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Research Article

Stagnation-point flow of a viscous fluid towards a stretching surface with variable thickness and thermal radiation

B. C. Prasanna Kumara *, G. K. Ramesh †, A. J. Chamkha ‡§, B. J. Gireesha ¶

Abstract

In the present analysis, we study the boundary layer flow of an incompressible viscous fluid near the two-dimensional stagnation-point flow over a stretching surface. The effects of variable thickness and radiation are also taken into account and assumed that the sheet is non-flat. Using suitable transformations, the governing partial differential equations are first converted to ordinary one and then solved numerically by fourth and fifth order Runge-Kutta-Fehlberg method with shooting technique. The influence of the various interesting parameters on the flow and heat transfer is analyzed and discussed through graphs in detail. Comparison of the present results with known numerical results is shown and a good agreement is observed. It is found that boundary layer is formed when $\lambda > 1$. On the other hand, an inverted boundary layer is formed when $\lambda < 1$.

Keywords : Stagnation-point flow; Variable thickness; Stretching sheet; Thermal radiation; Numerical solution.

1 Introduction

THe interest in the study of boundary layer flows over a stretching sheet has been significantly increased in view of their numerous applications in various fields of science and engineering for example, the hot rolling, continuous stretching, wire drawing, glass-fiber production, the aerodynamic extrusion of plastic sheets, the cooling process of metallic plate in cooling bath and polymer industries, etc. The pioneering work in this area was conducted by Sakiadis [23] put forward the very basic governing equation on the continuous moving solid surface problem. Now the literature is very rich in analyzing the various aspects of Sakiadis's problem. Crane [3] and Gupta and Gupta [7] have discussed the continuous moving surface problem with constant surface temperature. There are several extensions to this problem, which include consideration of more general stretching velocity and the study of heat transfer ([4] - [19]).

Flow near stagnation-point is very interesting in fluid dynamics. Actually, the stagnation flow takes place whenever the flow impinges to any solid object and the local velocity of the fluid at the stagnation-point is zero. It is an important bearing on several industrial and technical applications such as cooling of electronic devices by fans, cooling of nuclear reactors during emergency shutdown, heat exchangers placed in a low-velocity environment, solar central receivers ex-

*Department of Mathematics, Government First Grade College, Koppa, Karnataka, India.
†Department of Studies in Mathematics, Kuvempu University, Shimoga, Karnataka, India.
‡Corresponding author. achamkha@pmu.edu.sa
§Mechanical Engineering Department, Prince Mohammad Bin Fahd University (PMU) P.O. Box 1664, Al-Khobar 31952, Kingdom of Saudi Arabia.
¶Department of Studies in Mathematics, Kuvempu University, Shimoga, Karnataka, India.

Table 1: Comparison of the values of skin friction coefficient $f''(0)$ for various values of m in the case of $\lambda = Pr = Nr = 0$ and $\beta = 0.5$.

m	Fang et al. [6]	Present result
10	-1.0603	-1.06034
9	-1.0589	-1.05893
7	-1.0550	-1.05506
5	-1.0486	-1.04862
3	-1.0359	-1.03588
2	-1.0234	-1.02342
1	-1.0000	-1.00000
0.5	-0.9799	-0.97994
0.0	-0.9576	-0.95764

Table 2: Computations values of $-\theta'(0)$ for different values of Pr when $\beta = 0.5, m = 0.2, Nr = 0$ and $\lambda = 0.0$.

Pr	$-\theta'(0)$
0.72	0.64087
1	0.77462
2	1.14016
3	1.40436
4	1.61409
5	1.78925
6	1.94010
7	2.07269
8	2.19096
10	2.39469

posed to wind currents, and many hydrodynamic processes.The two-dimensional flow of a fluid near a stagnation point was first examined by Hiemenz [8]. Later Chiam [5] analyzed steady two dimensional stagnation-point flow of an incompressible viscous fluid towards a stretching surface. Mahapatra and Gupta [11] studied the stagnation-point flow towards a stretching sheet taking different stretching and straining velocities. Various important aspects of the stagnation-point flow over stretching sheet under were presented by many investigators ([13] - [2]).

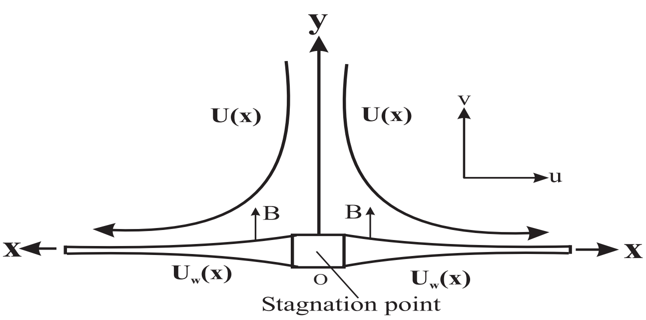

Figure 1: Schematic diagram of the flow

At high operating temperature, radiation effect can be quite significant. Many processes in engineering areas occur at high temperature and knowledge of radiation heat transfer becomes very important for the design of the pertinent equipment. Nuclear power plants, gas turbines and the various propulsion devices for aircraft, missiles, satellites and space vehicles are examples of such engineering areas. Pop et al. [16] studied theoretically the steady two-dimensional stagnation-point flow of an incompressible fluid over a stretching sheet by taking into account of radiation effects using the Rosseland approxima-

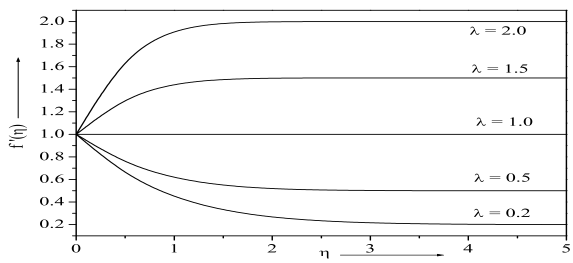

Figure 2: Effect of m on temperature profiles when $\lambda = 0.5, m = 2, Nr = 2$ and $Pr = 3.0$.

Table 3: Computations values of $-f''(0)$ and $-\theta'(0)$ for different values of λ when $\beta = 0.5, m = 0.2, Nr = 0$ and $Pr = 3.0$.

λ	$-f''(0)$	$-\theta'(0)$
0.0	1.02469	1.40436
0.2	0.95280	1.43499
0.4	0.802036	1.48348
0.5	0.702016	1.51094
0.6	0.587339	1.53960
0.8	0.317894	1.59893
1.0	0.0	1.659403
1.2	-0.361687	1.719941
1.4	-0.763560	1.779992
1.6	-1.202722	1.839253
1.8	-1.676789	1.897569
2.0	-2.183760	1.954864

Table 4: Values of skin friction coefficient $-f''(0)$ and Nusselt number $-\theta'(0)$ for different values of the physical parameters.

				$\lambda = 0.5$		$\lambda = 1.5$	
β	Pr	Nr	m	$-f''(0)$	$-\theta'(0)$	$-f''(0)$	$-\theta'(0)$
0.5	3	2	2	0.70201	0.80713	0.97863	1.04799
1.0				0.66180	0.73627	0.93571	0.97242
1.5				0.62391	0.67048	0.89445	0.90055
0.5	2	2	2	0.70201	0.64754	0.97863	0.87641
	3			0.70201	0.80713	0.97863	1.04799
	4			0.70201	0.93373	0.97863	1.18726
0.5	3	1	2	0.70201	1.01264	0.97863	1.27387
		2		0.70201	0.80713	0.97863	1.04799
		3		0.70201	0.68353	0.97863	0.91430
0.5	3	2	-0.25	0.38926	1.00692	0.15527	1.09113
			2	0.70201	0.80713	0.97863	1.047992
			4	0.72704	0.79427	1.02752	1.045913

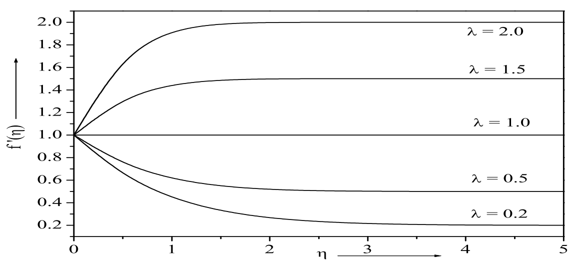

Figure 3: Effect of m on temperature profiles when $\lambda = 0.5, m = 2, Nr = 2$ and $Pr = 3.0$.

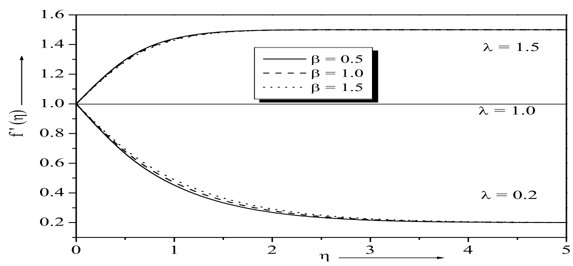

Figure 4: Effect of β on velocity profiles when $m = 2, Nr = 2$ and $Pr = 3.0$.

tion to model the radiative heat transfer. Pop et al. [17] discussed the radiation effects on the MHD flow near the stagnation point of a stretching sheet. Pal [18] analyzed the heat and mass transfer in stagnation point flow towards a stretching sheet in the presence of buoyancy force and thermal radiation. Incompressible water based nanofluid flow over a stretching sheet in the presence of transverse magnetic field with thermal radiation and buoyancy effects are studied by Rashidi et al [20]. Recently Ramesh et al. [22] examined the stagnation point flow of a

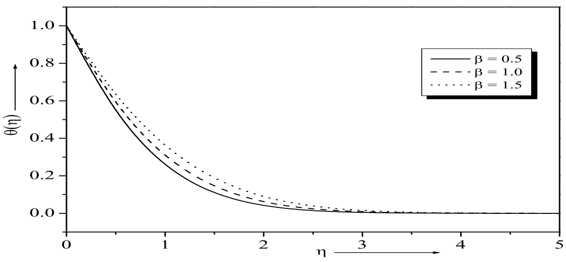

Figure 5: Effect of β on temperature profiles when $\lambda = 0.5, m = 2, Nr = 2$ and $Pr = 3.0$.

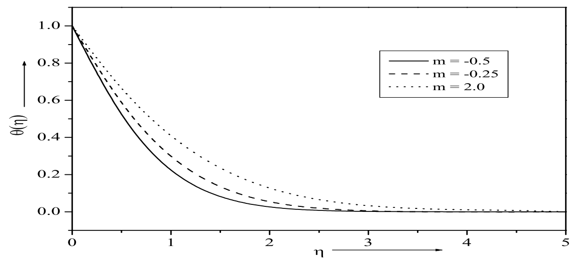

Figure 7: Effect of m on temperature profiles when $\lambda = 0.5, m = 2, Nr = 2$ and $Pr = 3.0$.

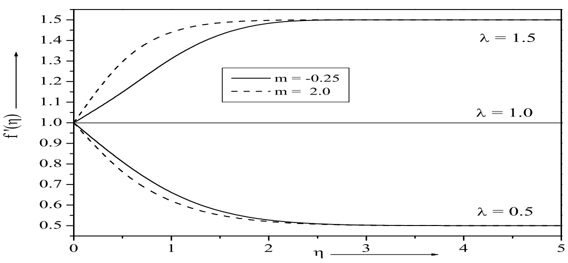

Figure 6: Effect of m on velocity profiles when $m = 2, Nr = 2$ and $Pr = 3.0$.

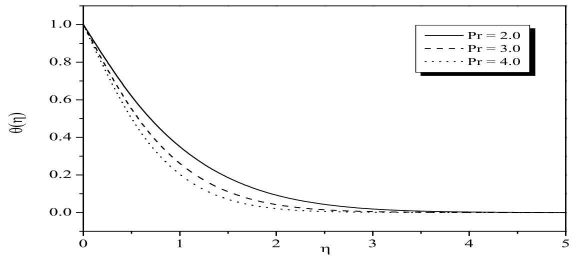

Figure 8: Effect of Pr on temperature profiles when $\lambda = 0.5, m = 2, Nr = 2$ and $\beta = 0.5$.

MHD dusty fluid towards a stretching sheet with radiation and found that fluid particle is always higher than the dust particles and they are parallel to each other. In all these studies, the boundary layer flow is investigated for a flat stretching sheet only.

They are very few studies can be found on the flow over a non-flatness stretching sheet. Study of flow and heat transfer of viscous fluids over stretching sheet with a variable thickness (non-flatness) can be more relevant to the situation in practical applications. For the first time Fang et al. [6] obtain an elegant analytical and numerical solution to the two-dimensional boundary layer flow due to a non-flatness stretching sheet. Further this problem was extended by Subhashini et al. [24] by including the energy equation and found that thermal boundary layer thicknesses for the first solution were thinner than those of the second solution. Numerical solution for the flow of a Newtonian fluid over a stretching sheet with a power law surface velocity, slip velocity and variable thickness was studied by Khaddar et al [10]. Akbar et al. [?] obtained the numerical solution for magnetohydrodynamics boundary layer flow of tangent hyperbolic fluid over a stretching sheet using fourth order Runga-Kutta method.

The purpose of current study is to analyze the characteristics of radiative heat transfer on the boundary layer stagnation-point flow of viscous fluid over a non-flatness stretching sheet. This work is extension of Subhashini et al. [24]. Similarity transforms are used for this problem, and non dimensionalized equations are solved numerically. Graphical results for various values of the parameters are presented to gain thorough insight towards the physics of the problem. To the best of our knowledge, this problem has not been studied before.

2 Mathematical Analysis

Consider the flow of an incompressible viscous fluid driven by a stretching surface located at $y = A(x+b)^{\frac{1-m}{2}}$ with a fixed stagnation point at $x = 0$ as shown in figure 1. We assume that wall is impermeable, non-flat with a given profile and the coefficient A being small so that the sheet is sufficiently thin. The stretching velocity $U_w(x)$ and the ambient fluid velocity $U(x)$ are assumed to be thickness of the stretched sheet from the stagnation point i.e., $U_w(x) = U_0(x+b)^m$ and $U(x) = U_1(x+b)^m$, where m is the velocity power index. Due to the acceleration or deceleration of

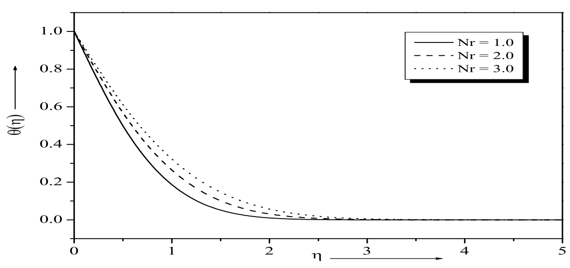

Figure 9: Effect of Nr on temperature profiles when $\lambda = 0.5, m = 2, Pr = 3$ and $\beta = 0.5$.

the sheet, the thickness of the stretched sheet may decrease or increase with distance from the slot, which is dependent on the value of the velocity power index. With the above assumptions, the boundary layer equations governing the flow and temperature fields are given by [24],

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{2.1}$$

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \mu\frac{\partial^2 u}{\partial y^2} - \frac{dp}{dx}, \tag{2.2}$$

$$\rho c_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = k\frac{\partial^2 T}{\partial y^2} - \frac{\partial q_r}{\partial y} \tag{2.3}$$

where u and v are the velocity components of the fluid along x and y directions respectively.μ, ρ, k and c_p are the co-efficient of viscosity of the fluid, density of the fluid, thermal conductivity and specific heat of the fluid respectively. The associated boundary conditions for the present problem are (see([6],[24]))

$$\begin{cases} u = U_w(x), v = 0, \\ T_w(x) = T_\infty + T_0(x+b)^{\frac{m}{2}} \\ \text{at} \quad y = A(x+b)^{\frac{1-m}{2}} \\ u \to U(x) \quad T \to T_\infty \quad \text{as} \quad y \to \infty \end{cases} \tag{2.4}$$

where $U_w(x) = U_0(x+b)^m$ is the stretching velocity, U_0 and b are the physical parameter related with stretched surfaces. T_w and T_∞ denote the temperature at the wall and at large distance from the wall respectively and T_0 is the characteristic temperature.

To employing the generalized Bernoulli's equation, in the free stream $U(x) = U_1(x+b)^m$ the equation (2.2) reduces to

$$U\frac{dU}{dx} = -\frac{1}{\rho}\frac{dp}{dx}. \tag{2.5}$$

Using (2.5) into (2.2) one can obtain

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \mu\frac{\partial^2 u}{\partial y^2} + U\frac{dU}{dx} \tag{2.6}$$

Using the Rosseland approximation for radiation [17], radiation heat flux is simplified as

$$q_r = -\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y} \tag{2.7}$$

where σ^* and k^* are the Stefan-Boltzman constant and mean absorption co-efficient, respectively. Assuming that the temperature differences within the flow such that the term T^4 may be expressed as a linear function of the temperature, we expand T^4 in a Taylor series about T_∞ and neglecting the higher order terms beyond the first degree in $(T - T_\infty)$ we get

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4 \tag{2.8}$$

Substituting equations (2.7) and (2.8) in equation (2.3) reduces to

$$\rho c_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = k\frac{\partial^2 T}{\partial y^2} - \frac{16\sigma^* T_\infty^3}{3k^*}\frac{\partial^2 T}{\partial y^2}. \tag{2.9}$$

The momentum and energy equations can be transformed using the following similarity transformation (see [6])

$$\begin{cases} \eta = \sqrt{\frac{(m+1)U_0}{2\nu}}\left[y(x+b)^{\frac{m-1}{2}} - A\right] \\ f = \frac{\psi}{\sqrt{\frac{2\nu U_0}{m+1}(x+b^{m+1})}} \\ \theta = \frac{T-T_\infty}{T_w-T_\infty} \end{cases} \tag{2.10}$$

where η is the similarity variable and ψ is the stream function defined as $u = \frac{\partial\psi}{\partial y}$ and $v = \frac{\partial\psi}{\partial x}$ which identically satisfies equation (2.1). Employing the similarity variables (2.10), equations (2.6) and (2.9) reduce to the following ordinary differential equations

$$f'''(\eta) + f(\eta)f''(\eta) + \frac{2m}{m+1}[\lambda^2 - f'^2(\eta)] = 0 \tag{2.11}$$

$$\left(1 + \frac{4}{3}Nr\right)\frac{1}{Pr}\theta''(\eta) + f(\eta)\theta'(\eta) - \frac{m}{m+1}f'(\eta)\theta(\eta) = 0 \tag{2.12}$$

Subjected to the boundary conditions (2.4) which becomes

$$\begin{cases} f(\eta) = \beta\left(\frac{1-m}{1+m}\right), f'(\eta) = 1, \\ \theta(\eta) = 1 \quad \text{at} \quad \eta = 0 \\ f'(\eta) = \lambda, \theta(\eta) = 0 \quad \text{as} \quad \eta \to \infty \end{cases} \tag{2.13}$$

In the above equations, primes denote differentiation with respect to η and $\beta = A\sqrt{\frac{U_0(m+1)}{2\nu}}$ is the wall thickness parameter, m is the velocity power index, $\lambda = \frac{U_1}{U_0}$ is the ratio of rates of velocities, $Nr = \frac{4\sigma^* T_\infty^3}{kk^*}$ is the radiation parameter and $Pr = \frac{\mu c_p}{k}$ is the Prandtl number.

3 Numerical Solutions

The analytical solutions of the set of differential equations given by (2.11) and (2.12) are generally intractable because these equations are highly non-linear. So we adopted the most effective shooting method with fourth and fifth-order Runge-Kutta-Fehlberg integration scheme to solve the boundary value problems. Important factor of this method is to choose the appropriate finite values of η Once the finite value of η is determined then the coupled boundary value problem given by equations (2.11), (2.12) and (2.13) are solved numerically using the method of superposition ([15]). The coupled boundary value problem of third-order in $f(\eta)$ and second-order in $\theta(\eta)$ has been reduced to a system of five simultaneous equations of first-order for five unknowns as follows:

$$\begin{cases} f' = p, p' = q, \\ q' = -fq - \frac{2m}{m+1}[\lambda^2 - p^2] \\ \theta' = r, \\ r' = \left(\frac{m}{m+1}\right)\frac{p\theta Pr}{1+4/3Nr} - \frac{frPr}{1+4/3Nr} \end{cases} \tag{3.14}$$

and the boundary condition becomes

$$\begin{cases} f(0) = \beta\left(\frac{1-m}{1+m}\right), p(0) = 1, \theta(0) = 1 \\ p(\eta_\infty) = \lambda, \theta(\eta_\infty) = 0. \end{cases} \tag{3.15}$$

In this study, the boundary value problem is first converted into an initial value problem (IVP). Then the IVP is solved by appropriately guessing the missing initial value $f''(0)$ and $\theta'(0)$ using the shooting method for several sets of parameters. The step size is $h = 0.1$ used for the computational purpose. The error tolerance of 10^{-6} is also being used. The results obtained are presented through tables and graphs, and the main features of the problems are discussed and analyzed.

4 Result and Discussions

For the verification of accuracy of the applied numerical scheme, a comparison with available results corresponding to the skin-friction coefficient $f''(0)$ for $\lambda = Nr = Pr = 0$ with the available published results of Fang et al. [6] for various values of m is made and presented in Table 1. This shows a favorable agreement thus gives confidence that the numerical results obtained are accurate. Also we provide a sample of our results for the $-\theta'(0)$ when Pr varies and other parameters are fixed is presented in Table 2. Similarly in Table 3 presented a computation values of $-\theta'(0)$ when λ varies. Now move on to the discussion part.

Figure 2 describes the velocity profiles for several values of λ. It is found that when the stretching velocity is less than the free stream velocity i.e., $\lambda > 1$, the flow has a boundary layer structure, physically saying that the straining motion near the stagnation region increases so the acceleration of the external stream increases which leads to decrease in the thickness of the boundary layer with increase in λ. When the stretching velocity $U_0(x+b)^m$ of the surface exceeds the free stream velocity $U_1(x+b)^m$ i.e., $\lambda < 1$ inverted boundary layer structure is formed and for $\lambda = 1$ there is no boundary layer formation because the stretching velocity is equal to the free stream velocity. The temperature profiles for different values of λ with other fixed parameter are presented in the figures 3. It is evident from the graph that the boundary layer thickness decreases with increase in λ.

Figure 4 exhibits the variation in the velocity profiles for different values of β. This figure indicates that if β increases the fluid velocity $f'(\eta)$ is increases for a fixed value of $\lambda < 1$. On the other hand when $\lambda > 1$ the fluid velocity decreases with the increase of β. This is because for higher value of β the boundary layer becomes thicker. The temperature profile for different values of β for a fixed value of λ is plotted in figure 5. As it can be noticed, an increase in the wall thickness parameter results in an increase of the temperature of fluid. The graph of velocity profiles for different values of m is depicted in figure 6. One can clearly observed in equation (2.13) that when increase of m, simultaneously β will also increases, whereas m, β also decreases, therefore β is depends on m. Figure 6 shows that when increase of m from -0.25 to 2.0 we can see that decrease in fluid velocity at $\lambda < 1$ and increase of velocity at $\lambda > 1$. For a fixed value of $\lambda = 0.5$ temperature increases with increase of m which is depicted in figure 7. From equation (2.13), one knows that if $m = 1$ the problem reduces to flat sheet problem.

Figure 8 depicts the effect of Prandtl number Pr on temperature distributions for a fixed value of $\lambda(\lambda = 0.5)$. An increase in Prandtl number reduces the thermal boundary layer thickness. Prandtl number signifies the ratio of momentum diffusivity to thermal diffusivity. Fluids with lower Prandtl number will possess higher thermal conductivities (thicker thermal boundary layer structures) so that heat can diffuse from the wall faster than for higher Pr fluids (thinner boundary layers). Hence Prandtl number can be used to increasethe rate of cooling in conducting flows. The effect of thermal radiation on temperature profiles is presented in figure 9. It is found that temperature increases as the radiation parameter Nr increases. This is in agreement with the physical fact that the thermal boundary layer thickness increases with increasing Nr The effect of radiation in the thermal boundary layer equation (2.12) is equivalent with an increased thermal diffusivity, i.e., $\left(1 + \frac{4}{3}Nr\right)\frac{1}{Pr}$. Thus the radiation should be at its minimum in order to facilitate the cooling process.

From the above discussion, the physics of stagnation point flow over a non-flatness stretching surface can be utilized as the basis for many engineering and scientific applications with this model. The findings of the present problem are also of great interest in different areas of science and technology, where the surface layers are being stretched.

5 Conclusions

We have studied the effects of the wall thickness parameter, velocity power index, ratio of rates of velocities, radiation parameter and the Prandtl number on the skin friction coefficient and the local Nusselt number, which represents the heat transfer rate at the surface, for the steady stagnation point flow and heat transfer towards stretching surface. It is found that boundary layer is formed when $\lambda > 1$. On the other hand, an inverted boundary layer is formed when $\lambda < 1$. Some results of thermal characteristics at the wall are usually analyzed from the numerical results and the same are documented in the Table 4. From this table, it is found that increasing the values of β and m is to decreases $f''(0)$ and the effect of increasing the values of Pr is to increase the $-\theta'(0)$ while increasing the values of Nr is to decrease the $-\theta'(0)$. Also one can observe that there is no changes in $f''(0)$ when Nr and Pr vary.

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Dr. B. C. Prasanna Kumara born in Kudlur village, Karnataka, India, received his Master's degree in Mathematics and Ph.D. in Fluid Mechanics from Kuvempu University, Shimoga, India, in 2000 and 2007 respectively. Currently he is working as Assistant Professor in the Department of Mathematics, Government First Grade College, KOPPA,Chikmagalur(District),Karnataka India. He has authored and coauthored 16 national and international journal papers and 4 conference papers. He has attended/presented the papers in 10 International/National conferences. He is a member of several academic bodies. His research interests include the areas of Fluid

Mechanics, Differential Geometry and Computer simulation.

Dr. G. K. Ramesh was born and brought up in the district of Chitradurga, Karnataka, India. He obtained the B.Sc. Degree, M.Sc. Degree in Mathematics, in 2006 and 2008 and Ph.D. in Mathematics from Kuvempu University in 2013. He has published 22 research papers. His field of interest covers the areas of the application of boundary layer flows of Newtonian/non-Newtonian fluids, nanofluids and dusty fluid flow and stretching sheet problems.

Dr. Ali J. Chamkha is Professor and Chairman of the Mechanical Engineering Department at the Prince Mohammad Bin Fahd University (PMU) in the Kingdom of Saudi Arabia. He earned his Ph.D. in Mechanical Engineering from Tennessee Technological University, USA, in 1989. His research interests include multiphase fluid-particle dynamics, fluid flow in porous media, heat and mass transfer, magnetohydrodynamics and fluid-particle separation. He has served as an Associate Editor for many journals such as International Journal of Numerical Method for Heat and Fluid Flow, Journal of Applied Fluid Mechanics, International Journal for Microscale and Nanoscale Thermal and Fluid Transport Phenomena and he is currently the Deputy Editor-in-Chief for the International Journal of Energy Technology. He has authored and co-authored over 450 publications in archival journals and conferences.

Dr. B. J. Gireesha born in Kolalu village, Karnataka, India on 1st June 1974 received his Master's degree in Mathematics, M.Phil., in Applied Mathematics and Ph.D. in Fluid Mechanics from Kuvempu University, Shimoga, India, in 1997, 1999 and 2002 respectively. Currently he is working as faculty in the Department of Mathematics, Kuvempu University. He has authored and coauthored 6 books, 120 national and international journal papers, 9 conference papers and editor of 2 conference proceedings. He has attended/presented the papers in 25 International/National conferences. He is a member of several bodies, Editorial Board member for several Journals. His research interests include the areas of Fluid Mechanics, Differential Geometry and Computer simulation. 8 students are awarded Ph.D., degree and 7 students are awarded M. Phil degree under his supervision. Presently he is guiding 8 Students for Ph.D.