proposed for ranking special kinds of fuzzy numbers with different flaws [19, 29, 12, 6, 7, 27, 8, 9, 14, 10, 11, 2, 3, 4, 22, 21, 20, 23, 24, 26, 28, 30].
All fuzzy set ranking methods can be categorized into two classes (after Yuan [31]):
1) Methods which convert a fuzzy number to a crisp number by applying a mapping function F (i.e., if A is a fuzzy number, then F(A) = a, where a is a crisp number). Fuzzy numbers are then sorted by ranking crisp numbers produced by the mapping.

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2) Methods which use fuzzy relations to compare pairs of fuzzy numbers, and then construct a relationship which produces a linguistic meaning of the comparison. The ordering results are something like fuzzy number A is slightly better than fuzzy number B.
However, each methodology has its own advantages and disadvantages.
With 1, it has been argued that by reducing the whole of our analysis to a single (crisp) number, we are loosing much of the information we have purposely been keeping throughout our calculations [15]. This methodology, on the other hand, produces a consistent ranking of all fuzzy sets considered (i.e., if A is ranked greater than B, and B is ranked greater than C, then A will always be much greater than C). Also, there will always exist a fuzzy set which is ranked as best, second best, third best, and so on.
With 2, by keeping the comparisons linguistic, we are preserving the inherit fuzzy information of the problem. However, as Yuan [31] points out, it may not always be possible to construct total ordering among all alternatives based on pairwise fuzzy preference relations. This means that even if A is better than B, and B is better than C, A may not always be better than C.
Discouraging facts about fuzzy set ranking methods, unfortunately, do not end here. In their review, Bortolan and Degani [5] find that for simple cases, most fuzzy set ranking methods produce consistent rankings. Difficult cases however, produce different rankings for different methods. This means that if membership functions overlap (or intersect) for some values of x, or if the supports of fuzzy numbers differ even slightly, different methods will most likely produce different rankings.
There is not a unique method for comparing fuzzy numbers. As a result, it is reasonable to expect that different ranking methods can produce different ranking order for the same sample of fuzzy numbers and some of them seem to be good in a particular context but not in general. Intricacies like these make ranking fuzzy numbers rather difficult.
In this paper, with a new attitude coupled with the basic thinking to ordering, we define a fuzzy linear (total) order on the fuzzy numbers set. Accordingly, we introduce the fuzzy numbers set as a fuzzy comparable set, namely, we can ordered any number of the fuzzy numbers. Finally, in order to the description new proposed attitude, several basic properties and illustrative examples presented for ranking fuzzy numbers. In later paper, with the new proposed method for ranking fuzzy numbers, we will have a new attitude to fuzzy distance.
The paper is organized as follows. In section 2, we present the basic definitions and concepts related to the subject. In section 3, a new definition of fuzzy partial order, fuzzy comparable and fuzzy linear (total) order are provided and its basic properties are investigated in section 4. In section 5, the proposed method has been explained with examples and shows the results of comparing our method to others. Finally, conclusions and future research are drawn in section 6.

2 Preliminaries and notations

In this section, some notations and background about the concept are brought.

Definition 2.1 *[16] (Fuzzy number) A fuzzy set A in $\mathbb{R}$ is called a fuzzy number if it satisfies the following conditions*
i) A is normal,
ii) $A_\alpha$is a closed interval for every $\alpha \in (0, 1]$,
iii) the support of A is bounded.

According to definition of fuzzy number mentioned above, we use define a pseudo-geometric fuzzy number in two case as follows:

Definition 2.2 *[18] (Pseudo-triangular fuzzy number) A fuzzy number $\widetilde{A}$ is called a pseudo-triangular fuzzy number if its membership function function $\mu_{\widetilde{A}}(x)$ is given by*

$$\mu_{\widetilde{A}}(x) = \begin{cases} l_{\widetilde{A}}(x), & \underline{a} \le x \le a, \\ r_{\widetilde{A}}(x), & a \le x \le \overline{a}, \\ 0, & otherwise. \end{cases} \quad (2.1)$$

Where $l_{\widetilde{A}}(x)$ and $r_{\widetilde{A}}(x)$ are non-decreasing and non-increasing functions, respectively. The pseudo-triangular fuzzy number $\widetilde{A}$ is denoted by the quintuplet

$$\widetilde{A} = (\underline{a}, a, \overline{a}, l_{\widetilde{A}}(x), r_{\widetilde{A}}(x)),$$

and the triangular fuzzy number by the quintuplet

$$(\underline{a}, a, \overline{a}, -, -).$$

Definition 2.3 *[18] (Pseudo-trapezoidal fuzzy number) A fuzzy number $\widetilde{A}$ is called a pseudo-trapezoidal fuzzy number if its membership function $\mu_{\widetilde{A}}(x)$ is given by*

$$\mu_{\widetilde{A}}(x) = \begin{cases} l_{\widetilde{A}}(x), & \underline{a} \leq x \leq a_1, \\ 1, & a_1 \leq x \leq a_2, \\ r_{\widetilde{A}}(x), & a_2 \leq x \leq \overline{a}, \\ 0, & otherwise. \end{cases} \quad (2.2)$$

Where $l_{\widetilde{A}}(x)$ and $r_{\widetilde{A}}(x)$ are nondecreasing and non increasing functions, respectively. The pseudo-trapezoidal fuzzy number $\widetilde{A}$ is denoted by the Senary

$$\widetilde{A} = (\underline{a}, a_1, a_2, \overline{a}, l_{\widetilde{A}}(x), r_{\widetilde{A}}(x)),$$

and the trapezoidal fuzzy number by the Senary

$$(\underline{a}, a_1, a_2, \overline{a}, -, -).$$

Definition 2.4 *[13] (Equal fuzzy sets) Two fuzzy sets $\widetilde{A}$ and $\widetilde{B}$ are said to be equal(denoted $\widetilde{A} = \widetilde{B}$) if and only if*

$$\forall x \in X, \qquad \mu_{\widetilde{A}}(x) = \mu_{\widetilde{B}}(x). \quad (2.3)$$

3 Description of the new attitude to ranking fuzzy numbers

It is useful, in dealing with linear orderings, to visualize what they look like. Although this is difficult to do with arbitrary mathematical structures, it is possible with linear orderings. In spite of many ranking methods, no one can rank fuzzy numbers with human intuition consistently in all cases. To overcome ordering issues, with a new attitude coupled with the basic thinking to ordering, we define a fuzzy linear (total) order on the fuzzy numbers set. Accordingly, we introduce the fuzzy numbers set as a fuzzy comparable set, namely, we can ordered any number of the fuzzy numbers. Finally, in order to the description new proposed attitude, several basic properties and illustrative examples will present for ranking fuzzy numbers.

Definition 3.1 *Let $\widetilde{A}$ be a normal, convex and continuous (NCC) fuzzy set on the universal set U. Then,*
i) $a.c(\widetilde{A}) = \frac{1}{2}\{min(core(\tilde{A})) + max(core(\tilde{A}))\}$,
ii) $\underline{support}(\widetilde{A}) = \{x \in U \mid x \leq a.c(\widetilde{A})\}$,
iii) $\overline{support}(\widetilde{A}) = \{x \in U \mid x \geq a.c(\widetilde{A})\}$,
iv) $\underline{p}_{\widetilde{A}} = \{x \in support(\widetilde{A}) \mid inf(support(\widetilde{A})) \leq x \leq min(core(\widetilde{A}))\}$,
v) $\overline{p}_{\widetilde{A}} = \{x \in support(\widetilde{A}) \mid max(core(\widetilde{A})) \leq x \leq sup(support(\widetilde{A}))\}$,
vi) $\mu_{\widetilde{A}}(x) = \begin{cases} \underline{\mu}_{\widetilde{A}}(x), & x \in \underline{p}_{\widetilde{A}}, \\ 1, & x \in core(\widetilde{A}), \\ \overline{\mu}_{\widetilde{A}}(x), & x \in \overline{p}_{\widetilde{A}}, \\ 0, & otherwise. \end{cases}$

Note: if $U = \mathbb{R}$ in the above definition, then NCC fuzzy set is fuzzy number.

Definition 3.2 *(Fuzzy less) Let $\widetilde{A}, \widetilde{B}$ are two the pseudo-geometric fuzzy numbers. Then,*
1) $\widetilde{A} \prec_{a.s} \widetilde{B}$ iff $a.c(\widetilde{A}) < a.c(\widetilde{B})$ and $inf(supp(\widetilde{B})) \leq inf(supp(\widetilde{A}))$,
2) $\widetilde{A} \prec_{b.s} \widetilde{B}$ iff $a.c(\widetilde{A}) < a.c(\widetilde{B})$ and $inf(supp(\widetilde{A})) \leq inf(supp(\widetilde{B})) \leq sup(supp(\widetilde{A}))$,
3) $\widetilde{A} \prec_{n.s} \widetilde{B}$ iff $a.c(\widetilde{A}) < a.c(\widetilde{B})$ and $inf(supp(\widetilde{B})) \geq sup(supp(\widetilde{A}))$,
4) $\widetilde{A} \prec_{a.b.s} \widetilde{B}$ iff $\widetilde{A} \prec_{a.s} \widetilde{B}$ and $\widetilde{A} \prec_{b.s} \widetilde{B}$,
5) $\widetilde{A} \prec_{b.n.s} \widetilde{B}$ iff $\widetilde{A} \prec_{b.s} \widetilde{B}$ and $\widetilde{A} \prec_{n.s} \widetilde{B}$,
finally,

$$\boldsymbol{\widetilde{A} \prec \widetilde{B}} \ \textbf{\textit{iff}} \ \boldsymbol{\widetilde{A} \prec_{a.s} \widetilde{B}} \ \textbf{\textit{or}} \ \boldsymbol{\widetilde{A} \prec_{b.s} \widetilde{B}} \ \textbf{\textit{or}} \ \boldsymbol{\widetilde{A} \prec_{n.s} \widetilde{B}}.$$

Remark 3.1 *The $\boldsymbol{\widetilde{A}} \prec_{a.s} \boldsymbol{\widetilde{B}}$ and $\boldsymbol{\widetilde{A}} \prec_{n.s} \boldsymbol{\widetilde{B}}$ can not occur together.*

Definition 3.3 *(Fuzzy approximation) Let $\widetilde{A}, \widetilde{B}$ are two the pseudo-geometric fuzzy numbers. Then,*
1) $\widetilde{A} \precapprox_{a.s} \widetilde{B}$ iff $a.c(\widetilde{A}) = a.c(\widetilde{B})$ and $inf(supp(\widetilde{B})) \leq inf(supp(\widetilde{A}))$,
2) $\widetilde{A} \cong \widetilde{B}$ iff $a.c(\widetilde{A}) = a.c(\widetilde{B})$ and $inf(supp(\widetilde{B})) = inf(supp(\widetilde{A}))$,
3) $\widetilde{A} \precapprox_{b.s} \widetilde{B}$ iff $a.c(\widetilde{A}) = a.c(\widetilde{B})$ and $inf(supp(\widetilde{B})) \geq inf(supp(\widetilde{A}))$,
finally,

$$\boldsymbol{\widetilde{A} \simeq \widetilde{B}} \ \textbf{\textit{iff}} \ \boldsymbol{\widetilde{A} \cong \widetilde{B}} \ \textbf{\textit{or}} \ \boldsymbol{\widetilde{A} \precapprox_{a.s} \widetilde{B}} \ \textbf{\textit{or}} \ \boldsymbol{\widetilde{A} \precapprox_{b.s} \widetilde{B}}.$$

Remark 3.2 *Let $\widetilde{A}, \widetilde{B}$ are two the pseudo-geometric fuzzy numbers. Then,*

$$\widetilde{A} \precapprox_{a.s} \widetilde{B} \quad \textit{iff} \quad \widetilde{B} \precapprox_{b.s} \widetilde{A}.$$

Definition 3.4 *(Fuzzy less then or approximation) Let $\widetilde{A}, \widetilde{B}$ are two the pseudo-geometric fuzzy numbers. Then,*

$$\widetilde{A} \precsim \widetilde{B} \quad \text{iff} \quad \widetilde{A} \prec \widetilde{B} \quad \text{or} \quad \widetilde{A} \simeq \widetilde{B}.$$

Remark 3.3 *(Fuzzy positive) Let $F_c(\mathbb{R})$ be a set of the pseudo-geometric fuzzy numbers, and $F_0(\mathbb{R}) = \{\widetilde{A} \in F_c(\mathbb{R}) \mid a.c(\widetilde{A}) = 0\}$. Then,*
$\forall \widetilde{A} \in F_c(\mathbb{R}), \qquad \widetilde{0} \in F_0(\mathbb{R})$;
1) $\widetilde{0} \prec_{a.s} \widetilde{A}$ *iff* $0 < a.c(\widetilde{A})$ *and* $inf(supp(\widetilde{A})) \leq inf(supp(\widetilde{0}))$,
2) $\widetilde{0} \prec_{b.s} \widetilde{A}$ *iff* $0 < a.c(\widetilde{A})$ *and* $inf(supp(\widetilde{0})) \leq inf(supp(\widetilde{A})) \leq sup(supp(\widetilde{0}))$,
3) $\widetilde{0} \prec_{n.s} \widetilde{A}$ *iff* $0 < a.c(\widetilde{A})$ *and* $inf(supp(\widetilde{A})) \geq sup(supp(\widetilde{0}))$,
4) $\widetilde{A} \cong \widetilde{0}$ *iff* $a.c(\widetilde{A}) = 0$ *and* $inf(supp(\widetilde{0})) = inf(supp(\widetilde{A}))$,
5) $\widetilde{A} \gtrapprox_{a.s} \widetilde{0}$ *iff* $a.c(\widetilde{A}) = 0$ *and* $inf(supp(\widetilde{0})) \leq inf(supp(\widetilde{A}))$,
6) $\widetilde{A} \gtrapprox_{b.s} \widetilde{0}$ *iff* $a.c(\widetilde{A}) = 0$ *and* $inf(supp(\widetilde{0})) \geq inf(supp(\widetilde{A}))$,
finally,

$$\widetilde{0} \prec \widetilde{A} \text{ iff } \widetilde{0} \prec_{a.s} \widetilde{A} \text{ or } \widetilde{0} \prec_{b.s} \widetilde{A} \text{ or } \widetilde{0} \prec_{n.s} \widetilde{A},$$
$$\widetilde{A} \simeq \widetilde{0} \text{ iff } \widetilde{A} \cong \widetilde{0} \text{ or } \widetilde{A} \gtrapprox_{a.s} \widetilde{0} \text{ or } \widetilde{A} \gtrapprox_{b.s} \widetilde{0}.$$

Definition 3.5 *(Fuzzy partial order) A binary relation (here denoted by infix $\precsim$) on a fuzzy set $\widetilde{X}$ is a fuzzy partial order if and only if it is*
1) reflexive,
2) fuzzy anti-symmetric,
3) transitive.
The pair $(\widetilde{X}, \precsim)$ is called a fuzzy partially ordered set (fuzzy poset). Incidentally,
for all $\widetilde{A}, \widetilde{B}, \widetilde{C} \in \widetilde{X}$:
a. $\widetilde{A} \precsim \widetilde{A}$, the reflexive property,
b. If $\widetilde{A} \precsim \widetilde{B}$ and $\widetilde{B} \precsim \widetilde{A}$ then $\widetilde{A} \simeq \widetilde{B}$, the fuzzy anti-symmetric property,
c. If $\widetilde{A} \precsim \widetilde{B}$ and $\widetilde{B} \precsim \widetilde{C}$ then $\widetilde{A} \precsim \widetilde{C}$, the transitive property.

Definition 3.6 *(Fuzzy comparable) Any two elements $\widetilde{A}$ and $\widetilde{B}$ of a fuzzy set $\widetilde{X}$ that is fuzzy partially ordered by a binary relation $\precsim$, are fuzzy comparable when $\widetilde{A} \precsim \widetilde{B}$ or $\widetilde{B} \precsim \widetilde{A}$. If it is not the case that $\widetilde{A}$ and $\widetilde{B}$ are fuzzy comparable, then they are called fuzzy incomparable.*

Definition 3.7 *(Fuzzy total order) A binary relation $\precsim$ on a fuzzy set $\widetilde{X}$ is a fuzzy total order if and only if it is*
1) a fuzzy partial order,
2) for any pair of elements $\widetilde{A}$ and $\widetilde{B}$ of $\widetilde{X}$, $\widetilde{A} \precsim \widetilde{B}$ or $\widetilde{B} \precsim \widetilde{A}$ (the totality property).
A fuzzy total order is also called a fuzzy linear order. Thus, a fuzzy totally ordered set is exactly a fuzzy poset in which every pair of elements is fuzzy comparable.

4 Theorems and properties

In this section, we introduce the pseudo-geometric fuzzy numbers set as a fuzzy totally ordered set and discuss some of its properties.

Lemma 4.1 *Let $F_c(\mathbb{R})$ be a set of the pseudo-geometric fuzzy numbers, then*
$\forall \widetilde{A}, \widetilde{B} \in F_c(\mathbb{R})$;
i) $\widetilde{A} \precsim \widetilde{B}$ or $\widetilde{B} \precsim \widetilde{A}$,
ii) $\widetilde{A} \precsim \widetilde{A}$,
iii) If $\widetilde{A} \precsim \widetilde{B}$ and $\widetilde{B} \precsim \widetilde{A}$ then $\widetilde{A} \simeq \widetilde{B}$,
iv) If $\widetilde{A} \precsim \widetilde{B}$ and $\widetilde{B} \precsim \widetilde{C}$ then $\widetilde{A} \precsim \widetilde{C}$.

Proof. We have the above cases, according to the definitions (3.4), (3.3) and (3.2).

Theorem 4.1 *Let $F_c(\mathbb{R})$ be a set of the pseudo-geometric fuzzy numbers, then the pair $(F_c(\mathbb{R}), \precsim)$ is a fuzzy totally ordered set.*

Proof. We have the above case, according to the lemma (4.1) and the definition (3.7).

Therefore, the pair $(F_c(\mathbb{R}), \precsim)$ as a fuzzy totally ordered set means that any pair of elements in the $F_c(\mathbb{R})$ of the relation are fuzzy comparable under the relation. This also means that the $F_c(\mathbb{R})$ can be diagrammed as a quasi-line of elements, giving it the name quasi-linear.
We define a parameter as ambiguity rank, that as a comparison coefficient between ranking fuzzy numbers and ranking crisp numbers can be considered. Then, we show that, total order is a special case of the proposed fuzzy total order.

Definition 4.1 *(Binary relation with ambiguity rank) Let $F_c(\mathbb{R})$ be a set of the pseudo-geometric fuzzy numbers. Then, ambiguity rank of $\widetilde{A} \precsim \widetilde{B}$ is defined with $ar = (l_{1\widetilde{A},\widetilde{B}}, r_{1\widetilde{A},\widetilde{B}}, l_{2\widetilde{A},\widetilde{B}}, r_{2\widetilde{A},\widetilde{B}})$ as follows:*

$l_{1\widetilde{A},\widetilde{B}} = d(sup\{d(\underline{\mu}_{\widetilde{A}}^{-1}(\alpha), a.c(\widetilde{A})) \mid \alpha \in (0,1]\}, sup\{d(\underline{\mu}_{\widetilde{B}}^{-1}(\alpha), a.c(\widetilde{B})) \mid \alpha \in (0,1]\}),$
$r_{1\widetilde{A},\widetilde{B}} = d(sup\{d(\overline{\mu}_{\widetilde{A}}^{-1}(\alpha), a.c(\widetilde{A})) \mid \alpha \in (0,1]\}, sup\{d(\overline{\mu}_{\widetilde{B}}^{-1}(\alpha), a.c(\widetilde{B})) \mid \alpha \in (0,1]\}),$
$l_{2\widetilde{A},\widetilde{B}} = sup\{d(\underline{\mu}_{\widetilde{A}}(x), \underline{\mu}_{\widetilde{B}}(y)) \mid \forall x \in \underline{support}(\widetilde{A}), \forall y \in \underline{support}(\widetilde{B}); d(x, a.c(\widetilde{A})) = d(y, a.c(\widetilde{B}))\},$
$r_{2\widetilde{A},\widetilde{B}} = sup\{d(\overline{\mu}_{\widetilde{A}}(x), \overline{\mu}_{\widetilde{B}}(y)) \mid \forall x \in \overline{support}(\widetilde{A}), \forall y \in \overline{support}(\widetilde{B}); d(x, a.c(\widetilde{A})) = d(y, a.c(\widetilde{B}))\}.$
We denote the $\precsim$ with ambiguity rank by:

$$\widetilde{A} \precsim_{ar} \widetilde{B}. \tag{4.4}$$

Definition 4.2 *(Ambiguity rank of fuzzy totally ordered set) Let $F_c(\mathbb{R})$ be a set of the pseudo-geometric fuzzy numbers. Then, ambiguity rank of the pair $(F_c(\mathbb{R}), \precsim)$ is defined with $F_c ar = (l_{1F_c(\mathbb{R})}, r_{1F_c(\mathbb{R})}, l_{2F_c(\mathbb{R})}, r_{2F_c(\mathbb{R})})$ as follows:*

$$l_{1F_c(\mathbb{R})} = sup\{l_{1\widetilde{A},\widetilde{B}} \mid \widetilde{A}, \widetilde{B} \in F_c(\mathbb{R})\},$$
$$r_{1F_c(\mathbb{R})} = sup\{r_{1\widetilde{A},\widetilde{B}} \mid \widetilde{A}, \widetilde{B} \in F_c(\mathbb{R})\},$$
$$l_{2F_c(\mathbb{R})} = sup\{l_{2\widetilde{A},\widetilde{B}} \mid \widetilde{A}, \widetilde{B} \in F_c(\mathbb{R})\},$$
$$r_{2F_c(\mathbb{R})} = sup\{r_{2\widetilde{A},\widetilde{B}} \mid \widetilde{A}, \widetilde{B} \in F_c(\mathbb{R})\}.$$

Remark 4.1 *Another natural characteristic of a fuzzy totally ordered set is the number of elements it contains. We call this the order of fuzzy totally ordered set. This number is, of course, most interesting when it is finite. In that case we say that fuzzy totally ordered set is finite. Then, ambiguity rank of fuzzy totally ordered set will be defined with the change sup to max in the definition (4.2).*

Definition 4.3 *(Type-1 fuzzy par) Let $\widetilde{A}, \widetilde{B}$ are two NCC fuzzy sets. Then,*
$\widetilde{A} \sim_1 \widetilde{B}$ if and only if
1. $\forall x \in \underline{support}(\widetilde{A}), \forall y \in \underline{support}(\widetilde{B})$; if $d(x, a.c(\widetilde{A})) = d(y, a.c(\widetilde{B}))$ then $\underline{\mu}_{\widetilde{A}}(x) = \underline{\mu}_{\widetilde{B}}(x)$,
2. $\forall x \in \overline{support}(\widetilde{A}), \forall y \in \overline{support}(\widetilde{B})$; if $d(x, a.c(\widetilde{A})) = d(y, a.c(\widetilde{B}))$ then $\overline{\mu}_{\widetilde{A}}(x) = \overline{\mu}_{\widetilde{B}}(x)$.

Lemma 4.2 *Let $\widetilde{A}, \widetilde{B}$ are two NCC fuzzy sets. Then,*

$$\widetilde{A} = \widetilde{B} \;\; iff \;\; \widetilde{A} \simeq \widetilde{B} \text{ and } \widetilde{A} \sim_1 \widetilde{B}.$$

Proof. According to the definitions, it is obvious.

Lemma 4.3 *Let $(F_c(\mathbb{R}), \precsim)$ be a fuzzy totally ordered set of the pseudo-geometric fuzzy numbers, with the ambiguity rank $F_c ar = (0,0,0,0)$. Then,*
$\forall \widetilde{A}, \widetilde{B} \in F_c(\mathbb{R}); \qquad \widetilde{A} \sim_1 \widetilde{B}.$

Proof.
Suppose $\widetilde{A}, \widetilde{B} \in F_c(\mathbb{R})$.
Then, we have from $(l_{1F_c(\mathbb{R})}, r_{1F_c(\mathbb{R})}) = (0,0)$:
$sup\{d(\underline{\mu}_{\widetilde{A}}^{-1}(\alpha), a.c(\widetilde{A})) \mid \alpha \in (0,1]\} = sup\{d(\underline{\mu}_{\widetilde{B}}^{-1}(\alpha), a.c(\widetilde{B})) \mid \alpha \in (0,1]\},$
$sup\{d(\overline{\mu}_{\widetilde{A}}^{-1}(\alpha), a.c(\widetilde{A})) \mid \alpha \in (0,1]\} = sup\{d(\overline{\mu}_{\widetilde{B}}^{-1}(\alpha), a.c(\widetilde{B})) \mid \alpha \in (0,1]\}.$
So,
$\mid \{x \in \underline{support}(\widetilde{A}) \mid \underline{\mu}_{\widetilde{A}}(x) > 0\} \mid = \mid \{x \in \underline{support}(\widetilde{B}) \mid \underline{\mu}_{\widetilde{B}}(x) > 0\} \mid,$
$\mid \{x \in \overline{support}(\widetilde{A}) \mid \overline{\mu}_{\widetilde{A}}(x) > 0\} \mid = \mid \{x \in \overline{support}(\widetilde{B}) \mid \overline{\mu}_{\widetilde{B}}(x) > 0\} \mid.$
(Note: $\mid \cdot \mid = sup(\cdot) - inf(\cdot)$)

Therefore, have Prerequisite for $\widetilde{A} \sim_1 \widetilde{B}$.

Also, we have from $(l_{2F_c(\mathbb{R})}, r_{2F_c(\mathbb{R})}) = (0,0)$:

$\forall x \in \underline{support}(\widetilde{A}), \forall y \in \underline{support}(\widetilde{B});$

$d(x, a.c(\widetilde{A})) = d(y, a.c(\widetilde{B})) \Rightarrow$

$$d(\underline{\mu}_{\widetilde{A}}(x), \underline{\mu}_{\widetilde{B}}(y)) = 0.$$

Namely,

$$\underline{\mu}_{\widetilde{A}}(x) = \underline{\mu}_{\widetilde{B}}(y). \tag{4.5}$$

And,

$\forall x \in \overline{support}(\widetilde{A}), \forall y \in \overline{support}(\widetilde{B});$

$d(x, a.c(\widetilde{A})) = d(y, a.c(\widetilde{B})) \Rightarrow$

$$d(\overline{\mu}_{\widetilde{A}}(x), \overline{\mu}_{\widetilde{B}}(y)) = 0.$$

Namely,

$$\overline{\mu}_{\widetilde{A}}(x) = \overline{\mu}_{\widetilde{B}}(y). \tag{4.6}$$

Finally, of (4.5), (4.6) and the definition (4.3) have $\widetilde{A} \sim_1 \widetilde{B}$.

Now, according to the cases mentioned above, we provide the relation between fuzzy totally ordered set and totally ordered set as a theorem in the following:

Theorem 4.2 *Let $(F_c(\mathbb{R}), \precsim)$ be a fuzzy totally ordered set of the pseudo-geometric fuzzy numbers, with the ambiguity rank $F_c ar = (0, 0, 0, 0)$. Then, the pair $(F_c(\mathbb{R}), \precsim)$ is totally ordered set.*

Proof. It is straightforward with the lemma (4.3) and (4.2).

In the future, with the new proposed ranking, we can have a new attitude to fuzzy distance. In this paper, we discuss a few related Lemma. Before it is necessary that the fuzzy arithmetic operations based on TA (in the domain of the transmission average of support) be introduced for pseudo-geometric fuzzy numbers.
As regards fuzzy arithmetic operations using of the extension principle (in the domain of the membership function) or the interval arithmetics (in the domain of the α- cuts), we have some problem in subtraction operator, division operator and obtaining the membership functions of operators.
Although with the revised definitions in [25] on subtraction and division, usage of an interval arithmetic for fuzzy operators have been permitted, because it always exists, but its not efficient, it means that result's support is major agent (dependence effect) and also complex calculations of interval arithmetic in determining the membership function of operators based on the extension principle , are not yet resolved.
We eliminated such deficiency with the fuzzy arithmetic operations based on TA in the [1] as follows:

Definition 4.4 *(The fuzzy arithmetic operations based on TA for pseudo-triangular fuzzy numbers)*
Consider two pseudo-triangular fuzzy number

$$\widetilde{A} = (\underline{a}, a, \overline{a}, l_{\widetilde{A}}(x), r_{\widetilde{A}}(x)),$$

$$\widetilde{B} = (\underline{b}, b, \overline{b}, l_{\widetilde{B}}(x), r_{\widetilde{B}}(x)).$$

With the following α-cut forms:

$$\tilde{A} = \bigcup_\alpha A_\alpha, A_\alpha = [\underline{A}_\alpha, \overline{A}_\alpha], 0 \le \alpha \le 1,$$
$$\tilde{B} = \bigcup_\alpha B_\alpha, B_\alpha = [\underline{B}_\alpha, \overline{B}_\alpha], 0 \le \alpha \le 1.$$

In the following, we define fuzzy arithmetic operations based on TA for addition, subtraction, multiplication and division:

$$\widetilde{A+B} = \bigcup_\alpha (\widetilde{A+B})_\alpha,$$
$$(\widetilde{A+B})_\alpha = [\tfrac{a+b}{2} + (\tfrac{\underline{A}_\alpha + \underline{B}_\alpha}{2}), \tfrac{a+b}{2} + (\tfrac{\overline{A}_\alpha + \overline{B}_\alpha}{2})]. \tag{4.7}$$

$$\widetilde{-A} = \bigcup_\alpha (\widetilde{-A})_\alpha, (\widetilde{-A})_\alpha = [-2a + \underline{A}_\alpha, -2a + \overline{A}_\alpha]. \tag{4.8}$$

$$\widetilde{A-B} = \bigcup_\alpha (\widetilde{A-B})_\alpha, (\widetilde{A-B})_\alpha = [\tfrac{a-3b}{2} + (\tfrac{\underline{A}_\alpha + \underline{B}_\alpha}{2}), \tfrac{a-3b}{2} + (\tfrac{\overline{A}_\alpha + \overline{B}_\alpha}{2})]. \tag{4.9}$$

$$\widetilde{A.B} = \bigcup_\alpha (\widetilde{A.B})_\alpha,$$
$$(\widetilde{A.B})_\alpha =$$
$$\begin{cases} [(\frac{b}{2})\underline{A}_\alpha + (\frac{a}{2})\underline{B}_\alpha,\ (\frac{b}{2})\overline{A}_\alpha + (\frac{a}{2})\overline{B}_\alpha], \\ a \ge 0, b \ge 0, \\ [(\frac{b}{2})\overline{A}_\alpha + (\frac{a}{2})\underline{B}_\alpha,\ (\frac{b}{2})\underline{A}_\alpha + (\frac{a}{2})\overline{B}_\alpha], \\ a \ge 0, b \le 0, \\ [(\frac{b}{2})\overline{A}_\alpha + (\frac{a}{2})\overline{B}_\alpha,\ (\frac{b}{2})\underline{A}_\alpha + (\frac{a}{2})\underline{B}_\alpha], \\ a \le 0, b \le 0, \\ [(\frac{b}{2})\underline{A}_\alpha + (\frac{a}{2})\overline{B}_\alpha,\ (\frac{b}{2})\overline{A}_\alpha + (\frac{a}{2})\underline{B}_\alpha], \\ a \le 0, b \ge 0. \end{cases} \tag{4.10}$$

$$\widetilde{A^{-1}} = \bigcup_\alpha (\widetilde{A^{-1}})_\alpha,\ (\widetilde{A^{-1}})_\alpha = [\frac{1}{a^2}\underline{A}_\alpha,\ \frac{1}{a^2}\overline{A}_\alpha]. \tag{4.11}$$

$$\widetilde{A.B^{-1}} = \bigcup_\alpha (\widetilde{A.B^{-1}})_\alpha,$$
$$(\widetilde{A.B^{-1}})_\alpha =$$
$$\begin{cases} [(\frac{1}{2b})\underline{A}_\alpha + (\frac{a}{2b^2})\underline{B}_\alpha,\ (\frac{1}{2b})\overline{A}_\alpha + (\frac{a}{2b^2})\overline{B}_\alpha], \\ \quad a \ge 0, b > 0, \\ [(\frac{1}{2b})\overline{A}_\alpha + (\frac{a}{2b^2})\underline{B}_\alpha,\ (\frac{1}{2b})\underline{A}_\alpha + (\frac{a}{2b^2})\overline{B}_\alpha], \\ \quad a \ge 0, b < 0, \\ [(\frac{1}{2b})\overline{A}_\alpha + (\frac{a}{2b^2})\overline{B}_\alpha,\ (\frac{1}{2b})\underline{A}_\alpha + (\frac{a}{2b^2})\underline{B}_\alpha], \\ \quad a \le 0, b < 0, \\ [(\frac{1}{2b})\underline{A}_\alpha + (\frac{a}{2b^2})\overline{B}_\alpha,\ (\frac{1}{2b})\overline{A}_\alpha + (\frac{a}{2b^2})\underline{B}_\alpha], \\ \quad a \le 0, b > 0. \end{cases} \tag{4.12}$$

Remark 4.2 *Fuzzy arithmetic operation division on pseudo-triangular fuzzy number $\tilde{0} = (\underline{0},\ 0,\ \overline{0},\ l_{\tilde{0}}(x),\ r_{\tilde{0}}(x))$ is not able to define.*

For a more comprehensive definition, we apply our proposed fuzzy arithmetic operations on pseudo-trapezoidal fuzzy numbers. It should be noted that the above definition is special case of below definition.

Definition 4.5 *(The fuzzy arithmetic operations based on TA for pseudo-trapezoidal fuzzy numbers)*
Consider two pseudo-trapezoidal fuzzy number
$\widetilde{A} = (\underline{a}, a_1, a_2, \overline{a}, l_{\widetilde{A}}(x), r_{\widetilde{A}}(x))$,
$\widetilde{B} = (\underline{b}, b_1, b_2, \overline{b}, l_{\widetilde{B}}(x), r_{\widetilde{B}}(x))$.
with the following α-cut forms:
$\tilde{A} = \bigcup_\alpha A_\alpha, A_\alpha = [\underline{A}_\alpha, \overline{A}_\alpha]$,
$0 \le \alpha \le 1, A_1 = [a_1,\ a_2]$,
$\tilde{B} = \bigcup_\alpha B_\alpha, B_\alpha = [\underline{B}_\alpha, \overline{B}_\alpha]$,
$0 \le \alpha \le 1, B_1 = [b_1,\ b_2]$. *Let*

$$\phi = \frac{a_1 + a_2}{2}, \qquad \psi = \frac{b_1 + b_2}{2}.$$

In the following, we define fuzzy arithmetic operations based on TA for addition, subtraction, multiplication and division:

$$\widetilde{A+B} = \bigcup_\alpha (\widetilde{A+B})_\alpha,$$
$$(\widetilde{A+B})_\alpha = [\tfrac{\phi+\psi}{2} + (\tfrac{\underline{A}_\alpha + \underline{B}_\alpha}{2}),\ \tfrac{\phi+\psi}{2} + (\tfrac{\overline{A}_\alpha + \overline{B}_\alpha}{2})]. \tag{4.13}$$

$$\widetilde{-A} = \bigcup_\alpha (\widetilde{-A})_\alpha, (\widetilde{-A})_\alpha = [-2\phi + \underline{A}_\alpha,\ -2\phi + \overline{A}_\alpha]. \tag{4.14}$$

$$\widetilde{A-B} = \bigcup_\alpha (\widetilde{A-B})_\alpha,$$
$$(\widetilde{A-B})_\alpha = [\tfrac{\phi-3\psi}{2} + (\tfrac{\underline{A}_\alpha + \underline{B}_\alpha}{2}),\ \tfrac{\phi-3\psi}{2} + (\tfrac{\overline{A}_\alpha + \overline{B}_\alpha}{2})]. \tag{4.15}$$

$$\widetilde{A.B} = \bigcup_\alpha (\widetilde{A.B})_\alpha,$$
$$(\widetilde{A.B})_\alpha =$$
$$\begin{cases} [(\frac{\psi}{2})\underline{A}_\alpha + (\frac{\phi}{2})\underline{B}_\alpha,\ (\frac{\psi}{2})\overline{A}_\alpha + (\frac{\phi}{2})\overline{B}_\alpha], \\ \quad \phi \ge 0, \psi \ge 0, \\ [(\frac{\psi}{2})\overline{A}_\alpha + (\frac{\phi}{2})\underline{B}_\alpha,\ (\frac{\psi}{2})\underline{A}_\alpha + (\frac{\phi}{2})\overline{B}_\alpha], \\ \quad \phi \ge 0,\ \psi \le 0, \\ [(\frac{\psi}{2})\overline{A}_\alpha + (\frac{\phi}{2})\overline{B}_\alpha,\ (\frac{\psi}{2})\underline{A}_\alpha + (\frac{\phi}{2})\underline{B}_\alpha], \\ \quad \phi \le 0,\ \psi \le 0, \\ [(\frac{\psi}{2})\underline{A}_\alpha + (\frac{\phi}{2})\overline{B}_\alpha,\ (\frac{\psi}{2})\overline{A}_\alpha + (\frac{\phi}{2})\underline{B}_\alpha], \\ \quad \phi \le 0,\ \psi \ge 0. \end{cases} \tag{4.16}$$

$$\widetilde{A^{-1}} = \bigcup_\alpha (\widetilde{A^{-1}})_\alpha,$$
$$(\widetilde{A^{-1}})_\alpha = [(\tfrac{1}{\phi^2})\underline{A}_\alpha\ ,\ (\tfrac{1}{\phi^2})\overline{A}_\alpha]. \tag{4.17}$$

$$\widetilde{A.B^{-1}} = \bigcup_\alpha (\widetilde{A.B^{-1}})_\alpha,$$
$$(\widetilde{A.B^{-1}})_\alpha =$$
$$\begin{cases} [(\frac{1}{2\psi})\underline{A}_\alpha + (\frac{\phi}{2\psi^2})\underline{B}_\alpha,\ (\frac{1}{2\psi})\overline{A}_\alpha + (\frac{\phi}{2\psi^2})\overline{B}_\alpha], \\ \quad \phi \ge 0,\ \psi > 0, \\ [(\frac{1}{2\psi})\overline{A}_\alpha + (\frac{\phi}{2\psi^2})\underline{B}_\alpha,\ (\frac{1}{2\psi})\underline{A}_\alpha + (\frac{\phi}{2\psi^2})\overline{B}_\alpha], \\ \quad \phi \ge 0,\ \psi < 0, \\ [(\frac{1}{2\psi})\overline{A}_\alpha + (\frac{\phi}{2\psi^2})\overline{B}_\alpha,\ (\frac{1}{2\psi})\underline{A}_\alpha + (\frac{\phi}{2\psi^2})\underline{B}_\alpha], \\ \quad \phi \le 0,\ \psi < 0, \\ [(\frac{1}{2\psi})\underline{A}_\alpha + (\frac{\phi}{2\psi^2})\overline{B}_\alpha,\ (\frac{1}{2\psi})\overline{A}_\alpha + (\frac{\phi}{2\psi^2})\underline{B}_\alpha], \\ \quad \phi \le 0,\ \psi > 0. \end{cases} \tag{4.18}$$

Remark 4.3 *The fuzzy arithmetic operation division on fuzzy number $\tilde{0}$ ($a.c(\tilde{0}) = 0$), is not able to define.*

Therefore, in order to introduce a new attitude coupled with the basic thinking to distance for fuzzy distance in the future, provided a lemma and theorem related to the subject as follows.

Lemma 4.4 *Let $F_c(\mathbb{R})$ be a set of the pseudo-geometric fuzzy numbers, and*
$F_0(\mathbb{R}) = \{\widetilde{A} \in F_c(\mathbb{R}) \mid a.c(\widetilde{A}) = 0\}$. *Then,*
$\forall \widetilde{A}, \widetilde{B}, \widetilde{C} \in F_c(\mathbb{R}), \qquad \tilde{0} \in F_0(\mathbb{R})$;
1) if $\widetilde{A} \precsim \widetilde{B}$ *then* $\widetilde{A \pm C} \precsim \widetilde{B \pm C}$,
2) if $\widetilde{A} \precsim \widetilde{B}$ *then*

$$\begin{cases} \widetilde{A.C} \precsim \widetilde{B.C}, & \tilde{0} \prec \widetilde{C}, \\ \widetilde{B.C} \precsim \widetilde{A.C}, & \widetilde{C} \prec \tilde{0}, \end{cases}$$

3) if $\widetilde{A} \precsim \widetilde{B}$ *and* $\widetilde{C} \precsim \widetilde{D}$ *then*

$$\widetilde{A+C} \precsim \widetilde{B+D},$$

4) if $\tilde{0} \prec \widetilde{A} \precsim \widetilde{B}$ *and* $\tilde{0} \prec \widetilde{C} \precsim \widetilde{D}$ *then*

$$\widetilde{A.C} \precsim \widetilde{B.D}.$$

Proof.we have the above cases, according to the definitions of (3.2), (3.3), (3.4), (4.4), (4.5) and remark (3.3).

Theorem 4.3 *Let $F_c(\mathbb{R})$ be a set of the pseudo-geometric fuzzy numbers, then*
$\forall \widetilde{A}, \widetilde{B} \in F_c(\mathbb{R})$;
1) If $\widetilde{A} \precsim \widetilde{B}$ *then* $\exists \widetilde{X}, \widetilde{Y} \in F_c(\mathbb{R})$; $\widetilde{A+X} \simeq \widetilde{B}$ *and* $\widetilde{B+Y} \simeq \widetilde{A}$,
2) If $\widetilde{A} \precsim \widetilde{B}$ *and* $\widetilde{A} \sim_1 \widetilde{B}$ *then* $\exists \widetilde{X}, \widetilde{Y} \in F_c(\mathbb{R})$; $\widetilde{A+X} = \widetilde{B}$ *and* $\widetilde{B+Y} = \widetilde{A}$.

Proof.we have the above cases, with the $\widetilde{X} = \widetilde{B-A}$ and $\widetilde{Y} = \widetilde{A-B}$.

5 Illustrative example and comparison study

It is reasonable to expect that different ranking methods can produce different ranking order for the same sample of fuzzy numbers and some of them seem to be good in a particular context but not in general. Intricacies like these make ranking fuzzy numbers rather difficult.
Although the proposed attitude for ranking fuzzy numbers is different from the other methods, however we use some comparative examples to illustrate the advantages of the proposed method.

Example 5.1 *Consider the four fuzzy numbers* $A = (-4, 1, 2, -, -), B = (\frac{-7}{4}, \frac{1}{4}, \frac{5}{4}, -, -)$, $C = (-7, 2, 3, -, -)$ *and* $D = (-2, 0, 1, 1, -, -)$, *taken from paper[2], with the following fig. 1.*

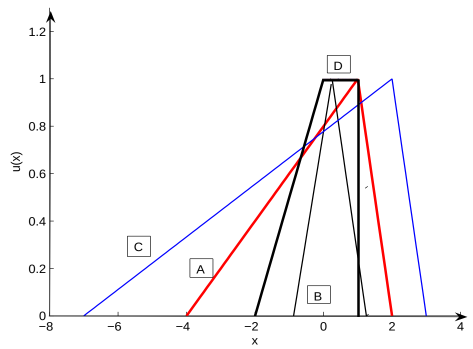

Figure 1

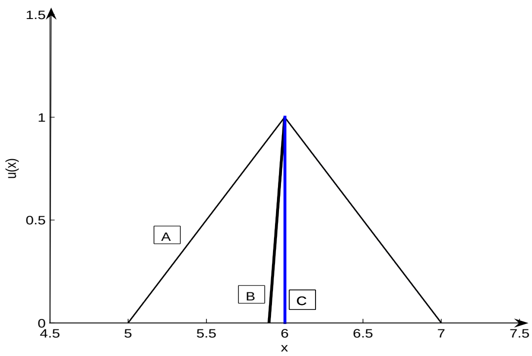

Figure 2: Velocity profiles for various values of λ.

Example 5.2 *Consider the three triangular fuzzy numbers,* $A = (5, 6, 7, -, -)$, $B = (5.9, 6, 7, -, -)$ *and* $C = (6, 6, 7, -, -)$, *taken from paper[2], with the following fig. 2.*

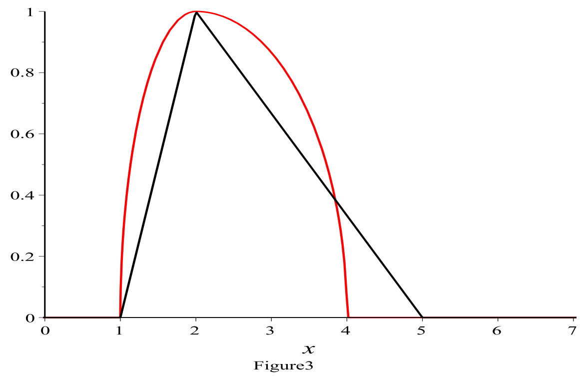

Figure 3

By using our method, $A \precsim_{a.s} B \precsim_{b.s} C$.
Thus the ranking order is
$A \simeq B \simeq C$.
To compare with some of other methods, the reader can refer to the following Table 2.

Example 5.3 *Consider the two fuzzy numbers,* $A = (1, 2, 5, -, -)$ *and* $B = (1, 2, 4, l_B(x), r_B(x))$, *as*
$l_B(x) = (1-(x-2)^2)^{\frac{1}{2}}, r_B(x) = (1-\frac{1}{4}(x-2)^2)^{\frac{1}{2}}$, *taken from paper[11], with the following fig. 3.*

By using our method, $A \cong B$. Thus the ranking order is
$A \simeq B$.
To compare with some of other methods, the reader can refer to the following Table 3.
All the above numerical examples show that the results of the proposed method, similar to the intuitively ranking order, can overcome the drawbacks of the other methods.

6 Conclusion

Ranking of fuzzy numbers is an important issue in the study of fuzzy set theory. In order to rank fuzzy numbers, one fuzzy number needs to be compared with the others but it is difficult to determine clearly which of them is larger or smaller. Numerous methods have been proposed in previous studies to rank fuzzy numbers. There is not a unique method for comparing fuzzy numbers. As a result, it is reasonable to expect that different ranking methods can produce different ranking order for the same sample of fuzzy numbers and some of them seem to be good in a particular context but not in general. Intricacies like these

Table 1: Comparative results of Example 5.1

Fn	Proposed Method	Abbasbandy and Hajjari	Sign Distance P=1	Sign Distance P=2	Distance minimization
A	-	0.6666	3.2000	2.5820	0.0000
B	-	0.1666	1.5313	1.2417	0.0000
C	-	1.3334	2.9444	4.0414	0.0000
D	-	0.3334	3.000	2.3094	0.0000
Results	$B \prec D \prec A \prec C$	$B \prec D \prec A \prec C$	$B \prec C \prec D \prec A$	$B \prec D \prec A \prec C$	$A \sim B \sim C \sim D$

Table 2: Comparative results of Example 5.2

Fuzzy number	proposed method	Abbasbandy and Hajjari	Sign Distance p=1	Sign Distance p=2	Chu and Tsao	Cheng Distance	CV index
A	-	6.0000	6.12	8.52	3	6.021	0.028
B	-	6.0750	12.45	8.82	3.126	6.349	0.0098
C	-	6.0834	12.5	8.85	3.085	6.3519	0.0089
Results	$A \simeq B \simeq C$	$A \prec B \prec C$	$A \prec B \prec C$	$A \prec B \prec C$	$A \prec C \prec B$	$A \prec B \prec C$	$C \prec B \prec A$

Table 3: Comparative results of Example 5.3

Fuzzy number	Proposed method	Liou and Wang $\alpha = 0$	Liou and Wang $\alpha = 1$	Chu - Tsao
A	-	1.5	3.5	1.2445
B	-	1.2	3.6	1.1821
Results	$A \simeq B$	$B \prec A$	$A \prec B$	$B \prec A$

make ranking fuzzy numbers rather difficult. In this paper, with the new attitude coupled with the basic thinking to ordering, we defined a fuzzy linear (total) order on the fuzzy numbers set. Accordingly, we introduced the fuzzy numbers set as a fuzzy comparable set, namely, we can ordered any number of the fuzzy numbers. Validity of the proposed method was compared with other ranking methods by some examples. These proposed method is relatively reasonable for fuzzy numbers based on the introduced axioms, especially in the future prospects of this new approach to fuzzy distance.

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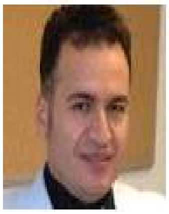

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