



# Performance evaluation of efficiency change and productivity growth in Supply Chain Management

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## Abstract

The performance of a supply chain can be evaluated in either a cross-sectional or a time series manner, and data envelopment analysis is a useful method for both types of evaluation. In this paper we develop an index and indicator of productivity change that can be used with radial and non-radial models for supply chain malmquist index. The supply chain malmquist productivity index (SCMPI) can be decomposed into two components: one is measuring the technical change (TC) and the other measuring technical efficiency change (TEC). So that we propose a supply chain DEA models that have supplier-manufacturer structures.

*Keywords* : Supply chain management; Data envelopment analysis; Productivity Malmquist index.

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## 1 Introduction

Performance evaluation is an important issue for effective supply chain management, to provide the best high-quality products and services at least cost. Supply chain managers have a tremendous impact on the success of an organization. These managers are engaged in every facet of the business process planning, purchasing, production, transportation, storage & distribution, customer service, and more! In short, these managers are the “glue” that connects the different parts of the organization. Their performance helps organizations control expenses, boost sales, and maximize profits. Two additional roles focus on facilitation and collaboration. Because supply chain managers

touch so many different parts of the business, they are in a unique position to help other functions execute their strategies. They are also called upon to diagnose and support the needs of external supply chain partners. Here are just a few examples of these cross-functional roles:

- Effective selection and management of suppliers support lean manufacturing processes.
- Efficient transportation & distribution practices bolster marketing campaigns.
- Timely customer communication and technology-enabled visibility allows companies to monitor product flows and collaboratively respond to potential delivery problems.

Meanwhile, supply chain management (SCM) has been a great importance in competitive strategy to enhance organizational productivity and profitability. In this regard, different models have been introduced to evaluate supply chain performance. The model proposed by [11] named logistic score cards and [7] model are examples of

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all inclusive models (overall efficiency of supply chain) and also [6] and [3] showed a number of applied researches on performance evaluation. Market globalization has made supply chain management one of the interesting topics to be discussed. On the other hand productivity growth and improving inefficient supply chains are key issues both at the firm and at the national level. Although the study of performance was enriched by researches but study on suitable supply chain performance measurement systems the technical change (TC) and the other measuring technical efficiency change (TEC) are still limit. The computation of productivity change by means of efficiency measures was first introduced by [1] and developed by [9] and by [4], in the context of parametric and non-parametric efficiency measurement, respectively. The [4] approach has become known as the measurement of productivity change through Malmquist indices. Recent years have seen a great variety of applications of DEA (Data Envelopment Analysis) for use in evaluating the performances of many different kinds of entities engaged in many different activities in many different contexts in many different countries. One reason is that DEA has opened up possibilities for use in cases which have been resistant to other approaches because of the complex (often unknown) nature of the relations between the multiple inputs and multiple outputs involved in many of these activities (which are often reported in non-commeasurable units). There are a number of ways to compute Malmquist Index, [4], [5] and [12]. Most of the performance evaluation of supply chain use DEA applications and can be applied to [8] and [10]. The specific approach used is called (radial) Malmquist productivity index in which radial DEA efficiency scores are used. Recent studies by [5], have shown that the Malmquist productivity index based on common-weights DEA. The DEA models used in the radial Malmquist productivity index can either be input or output oriented. In this paper we develop an index and indicator of productivity change that can be used with radial and non-radial model for supply chain malmquist index. The SCMPI supply chain Malmquist productivity index can be decomposed into two components: one is measuring the technical change (TC) and the other measuring technical efficiency change (TEC). So that we

propose a supply chain DEA model that have supplier-manufacturer structures. The rest of the paper is in 4 sections. Section 1 describes two method of supply chain the input-oriented radial Malmquist productivity index for supply chains with supplier-manufacturer structure. The proposed approach is validated by two numerical examples in Section 2. Finally, conclusions are drawn in Section 3.

## 2 Method

Let us assume time  $t=1,2,\dots,T$ . Therefore, consider a generic supplier- manufacturer process as shown in Fig.1. Suppose we have  $n$  supply chains (hereafter abbreviated SCs), and each  $SC_j$ , ( $j=1,2,\dots,n$ ), has  $P$  inputs to the supplier ( $S_j$ ),  $x_{pj}$  ( $p=1,2,\dots,P$ ), and  $K$  outputs from this  $S_j$ ,  $i_{kj}$  ( $k=1,2,\dots,K$ ). These  $K$  outputs become the inputs to the manufacturer ( $M_j$ ), and are referred to as intermediate products. The outputs from the  $M_j$  are denoted as  $y_{qj}$  ( $q=1,2,\dots,Q$ ). Also, we introduce  $i_{kd}$  ( $k=1,2,\dots,K$ ), representing a set of new intermediate products to be determined. We develop the Malmquist index was defined by [4] and [5] for supply chains with supplier-manufacturer structure as follows. Therefore, supplier-manufacturer supply chain



**Figure 1:** Supply Chain.

performance assessment model can be computed

using Model (2.1):

$$\begin{aligned}
 \theta_d^t(x_d^t, y_d^t) = & \text{Min } \theta \\
 \text{s.t. } & \sum_{j=1}^N \lambda_j x_{pj}^t \leq \theta \times x_{pd}^t \quad p = 1, \dots, P \\
 & \sum_{j=1}^N \lambda_j i_{kj}^t \geq i_{kd}^t \quad k = 1, \dots, K \\
 & \sum_{j=1}^N \eta_j i_{kj}^t \leq i_{kd}^t \quad k = 1, \dots, K \\
 & \sum_{j=1}^N \eta_j y_{qj}^t \geq y_{qd}^t \quad q = 1, \dots, Q \\
 & \lambda_j \geq 0, \eta_j \geq 0, i_{kd}^t \geq 0, \\
 & j = 1, 2, \dots, N, k = 1, 2, \dots, K. \quad (2.1)
 \end{aligned}$$

Consider two time periods t and t+1 and suppose we have production function in time period t as well as t+1. Malmquist productivity index calculation requires two single period and two mixed period measures. The two single period measures can be obtained by using the supplier-manufacturer supply chain DEA model (2.1) as follows:

$$\begin{aligned}
 \theta_d^{t+1}(x_d^t, y_d^t) = & \text{Min } \theta \\
 \text{s.t. } & \sum_{j=1}^N \lambda_j x_{pj}^{t+1} \leq \theta \times x_{pd}^t \quad p = 1, \dots, P \\
 & \sum_{j=1}^N \lambda_j i_{kj}^{t+1} \geq i_{kd}^t \quad k = 1, \dots, K \\
 & \sum_{j=1}^N \eta_j i_{kj}^{t+1} \leq i_{kd}^t \quad k = 1, \dots, K \\
 & \sum_{j=1}^N \eta_j y_{qj}^{t+1} \geq y_{qd}^t \quad q = 1, \dots, Q \\
 & \lambda_j \geq 0, \eta_j \geq 0, i_{kd}^t \geq 0, \\
 & j = 1, 2, \dots, N, k = 1, 2, \dots, K. \quad (2.2)
 \end{aligned}$$

The first of mixed period measures is calculated as optimal value to following linear programming:

$$\begin{aligned}
 \theta_d^t(x_d^{t+1}, y_d^{t+1}) = & \text{Min } \theta \\
 \text{s.t. } & \sum_{j=1}^N \lambda_j x_{pj}^t \leq \theta \times x_{pd}^{t+1} \quad p = 1, \dots, P \\
 & \sum_{j=1}^N \lambda_j i_{kj}^t \geq i_{kd}^{t+1} \quad k = 1, \dots, K \\
 & \sum_{j=1}^N \eta_j i_{kj}^t \leq i_{kd}^{t+1} \quad k = 1, \dots, K \\
 & \sum_{j=1}^N \eta_j y_{qj}^t \geq y_{qd}^{t+1} \quad q = 1, \dots, Q \\
 & \lambda_j \geq 0, \eta_j \geq 0, i_{kd}^{t+1} \geq 0, \\
 & j = 1, 2, \dots, N, k = 1, 2, \dots, K. \quad (2.3)
 \end{aligned}$$

Then Malmquist productivity index supply chains with supplier-manufacture structure are defined as:

$$SC.TPI_d = \left[ \frac{\theta_d^t(x_d^t, y_d^t) \theta_d^{t+1}(x_d^t, y_d^t)}{\theta_d^t(x_d^{t+1}, y_d^{t+1}) \theta_d^{t+1}(x_d^{t+1}, y_d^{t+1})} \right]^{1/2} \quad (2.4)$$

Therefore, if  $SC.TPI_d > 1$  indicates progress in the total factor productivity of the supply chain from period t to t+1, while  $SC.TPI_d = 1$  and  $SC.TPI_d < 1$  respectively the status quo (remain unchanged) and deterioration in the total factor productivity. This is equivalent to:

$$\begin{aligned}
 SC.TPI_d = & \underbrace{\frac{\theta_d^t(x_d^t, y_d^t)}{\theta_d^{t+1}(x_d^{t+1}, y_d^{t+1})}}_{SC.TEC} \times \\
 & \underbrace{\left[ \frac{\theta_d^{t+1}(x_d^{t+1}, y_d^{t+1}) \theta_d^{t+1}(x_d^t, y_d^t)}{\theta_d^t(x_d^{t+1}, y_d^{t+1}) \theta_d^t(x_d^t, y_d^t)} \right]^{1/2}}_{SC.TC} \quad (2.5)
 \end{aligned}$$

Where the first component is SC.TEC (supply chain technical efficiency change) between two time periods and second component is SC.TC (supply chain technology Change frontier) between time period t and t+1 that we have:

$SC.TEC > 1$ : Productivity growth

$SC.TEC = 1$ : Productivity unchanged

$SC.TEC < 1$ : Productivity decline

$SC.TC > 1$ : Technical progress

$SC.TC = 1$ : Technical without change

$SC.TC < 1$ : Technical regress

There are a number of ways to compute Malmquist Index, [4], [5] and [12]. However, the radial model suffers from one shortcoming, i.e., neglect of slacks . To overcome this shortcoming, SC.TPI can be computed using the slack based non-radial and oriented DEA model. Therefore we have:

$$\begin{aligned} \theta_d^{t+1}(x_d^t, y_d^t) &= \frac{1}{\sum_{p=1}^P w_p} \text{Min} \sum_{p=1}^P w_p \theta_{pd} \\ \text{s.t.} \quad &\sum_{j=1}^N \lambda_j x_{pj}^{t+1} \leq \theta_{pd} \times x_{pd}^t \quad p = 1, \dots, P \\ &\sum_{j=1}^N \lambda_j i_{kj}^{t+1} \geq i_{kd}^t \quad k = 1, \dots, K \\ &\sum_{j=1}^N \eta_j i_{kj}^{t+1} \leq i_{kd}^t \quad k = 1, \dots, K \\ &\sum_{j=1}^N \eta_j y_{qj}^{t+1} \geq y_{qd}^t \quad q = 1, \dots, Q \\ &\lambda_j \geq 0, \eta_j \geq 0, i_{kd}^t \geq 0, \\ &j = 1, 2, \dots, N, k = 1, 2, \dots, K. \end{aligned} \tag{2.6}$$

And we define,

$$w_p = \frac{x_{pd}}{\sum_{p=1}^P x_{pj}} \tag{2.7}$$

And we have,

$$\begin{aligned} \theta_d^t(x_d^{t+1}, y_d^{t+1}) &= \frac{1}{\sum_{p=1}^P w_p} \text{Min} \sum_{p=1}^P w_p \theta_{pd} \\ \text{s.t.} \quad &\sum_{j=1}^N \lambda_j x_{pj}^t \leq \theta_{pd} \times x_{pd}^{t+1} \quad p = 1, \dots, P \\ &\sum_{j=1}^N \lambda_j i_{kj}^t \geq i_{kd}^{t+1} \quad k = 1, \dots, K \\ &\sum_{j=1}^N \eta_j i_{kj}^t \leq i_{kd}^{t+1} \quad k = 1, \dots, K \\ &\sum_{j=1}^N \eta_j y_{qj}^t \geq y_{qd}^{t+1} \quad q = 1, \dots, Q \\ &\lambda_j \geq 0, \eta_j \geq 0, i_{kd}^t \geq 0, \\ &j = 1, 2, \dots, N, k = 1, 2, \dots, K. \end{aligned} \tag{2.8}$$

Then Malmquist productivity index supply chains with supplier-manufacturer structure are defined as:

$$\begin{aligned} SC.TPI_d^O &= \underbrace{\frac{\theta_d^t(x_d^t, y_d^t)}{\theta_d^{t+1}(x_d^{t+1}, y_d^{t+1})}}_{SC.TEC^O} \times \\ &\underbrace{\left[ \frac{\theta_d^{t+1}(x_d^{t+1}, y_d^{t+1}) \theta_d^{t+1}(x_d^t, y_d^t)}{\theta_d^t(x_d^{t+1}, y_d^{t+1}) \theta_d^t(x_d^t, y_d^t)} \right]^{1/2}}_{SC.TC^O} \end{aligned} \tag{2.9}$$

Therefore, we have:

$SC.TEC^O > 1$ : Productivity growth

$SC.TEC^O = 1$ : Productivity unchanged

$SC.TEC^O < 1$ : Productivity decline

$SC.TC^O > 1$ : Technical progress

$SC.TC^O = 1$ : Technical without change

$SC.TC^O < 1$ : Technical regress

**Table 1:** Data of 7 supply chain.

Time (1)						
NO	Input(1)	Input(2)	Input(3)	Intermediate product(1)	Output(1)	Output(2)
SC1	1.0168	1.2215	166.9755	8.3098	122.1954	3.7569
SC2	0.5915	0.4758	50.1164	1.7634	19.4829	0.6600
SC3	0.7237	0.6061	48.2831	3.4098	34.4120	0.7713
SC4	0.5150	0.3763	35.0704	2.3480	15.2804	0.3203
SC5	0.4775	0.3848	49.9174	5.4613	34.9897	0.8430
SC6	0.6125	0.3407	23.1052	1.2413	32.5778	0.4616
SC7	0.7911	0.4407	39.4590	1.1485	30.2331	0.6732

  

Time (2)						
NO	Input(1)	Input(2)	Input(3)	Intermediate product(1)	Output(1)	Output(2)
SC1	1.2363	0.5547	37.4954	4.0825	20.6013	0.4864
SC2	0.4460	0.3419	20.9846	0.6897	8.6332	0.1288
SC3	1.2481	0.4574	45.0508	1.7237	9.2354	0.3019
SC4	0.7050	0.4036	38.1625	2.2492	12.0171	0.3138
SC5	0.6446	0.4012	30.1676	2.3354	13.8130	0.3772
SC6	0.7239	0.3709	26.5391	1.3416	5.0961	0.1453
SC7	0.5538	0.3555	22.2093	0.9886	13.6085	0.3614

**Table 2:** TEC, TC, SC.TPI (Radial).

NO	TEC	State	TC	Move of Technology	SC.TPI	Results
SC1	0.5511	Decrease	1.4181	growth	0.7815	Productivity decline
SC2	0.4813	Decrease	1.1677	growth	0.5620	Productivity decline
SC3	0.7449	Decrease	1.4692	growth	1.0944	Productivity growth
SC4	0.7539	Decrease	1.5986	growth	1.2052	Productivity growth
SC5	0.5349	Decrease	1.6884	growth	0.9032	Productivity decline
SC6	1.1378	Increase	1.0202	growth	1.1608	Productivity growth
SC7	0.5204	Decrease	1.2043	growth	0.6268	Productivity decline

**Table 3:** TEC, TC, SC.TPI (Non-radial).

NO	TEC	state	TC	Move of Technology	SC.TPI	Results
SC1	0.3522	Decrease	0.9601	Decline	0.3381	Productivity decline
SC2	0.6738	Decrease	0.6301	Decline	0.4245	Productivity decline
SC3	1.0657	Increase	0.8815	Decline	0.9394	Productivity decline
SC4	1.2293	Increase	0.8887	Decline	1.0925	Productivity growth
SC5	0.7480	Decrease	0.8209	Decline	0.6140	Productivity decline
SC6	1.2892	Increase	0.8908	Decline	1.1484	Productivity growth
SC7	0.6384	Decrease	0.8881	Decline	0.5670	Productivity decline

### 3 Application

In this section, the proposed methodology is used for supply chain evaluation. Table 1 exhibits data for 7 of supply chain that is a typical two-member supply chain process. There are three inputs to the first stage are consumed to generate outputs in the second stage, and two observation periods.

Table 2 reports the overall efficiency scores of supply chains under radial model and TEC, TC and SC.TPI scores, reported as the 2th, 4th and 6th column of Table 2.

Table 3 reports the overall efficiency scores of supply chains under non-radial model.

## 4 Conclusions

Incorporating SCM successfully leads to a new kind of competition on the global market where competition is no longer of the company versus company form but rather takes on a supply chain versus supply chain form. In this paper we show that supply chain DEA approaches could lead to biased results due to the productivity effect in performance evaluation of supply chain. Meanwhile, this paper shows how DEA-based Malmquist productivity index for supply chains with supplier-manufacturer structure can be employed to evaluate the technology and productivity changes resulted from the economic development plans. In other words, it is the ability of a product design to generate demand by satisfying customer expectations.

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