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Research Article

Three-axis optimal control of satellite attitude based on Ponteryagin maximum principle

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Abstract

A long time ago, since the launch of the first artificial satellite in 1957, controling attitude of satellites has been considered by the designers and engineers of aerospace industry. Considering the importance of this issue various methods of control in response to this need have been presented and analyzed until now. In this paper, we propose and analyze a three-axis optimal control on the six-dimensional system which describes the kinetic and kinematic equations of a satellite subjected to deterministic external perturbations which induce chaotic motion. At first, the chaotic behavior of system using Lyapunov exponents (LE) and numerical simulations is investigated when no control is affected. Then, a three-axis optimal control is presented by the Pontryagin maximum principle (PMP). This optimal control stabilizes the satellite attitude around the equilibrium point of origin. Finally, we give some simulation results to visualize the effectiveness and feasibility of the proposed method.

Keywords : Lyapunov exponent; Satellite attitude; Pontryagin maximum principle; Optimal control.

1 Introduction

Artificial satellites are purposely placed into orbit around the Earth, other planets, or the Sun. Since the launching of the first artificial satellite in 1957, thousands of these man-made moons have been rocketed into Earth's orbit. Today, artificial satellites play key roles in the communications industry, military intelligence, and the scientific study of the Earth and the outer space [14].

A satellite has to keep the solar panels pointed toward the Sun. It has to keep its antennas' and sensors' point toward Earth or toward the object the satellite is observing.

Because the orientation of satellites in stationary orbits gradually is changed with time, attitude control is needed to maintain a satellite attitude in the desired direction. Many control design methods have been investigated to solve this problem until now. In general, these methods are classified as active or passive. Passive method has been utilized in [11]. Some of the active methods include generalized predictive control method [8], sliding-mode approach [3], control method based on Lyapunov [5], nonlinear control based on linear matrix inequality [15], linear time-delay feedback control [6], nonlinear H_∞ control [16], and Robust and optimal attitude control of spacecraft with disturbances [13].

Optimal control applications can be found in the minimizing issues such as energy consumption and fuel in systems [14], dose of drugs in the

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treatment of diseases [7], time in processes and etc. In connection with the satellites the optimal control problem can be applied to solve a variety of problems e.g. attitude control in the desired direction, lunar soft landing, orbital transfer, etc.

The main purpose of this paper is to introduce a control function on each of the three satellite's kinetic equations. These control functions are within the framework of an optimal control problem via PMP. Also, we show that these controls are able to return back the six satellite's attitude variables (including three variables of angular velocities and three variables of Euler angles) to equilibrium point origin, when satellite attitude is tilted of this point.

This paper is organized as follows: Section 2, expresses the governing equations of satellite attitude. Section 3, describes chaotic behavior of system using LEs and numerical simulations. Access to an attitude optimal control is investigated in Section 4 via PMP, and simulation results are shown at the end of this section. Finally, our concluding remarks are given.

2 Coordinate frame and governing equations

2.1 Coordinate frame

The frames that are used to describe the problem of satellite's attitude control are illustrated in Figure 1, which we discribe each of them [1].

- *Earth centered inertial frame*

  This frame has its origin in the center of the Earth. It is defined by the unit vectors $\vec{X}_i$,$\vec{Y}_i$ and $\vec{Z}_i$. The $\vec{Z}_i$ axis points at a 90-angle relative to the Earth's equatorial plane where it coincides with the Earth's rotational axis and continues through the celestial North Pole. The $\vec{X}_i$ axis points in the vernal equinox vector direction, which is the vector pointing from the center of the Sun to the center of the Earth at the vernal equinox. The vernal equinox is a time of the year when the Earth's orbital plane as it rotates around the Sun coincides with the equatorial plane, i.e. the center of the Sun lies in the same plane as the Earth's equator. Finally, the $\vec{Y}_i$ axis completes the three axis orthonormal frame according to the right-hand rule.

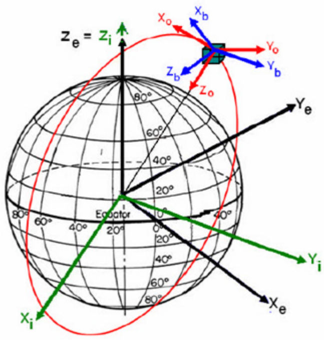

Figure 1: coordinates systems.

- *Orbit fixed frame*

  The orbit fixed frame follows the orbit trajectory and has its origin at the satellite's center of mass. It is defined by the unit vectors $\vec{X}_o$, $\vec{Y}_o$ and $\vec{Z}_o$. The $\vec{Z}_o$ axis points toward the center of the Earth, and the $\vec{X}_o$ axis points in the orbit normal direction, which is parallel to the orbital angular momentum vector. Again, the $\vec{Y}_o$ axis completes the right-hand orthonormal frame.

- *Body Frame*

  The attitude coordinates are chosen to be the (3-2-1) Euler angles: roll angle ϕ about $\vec{X}_b$ axis, pitch angle θ about $\vec{Y}_b$ axis, and yaw angle ψ about $\vec{Z}_b$ axis. The $\vec{X}_b$, $\vec{Y}_b$, and $\vec{Z}_b$ axes align with the principal body axes, and the body frame coincides with the orbit frame when the Euler angles are zeros.

2.2 Governing equations

The governing equations of satellite attitude are expressed by kinetic and kinematic equations.

2.2.1 Kinetic equations

Relation between angular velocity and torque in body frame is investigated by kinetic equations. By regarding the satellite as an ideal rigid body, the dynamic equations can be derived from a Newton-Euler formulation [4]

$$\begin{cases} \dot{w}_x = \frac{1}{I_x}\left[(I_y - I_z)w_y w_z + c_x\right] \\ \\ \dot{w}_y = \frac{1}{I_y}\left[(I_z - I_x)w_x w_z + c_y\right] \\ \\ \dot{w}_z = \frac{1}{I_z}\left[(I_x - I_y)w_x w_y + c_z\right], \end{cases} \tag{2.1}$$

where I_x, I_y, I_z are the inertial moments of the satellite about its principal axes, respectively, w_x, w_y, w_z are angular velocities about the same axes fixed in the rigid body, and c_x, c_y, c_z are torques applied about these axes at time t.

2.2.2 Kinematic equations

Kinematic equations of the satellite attitude based on Euler's angles are represented with

$$\begin{cases} \dot{\phi} = w_x + w_y \sin\phi \tan\theta + w_z \cos\phi \tan\theta \\ \dot{\theta} = w_y \cos\phi - w_z \sin\phi \\ \dot{\psi} = w_y \sin\phi \sec\theta + w_z \cos\phi \sec\theta, \end{cases} \tag{2.2}$$

where ϕ is the rotation about the X_b axis and θ is the rotation about the Y_b axis and ψ is the rotation about the Z_b axis in body frame [12].

The kinematic equations explain relationship between attitude and angular velocity in inertial frame.

In pursuit of our work, to refer to equations (2.1) and (2.2), we use the notation SA for satellite attitude.

3 Analysis of chaos in the SA system

In this section, chaotic behavior of the SA system, is investigated using simulation results and LEs. Early numerical simulation of system is done by using of the Maple under the perturbing torques [10, 17]

$$\begin{cases} c_x = -1200w_x + 500\sqrt{6}w_z \\ c_y = 350w_y \\ c_z = -1000\sqrt{6}w_x - 400w_z. \end{cases} \tag{3.3}$$

Assuming the conditions given in Table 1, chaotic behavior of the SA system is observed in attractors of Figure 2. The attractors are bounded but not fixed points and limit cycles which is a property of chaotic systems [2]. The LEs are quantities that characterize the rate of separation of infinitesimally close trajectories of a dynamical system. Figure 3 illustrates the LEs of the SA system. Furthermore the value of each of exponents are depicted in Table 2, and existing positive LEs, indicate that the system is chaotic.

Table 1: Initial conditions and constant values of the SA system.

Attitudes	Values	Constants	Values
$\phi_0(rad)$	0.5	$I_x(kgm^2)$	3000
$\theta_0(rad)$	0.5	$I_y(kgm^2)$	2000
$\psi_0(rad)$	0.7	$I_z(kgm^2)$	1000
$w_{x_0}(r/s)$	10		
$w_{y_0}(r/s)$	20		
$w_{z_0}(r/s)$	15		

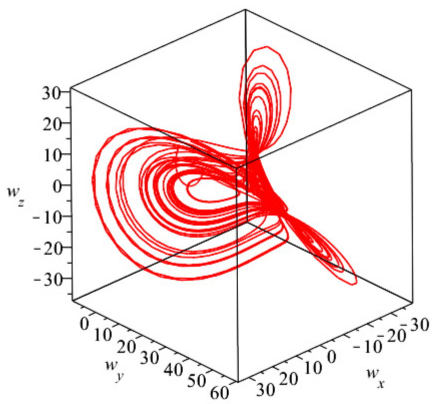

Figure 2: Phase portrait of the angular velocities of system (2.1), (2.2), whit perturbing torques (3.3).

4 Optimal control of satellite attitude

In this Section, we acquire optimal control of satellite attitude about its equilibrium point of origin, using the PMP [9].

Table 2: LEs Values of the SA system.

LEs	Values	LEs	Values
λ_ϕ	+0.76107	λ_{w_x}	-0.56619
λ_θ	+0.11558	λ_{w_y}	-1.3035
λ_ψ	+0.065799	λ_{w_z}	-5.186

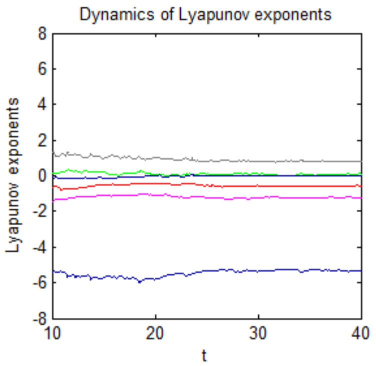

Figure 3: LEs of the SA system.

4.1 Theoretical results

Consider dynamical system

$$\dot{x}_i(t) = f_i(x_1(t), x_2(t), ..., x_n(t)), i = 1, ..., n. \tag{4.4}$$

where, $x_1, x_2, ..., x_n$ are the state variables, $f_i : \mathbb{R}^n \longrightarrow \mathbb{R}^n, (i = 1, ..., n)$ are continuous nonlinear functions. The controlled system with initial and final conditions is

$$\dot{x}_i(t) = f_i(x_1(t), x_2(t), ..., x_n(t)) + u_i(t), \quad i = 1, ..., n. \tag{4.5}$$

$$x_i(0) = x_{i,0}, x_i(t_f) = \bar{x}_i, \tag{4.6}$$

where, $\bar{x}_i, (i = 1, ..., n)$ are components equilibrium point $\bar{X}$, t_f is a constant final time and $u_i(t), (i = 1, ..., n)$ are control functions which minimize the cost function

$$J = \frac{1}{2}\int_0^{t_f} \sum_{i=1}^{n} (\alpha_i(x_i(t) - \bar{x}_i)^2 + \beta_i u_i^2(t))dt, \tag{4.7}$$

where, $\alpha_i, \beta_i, (i = 1, ..., n)$ are positive constants and J as a function of variables x_i and u_i. Optimal control functions are designed to achieve the equilibrium point in time t_f.

The corresponding Hamiltonian is

$$H = -\frac{1}{2} \sum_{i=1}^{n} (\alpha_i(x_i - \bar{x}_i)^2 + \beta_i {u_i}^2) + \sum_{i=1}^{n} \lambda_i(f_i + u_i), \tag{4.8}$$

where, $\lambda_i, (i = 1, ..., n)$ are costate variables. According to PMP, the costate equations and the necessary optimality conditions can be written as follows

$$\begin{cases} \dot{x}_i = \frac{\partial H}{\partial \lambda_i}, \\ \dot{\lambda}_i = -\frac{\partial H}{\partial x_i}, \\ \frac{\partial H}{\partial u_i} = 0. \end{cases} \tag{4.9}$$

Substituting Hamiltonian function (4.8) into (4.9)results

$$\begin{cases} \dot{x}_i(t) = f_i(x_1(t), x_2(t), ..., x_n(t)) + u_i(t), \\ \dot{\lambda}_i = \alpha_i(x_i - \bar{x}_i) - \frac{\partial(\sum_{i=1}^{n} \lambda_i(f_i + u_i))}{\partial x_i}, \end{cases} \tag{4.10}$$

and the optimal control functions

$$u_i^* = \frac{\lambda_i}{\beta_i}, \qquad i = 1, ..., n. \tag{4.11}$$

From (4.6), (4.10) and (4.11) we obtain

$$\begin{cases} \dot{x}_i(t) = f_i(x_1(t), x_2(t), ..., x_n(t)) + \frac{\lambda_i}{\beta_i}, \\ \dot{\lambda}_i = \alpha_i(x_i - \bar{x}_i) - \frac{\partial(\sum_{i=1}^{n} \lambda_i(f_i + u_i))}{\partial x_i}, \\ x_i(0) = x_{i,0}, x_i(t_f) = \bar{x}_i. \end{cases} \tag{4.12}$$

System (4.12) is a set of first order nonlinear ODEs with boundary conditions. Now, we apply system (4.12) on SA equations, so that the system corresponding to optimal control of satellite attitude is obtained

$$\begin{cases} \dot{\phi} = w_x + w_y \sin\phi \tan\theta + w_z \cos\phi \tan\theta, \\ \dot{\theta} = w_y \cos\phi - w_z \sin\phi, \\ \dot{\psi} = w_y \sin\phi \sec\theta + w_z \cos\phi \sec\theta, \\ \dot{w_x} = \frac{1}{I_x}[(I_y - I_z)w_y w_z - 1200 w_x \\ \quad +1225 w_z + \frac{\lambda_4}{\beta_4}], \\ \dot{w_y} = \frac{1}{I_y}[(I_z - I_x)w_x w_z + 350 w_y + \frac{\lambda_5}{\beta_5}], \\ \dot{w_z} = \frac{1}{I_z}[(I_x - I_y)w_x w_y - 2450 w_x \\ \quad -400 w_z - \frac{\lambda_6}{\beta_6}], \\ \dot{\lambda_1} = \alpha_1(\bar{\phi} - \phi) - \lambda_1(w_y \cos\phi \tan\theta) \\ \quad -\lambda_1(w_z \sin\phi \tan\theta) + \lambda_2(w_y \sin\phi) \\ \quad +\lambda_2(w_z \cos\phi) - \lambda_3(w_y \cos\phi \sec\theta) \\ \quad -\lambda_3(w_z \sin\phi \sec\theta), \\ \dot{\lambda_2} = \alpha_2(\bar{\theta} - \theta) - \lambda_1(w_y \sin\phi \sec^2\theta) \\ \quad +\lambda_1(w_z \cos\phi \sec^2\theta) \\ \quad -\lambda_3 \sec\theta \tan\theta(w_y \sin\phi + w_z \cos\phi), \\ \dot{\lambda_3} = \alpha_3(\bar{\psi} - \psi), \\ \dot{\lambda_4} = \alpha_4(\bar{w_x} - w_x) - \lambda_1 + 1200\lambda_4 \\ \quad -\lambda_5(\frac{1}{I_y}(I_z - I_x)w_z) \\ \quad -\lambda_6(\frac{1}{I_z}(I_x - I_y)w_y - 1000\sqrt{6}), \\ \dot{\lambda_5} = \alpha_5(\bar{w_y} - w_y) - \lambda_1 \sin\phi \tan\theta \\ \quad -\lambda_2 \cos\phi - \lambda_3 \sin\phi \sec\theta - 350\lambda_5 \\ \quad -\lambda_4 \frac{1}{I_x}(I_y - I_z)w_z - \lambda_6 \frac{1}{I_z}(I_x - I_y)w_x, \\ \dot{\lambda_6} = \alpha_6(\bar{w_z} - w_z) - \lambda_1 \cos\phi \tan\theta \\ \quad +\lambda_2 \sin\phi - \lambda_3 \cos\phi \sec\theta - 1225\lambda_4 \\ \quad -\lambda_4 \frac{1}{I_x}(I_y - I_z)w_y - \lambda_5 \frac{1}{I_y}(I_z - I_x)w_x \\ \quad +400\lambda_6, \\ \qquad X(O) = X_o, \quad X(t_f) = \bar{X}, \end{cases} \tag{4.13}$$

where, X and $\bar{X}$ are satellite's attitude vector $(\phi, \theta, \psi, w_x, w_y, w_z)$ and equilibrium point of SA system respectively, and t_f is a given time. Note that in (4.13) the control functions is only considered for kinematic equations. Next, by solving the nonlinear system (4.13) with given boundary conditions (4.6), we obtain the optimal control functions and the optimal state trajectory of the SA system.

4.2 Numerical simulation of optimal control

In this section, to predicate and verify the effectiveness of the theoretical results, we solve the system (4.13) with $t_f = 200s$, equilibrium point $\bar{X} = (0,0,0,0,0,0)$ and the initial conditions and the hypothetical constant values are given in Table 3 and Table 4. Numerical simulations are obtained using the Matlab's bvp4c solver. Figure 4 and Figure 5 illustrate the simulation results of the SA system based on the control functions (4.11). In these figures, time series responses corresponding to Euler angles and angular velocities demonstrates the appropriate performance of the optimal controllers with regard to the stabilization and suppression of chaos. Also time series responses for optimal controllers are depicted in Figure 6.

Table 3: Initial conditions and constant values of the SA system.

Attitudes	Values	Constants	Values
$\phi_0(rad)$	0.7	$I_x(kgm^2)$	3000
$\theta_0(rad)$	0.5	$I_y(kgm^2)$	2000
$\psi_0(rad)$	0.3	$I_z(kgm^2)$	1000
$w_{x0}(r/s)$	0.2		
$w_{y0}(r/s)$	0.1		
$w_{z0}(r/s)$	0.2		

Table 4: Constant values of the SA system.

Constants	Values	Constants	Values
α_1	15	β_4	0.2
α_2	15	β_5	0.4
α_3	15	β_6	0.2
α_4	10		
α_5	10		
α_6	10		

5 Conclusion

In this paper, the problem of three-axis optimal control of the chaotic satellite attitude has been developed. Optimal control functions were proposed based on the Pontryagin maximum principle. These control functions were powerful in order to align the body axes with the orbit axes when satellite attitude was confused to a disturbed torque. Moreover, angular velocities were diminished to zero by them. In the other words, satellite was stabilized around the equilibrium point of origin. Finally, numerical simulations were given to show the effectiveness of our method.

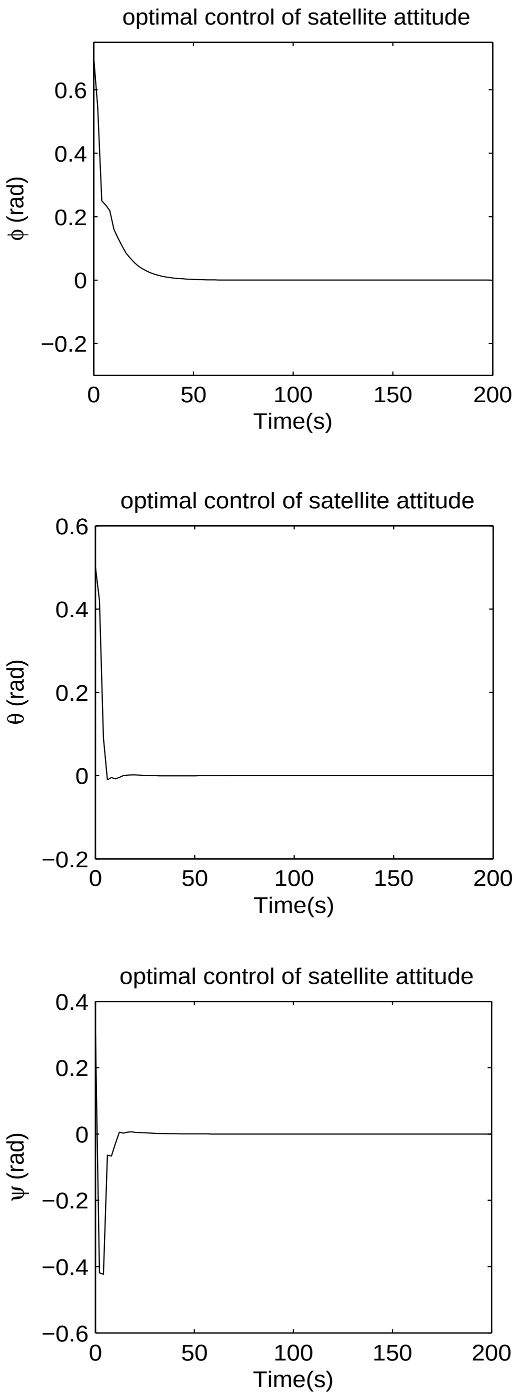

Figure 4: Time series responses corresponding to Euler angles in system (4.13) via optimal control.

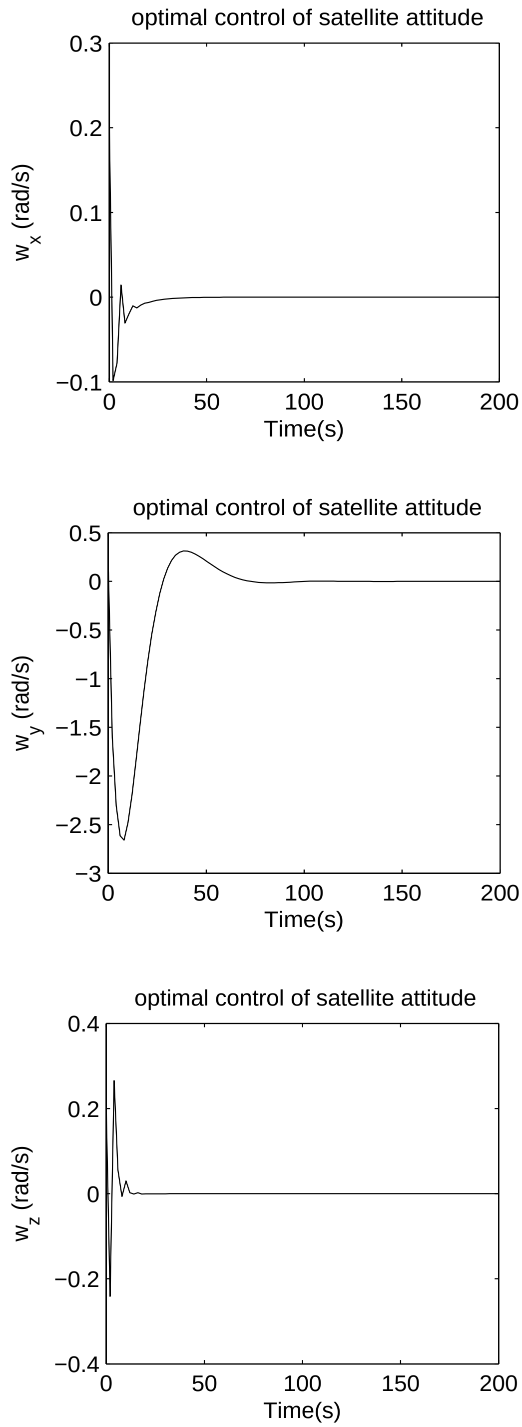

Figure 5: Time series responses corresponding to angular velocities in system (4.13) via optimal control.

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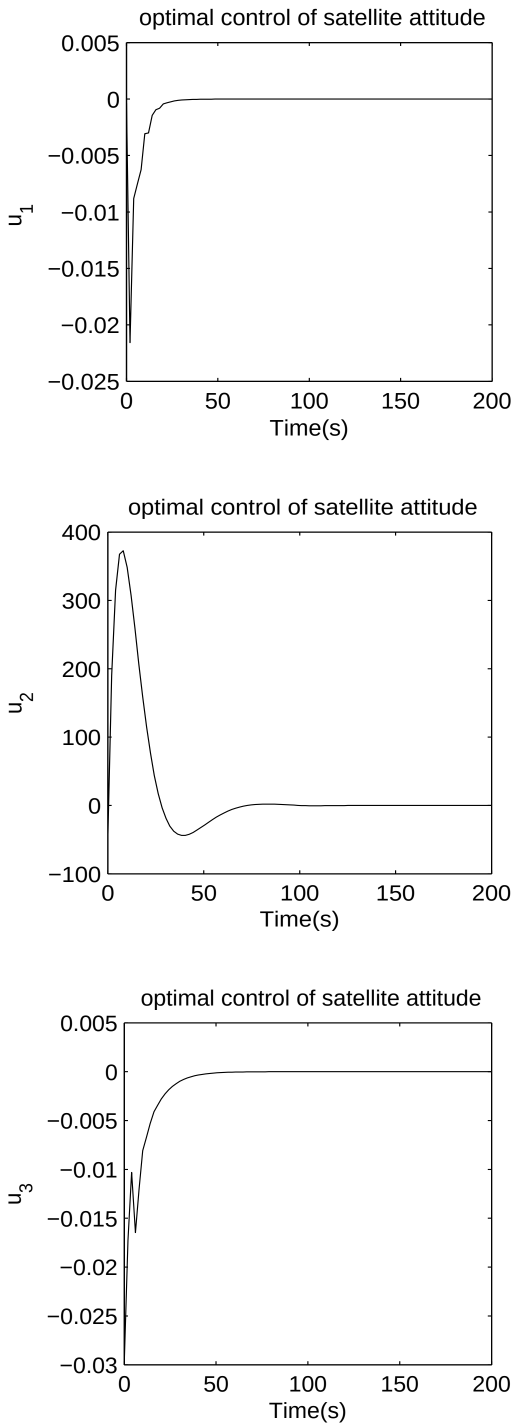

Figure 6: Time series responses corresponding to optimal controllers in system (4.11).

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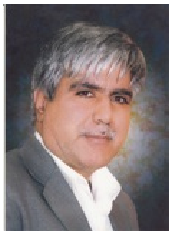

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