

Electromagnetism-like algorithm for fuzzy flow shop batch processing machines scheduling to minimize total weighted earliness and tardiness

S. Molla-Alizadeh-Zavardehi ^{*†}, R. Tavakkoli-Moghaddam ^{‡§}, F. Hosseinzadeh Lotfi [¶]

Abstract

In this paper, we study a flow shop batch processing machines scheduling problem. The fuzzy due dates are considered to make the problem more close to the reality. The objective function is taken as the weighted sum of fuzzy earliness and fuzzy tardiness. In order to tackle the given problem, we propose a hybrid electromagnetism-like (EM) algorithm, in which the EM is hybridized with a diversification mechanism and effective local search to enhance the efficiency of the algorithm. The proposed algorithms are evaluated by comparison against two existing well-known EMs in the literature. Additionally, we propose some heuristics based on the earliest due date (EDD) to solve the given problem. The proposed hybrid EM algorithm is tested on sets of various randomly generated instances. For this purpose, we investigate the impacts of the rise in problem sizes on the performance of the developed algorithm. Through the analysis of the experimental results, the highly effective performance of the proposed algorithm is shown against the two existing well-known EMs from the literature and proposed EDDs.

Keywords : Flow shop batch processing machines; Fuzzy due date; Hybrid electromagnetism-like algorithm; Fuzzy earliness/tardiness.

1 Introduction

Batching or grouping of jobs in many industries is a common policy. Batching occurs in two different versions: parallel-batching and serial-batching. On a parallel-batching machine, the jobs in a batch are processed simultaneously, so the processing time of a batch is the longest

processing time of jobs contained in the batch. On the serial batching machine, the processing time of a batch equals the sum of the processing times of jobs contained in the batch. In the literature, parallel batching scheduling is known as batch processing machine (BPM) scheduling.

The BPM scheduling problem is important and mainly motivated by the burn-in operation found in semiconductor manufacturing [31, 32]. Moreover, BPMs are encountered in various manufacturing environments (e.g., aircraft industry, shoe manufacturing industry, ion plating industry, iron and steel industry, steel-casting industry, glass container industry, and furniture manufacturing industry). A complete explanation of the product flow in the BPM and also semiconductor manu-

*Corresponding author. saber.alizadeh@gmail.com

[†]Department of Industrial Engineering, Science and Research Branch, Islamic Azad University, Tehran, Iran.

[‡]School of Industrial Engineering, College of Engineering, University of Tehran, Tehran, Iran.

[§]Research Center for Organizational Processes Improvement, Sari, Iran.

[¶]Department of Mathematics, Science Research Branch, Islamic Azad University, Tehran, Iran.

facturing can be seen in [31].

The BPM can process a group of jobs provided the sum of all the job sizes in the batch be less than or equal to the machine capacity. Once a batch is processed, it cannot be interrupted and no jobs can be added or removed until the process is completed.

Most existing models of scheduling problems neglect the presence of uncertainty within a manufacturing environment. In many real-world optimization problems, uncertainty in due date often do exist. This uncertainty may come about because of production problems (e.g., machine malfunctioning and defect in raw material) or problems with delivery itself (e.g., traffic jam and transportation delay). In some cases, due dates are considered as certain intervals and it may be appropriate to deal as fuzzy values. Fuzzy numbers are used to represent uncertain and incomplete information in optimization problems [1]. More and more, modeling techniques, control problems and operation research algorithms have been designed to fuzzy data since the concept of fuzzy number and arithmetic operations with these numbers was introduced and investigated first by Zadeh [24].

The remainder of the paper is organized as follows. Section 2 presents the literature on the problem under study. In Section 3, we describe the problem and present a fuzzy mathematical programming model. In Section 4, we propose our heuristics and hybrid electromagnetism-like (EM) algorithm and show the detailed implementation. In Section 5, we show the design of instances and report the computational results. Finally, the last section presents conclusions of our work.

2 Literature review

In this section, we focus on reviewing the studies with the assumptions of multi-BPM flow shop and the case of a fuzzy environment.

2.1 Multi-BPM flow shop

Most studies on flow shop scheduling to date have focused on discrete machines. Flow shop with BPMs has received very little attention. Danneberg et al. [9] proposed constructive and local search algorithms for minimizing the to-

tal weighted completion time and makespan in a multi-BPM flow shop with identical job sizes. They assumed batch-dependent setup and a limit number of jobs in a batch. A problem reduction procedure was proposed by Sung et al. [28] for removing dominated machines with respect to any regular performance measure. They studied a multi-BPM flow shop with the identical job sizes of one and constant batch processing times. The reduction procedure was combined with heuristics algorithms to minimize the total completion times and makespan. Husseinzadeh Kashan and Karimi [11] presented a mathematical model for multi-BPM flow shop to minimize the makespan and developed some lower bounds based on relaxing the problem assumptions.

Amin-Naseri and Beheshti-Nia [2] presented a Genetic Algorithm (GA) and three heuristics for minimizing the makespan in hybrid flow shop with BPMs in some stages. Lei and Guo [16] proposed a variable neighborhood search (VNS) for the objectives of the maximum tardiness, number of tardy jobs and total tardiness in multi-BPM flow shop. Lei and Wang [17] developed a neighborhood search algorithm for multi-BPM flow shop to minimize the maximum lateness. Behnamian et al. [3] studied the minimization of makespan in a three machine flow shop scheduling problem with a BPM among first and third stages (discrete-BPM- discrete) under the assumption of transportation capacity and times among machines. They proposed a mixed-integer programming (MIP) model and then a heuristic algorithm and GA. Damodaran et al. [8] proposed a Particle Swarm Optimization (PSO) algorithm to minimize the makespan and used some modified heuristics from the discrete flow shop problem to generate the initial population.

2.2 Fuzzy BPM scheduling

Fuzzy systems have gained more and more attention from researchers and practitioners of various fields [23, 25]. The concept of fuzzy due dates to scheduling problems introduced by Ishii et al. [12]. To the best of our knowledge, only three studies have been investigated the BPM scheduling problem with uncertainty. Harikrishnan and Ishii [11] proposed a polynomial time for bi-criteria scheduling to maximize the minimal satisfaction degree of due dates of jobs and minimize the total weighted resource

consumption. They studied the serial-batching problem with fuzzy due-dates in the presence of resource-dependent processing time and common setup. For fuzzy due dates, a membership function describing a non-decreasing satisfaction degree about job's completion time is defined.

Yimer and Demirli [34] applied a fuzzy goal programming problem for batch scheduling on parallel machines in a two-stage flow shop to minimize the total weighted flow time of jobs. They considered the uncertainty for processing and setup times by triangular fuzzy numbers. They also developed a GA for solving large-sized problems. Cheng et al. [7] proposed an ant colony optimization (ACO) algorithm for the fuzzy makespan on a single BPM with non-identical job sizes. The uncertainty of the jobs and the machine in the processing is denoted using triangular fuzzy numbers. For a further review on BPM scheduling problems, we refer to Potts and Kovalyov [22]. Besides, Mathirajan and Sivakumar [18] did a quite complete survey on scheduling with BPMs.

Minimizing the makespan is one of the most important criteria in scheduling problems. In the literature, there is no research that considers the objective of minimizing the fuzzy makespan, and proposes a new approach to solve a fuzzy multi-BPM flow shop scheduling problem. A trapezoidal fuzzy number is considered for processing times as an extended triangular fuzzy processing time. Here, we present a new fuzzy mathematical programming approach for the first time. Since the problem is NP-hard for solving the addressed problem, a new hybrid EM algorithm is proposed to obtain good solution results.

3 Fuzzy Mathematical Model

3.1 Deterministic Model

There are n jobs, J , to be processed on m different BPMs, M , with a processing time p_{jm} and a corresponding size, s_j . The BPM can process several jobs simultaneously as a batch b , as long as the total size of all the jobs in the batch does not exceed the machine capacity Cap . The processing time of a batch b on machine m is given by the longest job in the batch (i.e., $P_{bm} = \max \{p_{jm} | j \in \text{batch } b\}$). The formulation is as follows:

Notations:

Sets:

J	Jobs,	$j = 1, \dots, N$
B	Batches,	$b = 1, \dots, N$
M	Machines,	$m = 1, \dots, M$

Parameters:

$p_{j,m}$	Processing time of job j on machine m
s_j	Size of job j
Cap	Machines capacity
α_j	Earliness penalty (/unit/h) of job j
β_j	Tardiness penalty (/unit/h) of job j
d_j	Due date of job j
LN	Large number

Decision variables:

$X_{j,b}$	A binary variable indicates the assignment of job j to batch b
$P_{b,m}$	Processing time of batch b on machine m
c_j	Completion time of job j
$C_{b,m}$	Completion time of batch b on machine m
E_j	Earliness of job j
T_j	Tardiness of job j

The objective function consists of two sub-functions; earliness and tardiness. At first, the mixed integer linear program (MILP) model proposed by Husseinzadeh Kashan and Karimi [11] for the makespan is adapted for total weighted tardiness sub-function and then a mathematical model with the objective of the total weighted earliness and tardiness is proposed. According to the mentioned sets, parameters and decision variables, the mathematical formulation of the total weighted tardiness penalties of jobs can be writ-

ten as follows:

$$\text{Min } Z = \sum_{j \in J} \beta_j T_j s_j \tag{3.1}$$

s.t.

$$\sum_{b=1}^N X_{j,b} = 1 \quad j = 1, \dots, N \tag{3.2}$$

$$\sum_{j=1}^N s_j X_{j,b} \leq \text{Cap} \quad b = 1, \dots, N \tag{3.3}$$

$$P_{b,m} \geq p_{j,m} X_{j,b} \quad \begin{matrix} j = 1, \dots, N; \\ b = 1, \dots, N; \\ m = 1, \dots, M \end{matrix} \tag{3.4}$$

$$C_{b,1} = \sum_{b'=1}^b P_{b',1} \quad b = 1, \dots, N \tag{3.5}$$

$$C_{1,m} = \sum_{m'=1}^m P_{1,m'} \quad m = 2, \dots, M \tag{3.6}$$

$$C_{b,m} \geq C_{b-1,m} + P_{b,m} \quad \begin{matrix} b = 2, \dots, N; \\ m = 2, \dots, M \end{matrix} \tag{3.7}$$

$$C_{b,m} \geq C_{b,m-1} + P_{b,m} \quad \begin{matrix} b = 2, \dots, N; \\ m = 2, \dots, M \end{matrix} \tag{3.8}$$

$$c_j \geq C_{b,M} - LN(1 - X_{jb}) \quad \begin{matrix} j = 1, \dots, N; \\ b = 1, \dots, N; \end{matrix} \tag{3.9}$$

$$T_j \geq c_j - d_j \quad j = 1, \dots, N; \tag{3.10}$$

$$X_{j,b} \in \{0, 1\} \quad \begin{matrix} j = 1, \dots, N; \\ b = 1, \dots, N; \end{matrix} \tag{3.11}$$

The objective function is to minimize the total weighted tardiness penalties of the jobs. Constraint set (2) ensures that each job can be processed in only one batch. Constraint set (3) ensures that machine capacity is not exceeded when jobs are assigned to a batch. Constraint set (4) determines the processing time for the batch on machine m. Constraint set (5) determines the completion time of each batch. Constraint (5) determines the completion time of batches on machine 1. Constraint (6) determines the completion time of the first batch on other machines. Constraint (7) states that batch b is processed in machine m after its previous job. Constraint (8) ensures that batch b is processed in machine m only after its completion in machine m-1. Constraint set (9) defines the completion time of each job as the completion time of the batch in

last machine that it is processed in. Constraint set (10) defines the tardiness of a job. Constraint set (11) specifies the type of decision variable $X_{j,b}$.

Due to minimization of just only tardiness or total weighted tardiness penalties in the objective function, the model chooses the minimum P^b in the constraint sets (4) to reach the longest processing time among all the jobs in that batch. The smaller the completion time of jobs, the more desirable the objective function. Similarly, the model finds the minimum c_j and T_j in Constraint sets (6) and (7).

For the objective function of

$$\sum_{j \in J} (\beta_j T_j + \alpha_j E_j) s_j,$$

in addition to Constraint set (10), Constraint sets (12) and (13) are needed to calculate the earliness and tardiness of jobs.

$$E_j \geq d_j - c_j \quad j = 1, \dots, N; \tag{3.12}$$

$$c_j + E_j - T_j = d_j \quad j = 1, \dots, N; \tag{3.13}$$

Contrary to tardiness, earliness of jobs decreases when the completion time of jobs increases. So, the above model may not choose the minimum $C_{b,m}$, c_j in the constraint sets (4) and (7-9) in some situation. So, to tackle the dilemma, for the objective function with total weighted earliness-tardiness penalties of jobs, the nonlinear constraint sets (14) to (16) should be used instead of linear constraint sets (4), (7) to (9), (for $b = 1, \dots, N; m = 1, \dots, M$).

$$P_{b,m} = \max(\forall j \in J : p_{j,m} X_{j,b}) \tag{3.14}$$

$$C_{b,m} \geq \max(C_{b-1,m} + P_{b,m}, C_{b,m-1} + P_{b,m}) \tag{3.15}$$

$$c_j = \sum_{b=1}^n X_{j,b} C_b \tag{3.16}$$

These three constraint sets (14) to (16) convert the model to a mixed-integer nonlinear program and make it more complex.

3.2 Fuzzy model

We briefly introduce some basic concepts and results about fuzzy measure theory.

Definition 3.1 If X is a collection of objects denoted generically by x , then a fuzzy set in X is a set of the ordered pairs:

$$\tilde{d} = \left\{ x, \tilde{d}(x) | x \in X \right\},$$

where $\tilde{d}(x)$ is called the membership function that associates with each $x \in X$ a number in $[0, 1]$ indicating to what degree x is a number.

Definition 3.2 $\tilde{d}_j = (d_{j,1}, d_{j,2}, d_{j,3}, d_{j,4})$ denote a Trapezoidal Fuzzy Number (TFN) as shown in Fig. 1.

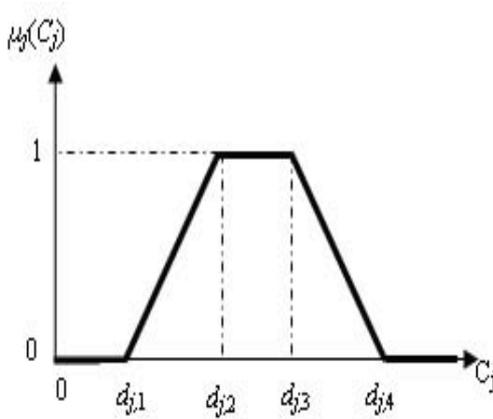


Figure 1: Trapezoidal membership function.

As mentioned in the literature, the concept of fuzzy due dates has been used in scheduling problems. Here, this concept is being firstly utilized in the BPM scheduling problem. In a fuzzy due date, the membership function assigned to each job represents the customer satisfaction degree for the delivery or completion time of that job. The membership function of a trapezoidal fuzzy due date of a job as a generalized triangular fuzzy due date is represented below.

$$\mu_j(c_j) = \begin{cases} 0 & \text{if } c_j \leq d_{j,1} \\ \frac{c_j - d_{j,1}}{d_{j,2} - d_{j,1}} & \text{if } d_{j,1} \leq c_j \leq d_{j,2} \\ 1 & \text{if } d_{j,2} \leq c_j \leq d_{j,3} \\ \frac{d_{j,4} - c_j}{d_{j,4} - d_{j,3}} & \text{if } d_{j,3} \leq c_j \leq d_{j,4} \\ 0 & \text{if } c_j > d_{j,4} \end{cases} \quad (3.17)$$

From Fig. 1, we can see that the full satisfaction (i.e., $\mu_j(C_j) = 1$) is attained if $d_{(j,2)} \leq c_j \leq d_{(j,3)}$, and the satisfaction grade is positive if $d_{(j,1)} \leq c_j \leq d_{(j,2)}$ or $d_{(j,3)} \leq c_j \leq d_{(j,4)}$ in the membership function (1). If $d_{(j,2)} = d_{(j,3)}$, the

fuzzy trapezoidal due date is called a triangular fuzzy due date and can be denoted by triplet $\tilde{d}_j = (d_{(j,1)}, d_{(j,2)}, d_{(j,3)})$.

In a particular situation (e.g., just-in-time production system), the full satisfaction is not attained if a completion time is too early or tardy. According to the mentioned fuzzy due date, the studied problem can be formulated as a maximization problem of the total degree of satisfaction over given jobs or equivalently, a minimization problem of the total degree of dissatisfaction. As mentioned above, the fuzzy mathematical formulation of the total degree of dissatisfaction is as follows.

$$\text{Min } Z = \sum_{j \in J} (T_j + E_j) s_j \quad (3.18)$$

$$E_j = 1 \quad \text{if } c_j \leq d_{j,1} \quad (3.19)$$

$$E_j = \frac{d_{j,2} - c_j}{d_{j,2} - d_{j,1}} \quad \text{if } d_{j,1} < c_j < d_{j,2} \quad (3.20)$$

$$T_j = \frac{c_j - d_{j,3}}{d_{j,4} - d_{j,3}} \quad \text{if } d_{j,3} < c_j < d_{j,4} \quad (3.21)$$

$$T_j = 1 \quad \text{if } c_j \geq d_{j,4} \quad (3.22)$$

We can also use the following objective function (23) to calculate the total degree of satisfaction instead of expressions (18) to (22).

$$\text{Min } Z = \sum_{j \in J} s_j \left[\left(\frac{\max(0, d_{j,1} - c_j)}{d_{j,1} - c_j} \right) + \left(\frac{\max(0, c_j - d_{j,1})}{c_j - d_{j,1}} \right) \left(\frac{\max(0, d_{j,2} - c_j)}{d_{j,2} - c_j} \right) \left(\frac{d_{j,2} - c_j}{d_{j,2} - d_{j,1}} \right) \right] + \sum_{j \in J} s_j \left[\left(\frac{\max(0, c_j - d_{j,3})}{c_j - d_{j,3}} \right) \times \left(\frac{\max(0, d_{j,4} - c_j)}{d_{j,4} - c_j} \right) \left(\frac{c_j - d_{j,3}}{d_{j,4} - d_{j,3}} \right) + \left(\frac{\max(0, c_j - d_{j,4})}{c_j - d_{j,4}} \right) \right] \quad (3.23)$$

Since each customer has a different degree of importance as a decision maker, different earliness tardiness weights or penalties are considered, which is the same as the extreme majority of related papers. Therefore, here we extend the previous model (i.e., Equation (23)) and reformulate

it with earliness tardiness penalties shown below.

$$\begin{aligned}
 & \text{Min } Z \\
 & = \\
 & \sum_{j \in J} \alpha_j s_j \left[\left(\frac{\max(0, d_{j,1} - c_j)}{d_{j,1} - c_j} \right) + \right. \\
 & \left. \left(\frac{\max(0, c_j - d_{j,1})}{c_j - d_{j,1}} \right) \left(\frac{\max(0, d_{j,2} - c_j)}{d_{j,2} - c_j} \right) \right. \\
 & \left. \left(\frac{d_{j,2} - c_j}{d_{j,2} - d_{j,1}} \right) \right] \\
 & + \sum_{j \in J} \beta_j s_j \left[\left(\frac{\max(0, c_j - d_{j,3})}{c_j - d_{j,3}} \right) \right. \\
 & \left. \left(\frac{\max(0, d_{j,4} - c_j)}{d_{j,4} - c_j} \right) \left(\frac{c_j - d_{j,3}}{d_{j,4} - d_{j,3}} \right) \right. \\
 & \left. + \left(\frac{\max(0, c_j - d_{j,4})}{c_j - d_{j,4}} \right) \right]
 \end{aligned} \tag{3.24}$$

4 Solution approach

4.1 Proposed earliest due date heuristics

In this subsection, we propose three constructive greedy heuristics based on the earliest due date (EDD) as a well-known heuristic method related to the due date. The details of these proposed heuristics are as follows:

- 1) Calculate the index of jobs to be scheduled.
- 2) Sort jobs in increasing order of their index.
- 3) Apply the first-first (FF) heuristic to group jobs into batches.

Accordingly, the details of these three variants of EDDs are as follows:

EDD Algorithm: In this variant, the indexes are equal to the EDD of the respective jobs. Ranking of fuzzy numbers play an important role in decision making, optimization, forecasting etc [26]. The centroid-based distance method is used for ranking fuzzy numbers as follows.

$$d = \frac{1}{3} \left(K_1 - \frac{K_2}{K_3} \right) \tag{4.25}$$

where

$$\begin{aligned}
 K_1 &= d_1 + d_2 + d_3 + d_4, \\
 K_2 &= d_3 d_4 - d_1 d_2, \\
 K_3 &= (d_3 + d_4) - (d_1 + d_2).
 \end{aligned}$$

The jobs are sorted in increasing order of their crisp due dates. So, the job that has the EDD will be allotted first.

EDD1 Algorithm: Sort jobs in increasing order of their d_1 .

EDD2 Algorithm: Sort jobs in increasing order of their d_2 .

EDD3 Algorithm: Sort jobs in increasing order of their d_3 .

EDD4 Algorithm: Sort jobs in increasing order of their d_4 .

4.2 Encoding scheme and initialization

A candidate solution is represented as an array. As mentioned earlier in the literature, the random key (RK) method is used for solving BPM scheduling problems. To generate a sequence by this method, random real numbers between zero and one are generated for each job. By ascending sorting of the value corresponding to each job, the job sequence is obtained and then the first-first (FF) heuristic is applied to group the jobs into batches.

After having a permutation and forming the batches, we can use it to compute the objective function value of this solution. Each job has a random real number between 0 and 1, and these numbers show the relative order of the jobs. In fact, the problem variables in the algorithms are limited between 0 and 1. For example, consider a problem with ten jobs. The encoding of this sequence through the random keys is shown in Fig. 2. The sequence at position 1 is 2, which means we schedule job 2 in the beginning position and job 8 at last.

	Jobs	1	2	3	4	5	6	7	8	9	10
Before	RKs	0.23	0.18	0.38	0.87	0.53	0.76	0.46	0.93	0.36	0.84
	Jobs sequence	2	1	9	3	7	5	6	10	4	8
After	RKs	0.18	0.23	0.36	0.38	0.46	0.53	0.76	0.84	0.87	0.93

Figure 2: Representation encoding for a candidate solution.

4.3 The proposed hybrid EM

Due to the NP-hard nature of BPM problems, heuristics and meta-heuristics are recommended

for seeking high-quality schedules within reasonable computation times. The EM was first introduced by Birbil and Fang [4] as a new stochastic population-based heuristic optimization tool to solve the problems with bounded variables. Although the EM approach has been designed for continuous optimization problems, here we adapt it to solve the discrete optimization problems. The EM approach has been recently applied to solve several combinatorial optimization problems, such as set covering problem [20], p-hub median problem [15], feature selection [27], single machine scheduling [6], flow shop [14], and job shop [29], and the like. To make meta-heuristics more efficient and robust in solving complex optimization problems, diverse and intense hybridization mechanisms have been successfully proposed.

4.3.1 Diversification phase

One of the important issues in designing the hybrid meta-heuristic algorithm is to keep the diversity to explore new unvisited regions of the solution space. The original EM often suffers from loss of diversity through premature convergence of the population, causing the search becoming trapped in a local optimum. In our procedure the diversification is applied when the best objective function does not change during the number of pre-specified consecutively iterations in the EM algorithm. Diversification refers to the process of replacing inferior solutions of the current population by replacing new randomly generated solutions. If the X^{best} is not improved for more than a pre-specified number of iterations (*no_change*), the restart phase triggers to regenerate the population by the following process:

- (1) Sort the population in ascending order of the objective function value;
- (2) The first $\gamma\%$ of individuals from the best individuals are chosen to skip; and
- (3) The remaining $1 - \gamma\%$ of individuals passed away from the population and regenerated randomly from the solution space to join the population such that the population size remains *popsize*.

4.3.2 Intensification phase

The right balance between intensification and diversification makes meta-heuristic algorithms

naturally effective to solve the complex problems. The basic idea of the proposed local search procedures is to successively use a set of various neighborhoods to get a better solution. It searches either at random or systematically two neighborhoods called interchange and inversion neighborhood to generate different landscapes and to escape from local optima. It exploits the fact that using different neighborhoods in local search may obtain better local optima and that the global optimum is one of the local optimum for a given neighborhood. Also, the results show that using the multiple neighborhoods structure is more effective way to get a better control in balancing intensification and diversification. The proposed powerful local search procedures are illustrated in the following Algorithm.

ALGORITHM. Local (*LSITER*, *interchange*, *insertion*)

```

1:  $I = 0$ 
2: for  $i = 1$  to popsize do
3:   counter  $\leftarrow 1$ 
4:   while counter  $\leq$  LSITER do
5:     if  $I = 0$  then
6:       randomly generate a solution Y from
interchange neighborhood structure on  $X^i$ 
7:     else
8:       randomly generate a solution Y from
interchange neighborhood structure on  $X^i$ 
9:     end if
10:    if  $f(Y) < f(X^i)$  then
11:       $X^i \leftarrow Y$ 
12:    else
13:       $I \leftarrow |I - 1|$ 
14:    end if
15:    counter  $\leftarrow$  counter + 1
16:  end while
17: end for
18:  $X^{best} \leftarrow \operatorname{argmin}\{f(X^i), \forall i\}$ 

```

5 Computational experiments

5.1 Instances

we generate the required data that can affect the performance of the algorithms including the number of jobs (n), number of machines (m), range of processing time (p_j), size (s_j), earliness costs (α_i), tardiness costs (β_i) and tightness of jobs due date (d_j).

30 different problem sizes with the combinations of number of machines (i.e., $m \in \{3, 5, 10\}$) and number of jobs (i.e., $n \in \{10, 20, 30, 50, 75, 100, 125, 150, 175, 200\}$) are considered for the experimental study. Three different types of processing times, size, earliness/tardiness costs and due date of jobs are generated randomly and Cap considered 20 in all instances. Hence, the number of parameter configurations in each problem size is equal to $3^4 = 81$.

The crisp due dates in Tavakkoli-Moghaddam et al. [30], test problems are generated from a uniform distribution. We use such a procedure with some modifications to adapt the procedure for our problem as shown below.

$$\bar{P} = \frac{\sum_{i=1}^m \sum_{j=1}^n P_{ij}}{m \times n} \tag{5.26}$$

$$\bar{B} = \frac{\sum_{j=1}^n s_j}{0.65 \times Cap} \tag{5.27}$$

$$BP = (2m + \bar{B} - 1) \times \bar{P} \tag{5.28}$$

After generating the BP , $d_{j,1}$, $d_{j,2}$, $d_{j,3}$ and $d_{j,4}$ are generated as explained in Table 1.

Table 1. Test problems characteristics.

Parameter	Level	Count
Number of jobs (N)	10, 20, 30, 50, 75, 100, 125, 150, 175, 200	10
Number of machines (M)	3, 5, and 10	3
Processing time of jobs (P)	Uniform distributions [1, 5], [1, 10], and [1, 20]	3
Size of jobs (S)	Uniform distributions [1, 5], [1, 10], and [1, 15]	3
Tardiness cost (T)	Uniform distributions [5, 10], [5, 15], and [5, 20]	3
Earliness cost (E)	Uniform distributions [1, 4]	1
$d_{j,4}$	Uniform distributions $(0.4, 1) \times BP$, $(0.4, 1.3) \times BP$, and $(0.4, 1.6) \times BP$	3
$d_{j,3}$	Uniform distributions $(0.8, 0.9) \times d_{j,4}$	1
$d_{j,2}$	Uniform distributions $(0.6, 0.7) \times d_{j,4}$	1
$d_{j,1}$	Uniform distributions $(0.4, 0.5) \times d_{j,4}$	1
Total number of problem instances		2430

Figure 3: Variation of β on u when $M = 1$ and $K = 1$.

5.2 Experimental parameters

The parameter values and operators are given as follows:

- LSITER: 30;
- Population size: 50;
- ν in revised EM: 0.5;
- Type of modification in revised EM: farthest point;
- no_change : 20;
- γ : $0.4 \times PopSize$;

5.3 Experimental results

A computational study was conducted to evaluate the efficiency and effectiveness of the proposed algorithm, which was coded in MATLAB and run on a PC with 2.8 GHz Intel Core 2 Duo and 4 GB of RAM memory. For this purpose, we present and compare the results of hybrid EM with the original EM, revised EM proposed by Birbil et al. (2004), EDD, EDD1, EDD2, EDD3 and EDD4. In order to be fair, the stopping criterion for all algorithms is equal to $6 \times (n + m)$ milliseconds.

This criterion is sensitive to a number of jobs and number of machines, so the searching time increases according to the rise of the problem size. Because the scale of objective functions in each instance is different, they cannot be used directly. In order to make the comparison easy and comprehensive, we use the relative percentage deviation (RPD) as a common and straight forward measure of comparing algorithms for each instance as follows.

$$RPD = \frac{Alg_{sol} - Min_{sol}}{Min_{sol}} \times 100$$

where Alg_{sol} and Min_{sol} are the obtained objective value and the obtained best solution for each replication of instance in a given test problem, respectively. Clearly, lower values of the RPD are preferable. The results of the experiments are transformed into the RPD.

To analyze the interaction between quality of the algorithms and different problem sizes, the RPD results are calculated for test problems and averaged for each problem size. The average RPDs obtained by each algorithm are shown in Figs. 3 to 5. Considering these figures, it is

revealed that the EDD rule produces intermediate solutions comparatively. Among heuristics, EDD1 and EDD4 have worst results, and it can be concluded that EDD3 is better than EDD2 and EDD. However, the comparison with hybrid EM, revised EM and original EM, they produce larger values, which are expected. As it can be seen, Hybrid EM keeps its robust performance in all the problem sizes. The revised EM has better performance than the original EM and outperforms it. It is noticeable that with increasing the problem size in terms of both number of jobs and number of machines, gradually the RPDs of the proposed EDDs decreases and can be complete especially in the three last sizes.

To verify the statistical validity of the results, we carry out the analysis of variance (ANOVA) technique to accurately analyze the results. The means plot and LSD intervals at the 95% confidence level for all the algorithms are shown in Fig. 6. According to the results, the average RPD obtained by the proposed hybrid EM is 18.09, while for the revised EM, original EM, EDD, EDD1, EDD2, EDD3 and EDD4 are 23.665, 28.714, 110.963, 116.984, 112.331, 108.264 and 113.757, respectively.

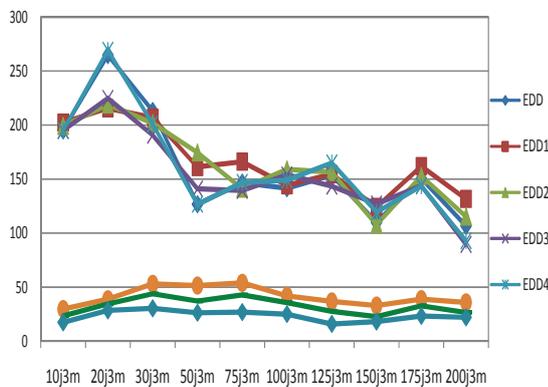


Figure 4: Interaction between the number of jobs and performance of algorithms for instances with three machines.

The related results demonstrate that there is a clear statistically significant difference between performances of heuristics and meta-heuristics. It shows that there are no statistically significant differences between the EDDs; however, the EDD3 and EDD2 have slightly better performance compared with the EDD, EDD1 and EDD4. It is clear that the upper and lower lim-

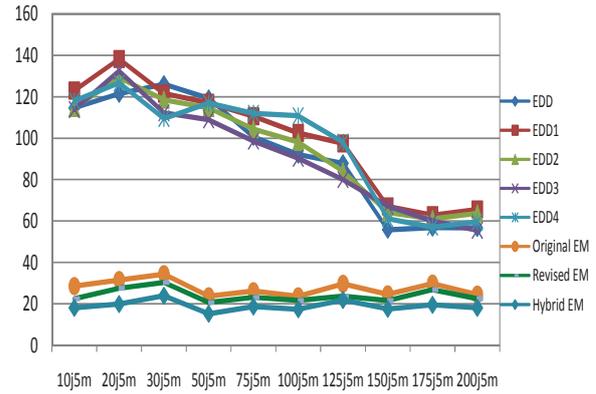


Figure 5: Interaction between the number of jobs and performance of algorithms for instances with five machines.

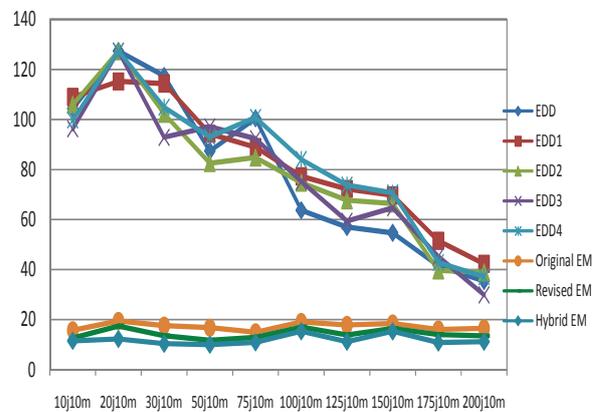


Figure 6: Interaction between the number of jobs and performance of algorithms for instances with ten machines.

its of the proposed hybrid EM do not have any overlapping.

6 Conclusions

This paper has considered a realistic variant of a multi-BPM flow shop scheduling environment, which aimed at minimizing the total weighted fuzzy earliness and tardiness. To solve this problem, a novel hybrid electromagnetism-like (EM) mechanism and some heuristics have been proposed based on the EDD. To the best of our knowledge, this has been the first reported application of the EM algorithm for solving the problem under consideration. In the proposed algorithm, the EM has been hybridized with a diversification mechanism, and an effective local

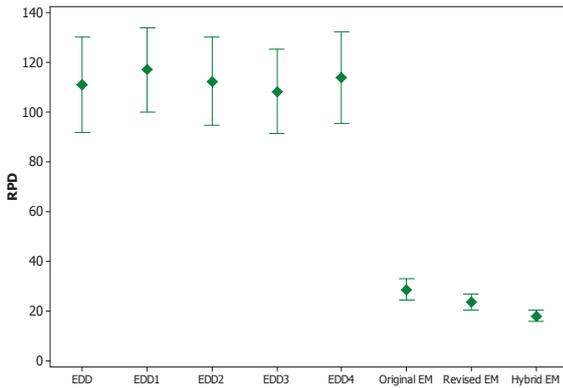


Figure 7: Means plot and LSD intervals for the proposed algorithms.

search to implement the local search and global search. So, to enhance the exploitation and exploration capabilities of the proposed algorithm, two local search and diversification mechanism have been proposed in EM. To evaluate performance of algorithms, a set of test problems has been designed. The experimentation has shown that EDDs obtains good solutions partly and their results have been better when the number of jobs and number of machines have increased. Among them, the EDD3 has performed better than others. The computational results and comparisons have demonstrated the effectiveness and robustness of the hybrid EM. Our future work is to investigate the other soft computing technique such as artificial neural networks [21,13,19] for the multi-BPM flow shop scheduling problems and generalize the application of the hybrid EM algorithm to solve other combinatorial optimization problems.

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Saber Molla-Alizadeh-Zavardehi is an assistant professor of Industrial Engineering at College of Engineering, Islamic Azad University (IAU), Masjed-Soleiman Branch in Iran. He is the Graduate Programs Manager of Industrial Engineering at the IAU, Masjed-Soleiman Branch, in Iran. His research interests are production scheduling, supply chain modeling, fuzzy set theory and meta-heuristic algorithms. He has about 8 years of teaching and research experience and has authored 25 papers published in various international journals.



Reza Tavakkoli-Moghaddam is a professor of Industrial Engineering at College of Engineering, University of Tehran in Iran. He obtained his Ph.D. in Industrial Engineering from Swinburne University of Technology in Melbourne (1998), his M.Sc. in Industrial Engineering from the University of Melbourne in Melbourne (1994) and his B.Sc. in Industrial Engineering from the Iran University of Science and Technology in Tehran (1989). His research interests include facility layouts and location design, cellular manufacturing systems, sequencing and scheduling, and us-

ing meta-heuristics for combinatorial optimization problems. He is the recipient of the 2009 and 2011 Distinguished Researcher Award and the 2010 Distinguished Applied Research Award at University of Tehran, Iran. Additionally, he has been selected as National Iranian Distinguished Researcher for two years (2008 and 2010). Professor Tavakkoli-Moghaddam has published 4 books, 15 book chapters and more than 500 papers in reputable academic journals and conferences



Farhad Hosseinzadeh-Lot is currently a professor of Mathematics at Islamic Azad University - Science Research Branch, Tehran, Iran. He obtained his Ph.D. in Applied Mathematics from Islamic Azad University (IAU), Science Research Branch in Iran in 1999. He has been selected as National Iranian Distinguished Researcher in 2008 and National Iranian Distinguished Research Manager in 2009. Professor Hosseinzadeh-Lot has published a number of books and papers