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Research Article

Evaluating Quasi-Monte Carlo (QMC) algorithms in blocks decomposition of de-trended

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Abstract

The length of equal minimal and maximal blocks has effected on logarithm-scale logarithm against sequential function on variance and bias of de-trended fluctuation analysis, by using Quasi Monte Carlo(QMC) simulation and Cholesky decompositions, minimal block couple and maximal are founded which are minimum the summation of mean error square in Horest power.

Keywords : De-trended fluctuation analysis; Long-range dependence; Cholesky decomposition; Quasi Monte Carlo simulation.

1 Introduction

IN last decade the de-trended fluctuation analysis (DFA) were introduced by Peng [9], which was as an important method for identifying the long-range dependence in data with multiple trends. The long-range dependence data are functions without summation auto covariance with hyperbolic descend. The long-range dependence is known as *long memory* and *1/f noise*. Horest (H) measures the power of long-range dependence with indicator H and is calculated by de-trended fluctuation analysis. fractional Brownian motion series (FBm) and fractional Gaussian noise (FGn) are ideals models, which are shown the long-range dependence. The DFA method is more credible than the traditional methods such as power spectrum, dependence analysis and Horest method for determining long-range dependence in invariant signals. The previous methods only use for fractional Gaussian noise signals or fractional Brownian motion. However, DFA should use for FBm or summation of FGn. The benefit of this method in comparison with other methods is that by using this it could be search the long-range dependence even in un-invariant series. [3]

In this survey the focus is on finding the best cutting block from de-trended fluctuation analysis minimized the mean square error of Horest power. This function is performance by sequential treatment with using Cholesky decomposition and Monte Carlo simulation. The Quasi Monte Carlo simulation works such as Monte Carlo simulation but it uses quasi-random numerical sequences. Calculating of Monte Carlo is easy and by repeating the accuracy decrease. Although, the rate of improvement is low. For instance, if we want increase the rate of accuracy to one decimal. The repetition of simulation should 100 times, and if it increases till 3 decimal numbers it should repeat 1 million times [10].

This error occurs because of using quasi numbers, although this is random but it is not uniform. The alternative method for increasing the rate of Monte Carlo method is changing the

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sequential. This changes need to product sequences in definite interval and distribute uniform. This number of sequences is named in term *quasi-random*. The methods, which use these semi-random numbers, are called quasi Monte Carlo methods [1]. In most of times the error of quasi Monte Carlo is less than the classic Monte Carlo method. The Monte Carlo convergences rank $O(N^{-\frac{1}{2}})$ is independent of dimension, whereas the quasi Monte Carlo convergence rank is $O(log\ N^d\ N^{-1})$, so it depends on the quasi dimension of simulation sequence. We use these quasi Monte Carlo in Cholesky decomposition and product the *FBm* series for analyzing DFA.

2 Quasi-random sequences

The simple example of quasi-random sequences is Wondercarpet sequences in one dimensional ($d = 1$). For producing this sequence n should write in binary base. The n^{th} point X_n is gained by inverting n numbers of the other side of decimal point [4].

2.1 Halton sequence

This is a basic sequence with lowest confusion with multiple dimension. This is a generalization of Wondercarpet sequences in d dimension. The nth number of Halton sequences in one dimension for the first number pd is gained by follow algorithm:

1. For all N , $n=1,2,\ldots,N$, n writes in base pd .

$$n = \sum_{i=0}^{l} a_i(n) p_d^i = a_0 p_d^0 + a_1 p_d^1 + ... + a_l p_d^l$$

2. Inverting the ratio decimal point numbers.

$$\phi_{p_d}(n) = \sum_{i=0}^{l} \frac{a_i(n)}{p^{i+1}}$$

3. In general a Halton sequences with d-dimension in base is:

$$x_n = (\varphi_2(n), \varphi_3(n), ..., \varphi_{p_d}(n))$$

2.2 Sobol sequence

The d dimension Sobol sequence for all of its dimension uses prime number 2 as a basic. The first dimension of this sequence is Wandercarpet in bases 2 and the higher dimension is permutation from sequence in one dimension. For producing jth factor of Sobol sequence needs to primer polynomial of degree n in Z.

$$x^n + a_1 x^{n-1} + a_2 x^{n-2} + \ldots + a_{n-1} x + 1,$$

which all coefficients are 0 or 1. The sequence of positive integer numbers $\{m_1, m_2, \ldots\}$ have return relation which is: $m_k = 2a_1 m_{k-1} \oplus 2^2 a_2 m_{k-2} \oplus \ldots \oplus 2^{n-1} a_{n-1} m_{k-n+1} \oplus 2^s m_{k-n} \oplus m_{k-n}$ That $\oplus$is an indicator of summation of bit by bit. The primary number $\{m_1, m_2, \ldots\}$ should choose in the way that each $m_{k\ ,\ 1<k<n}$ is an odd number less than 2k. The direct numbers $\{v_1, v_2, \ldots\}$ define $v_k = \frac{m_k}{2^k}$. Then, the jth factor from ith point or x_{ij} in one Sobol sequence as follow:

$$x_{i,j} = i_1 v_{1,j} \oplus i_2 v_{2,j} \oplus \ldots$$

which ik ,is kth number of right hand when i is on the base of $i = (...i_3 i_2 i_1)_2$ [7].

3 De-trended fluctuation analysis (DFA)

The steps of performance DFA are:

1. Suppose that X_t is a time series. The series makes plural with $Y(t) = \sum_{i=1}^{t}(X_i - \bar{X})$ which in $\left(\bar{X} = N^{-1}\sum_{i=1}^{N} X_i\right)$ is sample mean but it is not force to subtract the mean, because it deleted by final de-trended.

2. The plural series divide to N/m uncommon logarithm block with m size. On the other hand, because of N is not multiply of time parameter m, the small part of the end of series remains which it is not correct to ignore it. Therefore, this method should repeat. Hence, in this paper the m and N to the power of 2, so the number of final blocks N/m are integers [8].

3. In each block $k=1,2,\ldots,N/m$ the least square line $a_k + b_k t$ is fitted and the remained sample variance is calculated. $F^2(k,m) = \frac{1}{m-1}\sum_{t=1}^{m}\left(Y\left((k-1)m+t\right) - (a_k + b_k t)\right)^2,\ k = 1, ..., N/m$.

The trend which is fitted $a_k + b_k t$ could replace by polynomial degree 2, degree 3 or higher rank. In this case DFA2 is equivalent to the polynomial degree 2 , . . . and DFAr is correspond to the polynomial degree r.

4. By taking mean from $F^2(k,m)$ on all blocks [2].

The frequency function of qth is gained.

$$F_q(m) = \left\{ \frac{1}{N/m} \sum_{k=1}^{N/m} \left(F^2(k,m)\right)^{\frac{q}{2}} \right\}^{\frac{1}{q}},$$

which only define for $m \geq r+2$. This relation named multiple fractals DFA. In this paper we just focus on standard DFA. It means q=2, therefore

$$F(m) = \left\{ \frac{1}{N/m} \sum_{k=1}^{N/m} \left(F^2(k,m)\right) \right\}^{\frac{1}{2}}$$

remaining variance is proportional to m2H . Thus, the frequency function F(m) is proportional to mH.

$$F(m) \propto m^H$$

The behavior of scalable frequency function is surveying on logarithm-logarithm plots F(m) against m. Therefore, the slope of regression line $\log(F(m)) = c + H\log(m)$ is Horest estimated [5].

4 The Cholesky method for producing the fractional Brownian motion

The Cholesky decomposition uses for simulating systems with multiple dependence variables. Therefore, by using this method it could be possible to product the fractional Brownian motion series. Suppose $\Gamma = \frac{1}{2}\{|k-1|^{2H} - 2|k|^{2H} + |k+1|^{2H}\}$ is covariance matrix of fractional Brownian motion. We define Γ' is a matrix such as Γ without first column and row. Because Γ' is symmetric positive definite matrix, so the Cholesky decomposition could work, then we have $\Gamma' = LL'$. However, simulating of one sample FBm is equal to product the U vector with (N-1) independent variable and multiplying LU. In fact $\tilde{\beta} = \left(0, (LU)^T\right)^T$ which $\tilde{\beta}$ is a sample FBm. For producing FBm in surveying block decomposition of de-trended fluctuation analysis vector U is indicate as quasi-random sequences. We introduce these sequences as follow [2], [11].

5 The ChBlock decomposition of de-trended fluctuation analysis

For our treatments, we use Cholesky generation FBm by using Sobol and Halton sequences. Because in de-trended fluctuation analysis the block decomposition is only on the base of series length. The optimum criterion should not only minimum the bias but also variance for N .Hence, the sum mean square error is a scale for rating ability of method. If the bias square and variance are minimum then the MSE will be minimum. $MSE(\hat{H}) = E(\hat{H}-H)^2 = (E(\hat{H})-H)^2 + E(\hat{H}-E(\hat{H}))^2 = bias^2_{\hat{H}} + variance_{\hat{H}}$ $MSE(\hat{H})$ is considered as a function of both minimal and maximal block ($m+$, m-).

Suppose DFA establishes on different equal distance c* minimum on log-log block scales. The possible combination sets could define (m+, m-)

$\ell \equiv \{(m^-, m^+) = (2^l, 2^u) : u-l+1 \geq c^* \wedge l = l_1, ..., \log_2 N \wedge u = u_1, ..., \log_2 N\}$

The number of elements l in table 1 (see Appendix) is $\#\ell = a(a+1)/2$ where $a = \log_2 N + 2 - l_1 - c^*$

If the fitted trend is liner or sequence then the smallest block is $2^{l_1} = 4$ and for multinomial trend of degree 3 and 4 it is $2^{l_1} = 8$. Considering different minimal blocks are c*=4 and defining the following function:

$$\wp(m^-, m^+) \equiv \sum_{H \in H} MSE(\hat{H})(m^-, m^+)$$

Now, the trend of $MSE(\hat{H})$ for the Horest indicator with unknown posterior is defined. However, because the DFA bias for un-invariant processes is so strong, then we just consider an invariant process. Hence, we calculate $\wp(m^-, m^+)$ for H= $\{0.5, \ldots, 0.9\}$. It is obvious that $MSE(\hat{H})$ is sensitive to series length both N and H [6]. The target -among simulating numbers- is finding pair (m^-, m^+) which minimum the $\wp$ for long memory time series with length N.

$$(\tilde{m}^-, \tilde{m}^+) = \arg\min \; \wp(m^-, m^+)$$

In this way, we consider all possible shapes $(m^-, m^+)$for all N and use them in de-trended fluctuation analysis. This loop repeats for 1000 times till finding (m^-, m^+) with minimum $\wp$ The both blocks combination (4, 32) and (4, 64) are the best combination. The Table 2 (see Appendix) shows $(m^-, m^+)$for N=256 for each r=1 or r=2.

6 Conclusion

The Table 3 (see Appendix) shows the best combination blocks both minimal and maximal blocks $(\tilde{m}^-, \tilde{m}^+)$ for series *FBm* with length p=8,. . . , 12, N=*2p* and 1000 times repeat by using random quasi different generations. The *R* software is used. When the Halton generation used the blocks combination (4,64) in dimension 3, has minimized the $\wp(m^-, m^+)$ and after that the block combination (4,32) has minimized $\wp(m^-, m^+)$too . However, in dimension 8 the higher block combination $(\tilde{m}^-, \tilde{m}^+)$ is equal to (4,32) and the second higher block combination is (4,64) [9]. In Sobol generation results in dimension 3 it could see $(\tilde{m}^-, \tilde{m}^+)$ is a block combination (4,32) and after that (4,64), $\wp(m^-, m^+)$ is minimized. In dimension 8, the first higher block combination is (4,64) and the second higher combination is (4,32).Therefore, although it is different among different dimension in quasi-random generations of (4,32) and (4,64) in the best block combination. Also, by increasing the number of obvious, $(\tilde{m}^-, \tilde{m}^+)$ is not change. For generating and different dimension with equal N and rank $(\#1, \#2)$,$\wp(m^-, m^+)$ is so near to each other and this expresses tend of DFA is similar to simulating processes. For determining the bias, the standard deviation and mean root error $\hat{H}$, the Sobol generation with dimension 3 and 8 are chosen for producing *FBm*. Considering that $\wp(m^-, m^+)$ for H={0.5. . . 0.9} is calculated. In addition, the behavior of estimator in Table 4 (see Appendix) for Sobol sequence with dimension 3 and block combination (4,32) are studied. By increasing the series length from N=210 to N=212 for each H=1/2 the standard deviation from 0.035 to 0.014 and R-MSE for N=210,. . . ,212 from 0.040 to 0.023 are decreased. In Table 5 (see Appendix) for Sobol sequence with dimension 8 and better block combination (4,64) by increasing the series length the standard deviation and the mean root error $\hat{H}$ are decreased. For N=4096 and dimension 8 by increasing H from 0.5 to 0.9 the bias decrease from 0.018 to 0.003 [13]. The exit of Table 4 and 5 have brought in for more clarifying. By comparing figures (see Appendix) and tables for dimension 3 and 8 it could be infer that by increasing H from 0.5 to 0.9 the bias is decreased and by increasing N, the standard deviation and *R-MSE* are decreased [12].

Appendix

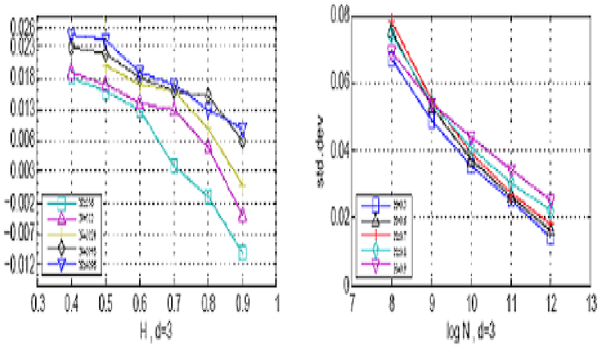

Figure 1: Standard deviation (right) and bias (left) Sobol generation with dimension 3.

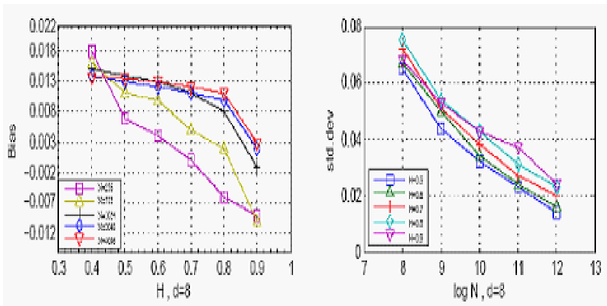

Figure 2: Standard deviation (right) and bias (left) Sobol generation with dimension 8.

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Table 1: The number of l for series with length N=28,...,212 for fitting polynomial trends in different degrees

p	$N = 2^p$	$\#\ell$	
		Fitting trend	
		Linear or quadratic	Cubic or fourth order
8	256	10	6
9	512	15	10
10	1024	21	15
11	2048	28	21
12	4096	36	28

Table 2: The possible positions (m^-, m^+) for N=256 for r=1 or r=2.

$N = 256$	(m^-, m^+)
	Linear or quadratic
$(4, 32)$ $(4, 64)$ $(4, 128)$ $(4, 256)$ $(8, 64)$ $(8, 128)$ $(8, 256)$ $(16, 128)$ $(16, 256)$ $(32, 256)$	

Table 3: The best combinations of 2 blocks $(\tilde{m}^+, \tilde{m}^-)$ for FBm with using Sobol sequence with dimension 3 and 8.

N		(m^+, m^-)		(m^+, m^-)		$\wp(\tilde{m}^+, \tilde{m}^-)$		$\wp(\tilde{m}^+, \tilde{m}^-)$	
		$d = 3$		$d = 8$		$d = 3$		$d = 8$	
		Sob	*Halt*	*Sob*	*Halt*	*Sob*	*Halt*	*Sob*	*Halt*
256	#1	(4,32)	(4,64)	(4,64)	(4,32)	0.0054	0.0053	0.0049	0.0054
256	#2	(4,64)	(4,32)	(4,32)	(4,64)	0.0055	0.0054	0.0051	0.0055
512	#1	(4,32)	(4,64)	(4,64)	(4,32)	0.0029	0.0031	0.0027	0.0030
512	#2	(4,64)	(4,32)	(4,32)	(4,64)	0.0030	0.0032	0.0029	0.0031
1024	#1	(4,32)	(4,64)	(4,64)	(4,32)	0.0018	0.0017	0.0016	0.0017
1024	#2	(4,64)	(4,32)	(4,32)	(4,64)	0.0020	0.0019	0.0017	0.0018
2048	#1	(4,32)	(4,64)	(4,64)	(4,32)	0.0012	0.0011	0.0010	0.0009
2048	#2	(4,64)	(4,32)	(4,32)	(4,64)	0.0014	0.0012	0.0011	0.0010
4096	#1	(4,32)	(4,64)	(4,64)	(4,32)	0.0009	0.0007	0.0006	0.0008
4096	#2	(4,64)	(4,32)	(4,32)	(4,64)	0.0010	0.0009	0.0007	0.0009

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Table 4: Bias, standard deviation, square mean root error, for the best 2 block combination and Sobol generation with dimension 3

N	d		$(\tilde{m}^-, \tilde{m}^+)$	$\wp$	Bias				
					0.5	0.6	0.7	0.8	0.9
256	3	#1	(4,32)	0.0054	0.016	0.013	0.004	-0.001	-0.010
		#2	(4,64)	0.053	0.015	0.013	0.008	-0.000	-0.010
512	3	#1	(4,32)	0.0029	0.017	0.014	0.013	0.007	-0.004
		#2	(4,64)	0.0030	0.019	0.016	0.012	0.009	-0.001
1024	3	#1	(4,32)	0.0018	0.020	0.017	0.016	0.010	0.001
		#2	(4,64)	0.0020	0.020	0.018	0.016	0.013	0.002
2048	3	#1	(4,32)	0.0012	0.022	0.018	0.016	0.015	0.008
		#2	(4,64)	0.0014	0.020	0.018	0.017	0.010	0.007
4096	3	#1	(4,32)	0.0009	0.024	0.019	0.017	0.013	0.010
		#2	(4,64)	0.0010	0.022	0.020	0.019	0.015	0.006

Table 4. Continue.

Std.dev					RMSE				
0.5	0.6	0.7	0.8	0.9	0.5	0.6	0.7	0.8	0.9
0.067	0.075	0.078	0.074	0.069	0.069	0.076	0.078	0.074	0.070
0.072	0.073	0.075	0.075	0.070	0.074	0.074	0.075	0.075	0.070
0.049	0.053	0.055	0.054	0.054	0.052	0.055	0.056	0.055	0.054
0.050	0.053	0.054	0.055	0.055	0.053	0.055	0.055	0.056	0.055
0.035	0.037	0.039	0.041	0.044	0.040	0.041	0.043	0.044	0.044
0.036	0.037	0.037	0.042	0.043	0.041	0.041	0.041	0.044	0.043
0.025	0.026	0.027	0.030	0.034	0.033	0.032	0.031	0.034	0.035
0.025	0.027	0.028	0.029	0.031	0.032	0.031	0.030	0.032	0.034
0.014	0.016	0.018	0.022	0.025	0.023	0.025	0.026	0.029	0.031
0.013	0.015	0.019	0.023	0.026	0.024	0.026	0.028	0.030	0.033

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Table 5: Bias, standard deviation, square mean root error, for the best 2 block combination and Sobol generation with dimension 8

N	d		$(\tilde{m}^-, \tilde{m}^+)$	$\wp$	Bias				
					0.5	0.6	0.7	0.8	0.9
256	5	#1	(4,32)	0.0049	0.007	0.004	0.000	-0.006	-0.009
		#2	(4,64)	0.0051	0.020	0.014	0.012	-0.005	-0.015
512	5	#1	(4,32)	0.0027	0.011	0.010	0.005	0.002	-0.010
		#2	(4,64)	0.0029	0.023	0.019	0.014	0.006	-0.000
1024	5	#1	(4,32)	0.0016	0.015	0.013	0.011	0.008	-0.001
		#2	(4,64)	0.0017	0.023	0.017	0.015	0.012	0.002
2048	5	#1	(4,32)	0.0010	0.017	0.012	0.011	0.010	0.002
		#2	(4,64)	0.0011	0.023	0.021	0.018	0.013	0.008
4096	5	#1	(4,32)	0.0006	0.018	0.013	0.012	0.011	0.003
		#2	(4,64)	0.0007	0.0225	0.019	0.018	0.014	0.007

Table 5. Continue.

Std.dev					RMSE				
0.5	0.6	0.7	0.8	0.9	0.5	0.6	0.7	0.8	0.9
0.065	0.067	0.072	0.075	0.068	0.065	0.067	0.072	0.075	0.070
0.067	0.072	0.073	0.072	0.066	0.070	0.073	0.074	0.072	0.060
0.044	0.050	0.051	0.054	0.053	0.045	0.052	0.052	0.054	0.054
0.047	0.051	0.053	0.057	0.052	0.052	0.055	0.053	0.057	0.052
0.032	0.034	0.038	0.043	0.043	0.035	0.037	0.039	0.043	0.043
0.035	0.036	0.038	0.039	0.041	0.042	0.040	0.041	0.041	0.041
0.023	0.024	0.027	0.031	0.037	0.028	0.027	0.029	0.033	0.037
0.023	0.025	0.027	0.032	0.034	0.032	0.033	0.033	0.034	0.035
0.014	0.016	0.020	0.023	0.024	0.020	0.019	0.022	0.023	0.026
0.017	0.018	0.019	0.024	0.026	0.025	0.023	0.025	0.026	0.028

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