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Research Article

On approximation of the fully fuzzy fixed charge transportation problem

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Abstract

In the literature hardly any attention is paid to solving a fuzzy fixed charge transportation problem. In this paper, we consider the fully fixed-charge transportation problem and try to find both the lower and upper bounds on the fuzzy optimal value of such a problem in which all of the parameters are triangular fuzzy numbers. To illustrate the proposed method, a numerical example is presented.

Keywords : Fixed charge transportation; Triangular fuzzy numbers; Fuzzy transportation problem; Ranking function.

1 Literature review

The Fixed-Charge Transportation Problem (FCTP) is an extension of the classical transportation problem (TP). The FCTP was first formulated and studied by Balinski [3]. In the FCTP, each route is associated with a fixed cost and a transportation cost per unit shipped. Since problems with fixed charge are usually NP-hard problems [9], the computational time to obtain exact solutions increases in the distinguished Class P of problems and very quickly becomes extremely long as the size of the problem increases [1], and the presence of the fixed cost in the objective function makes the problem difficult to solve. It seems useful to use a heuristic method further in order to make it more flexible to provide a good solution.

According to the available literature, a wide range of different strategies are used in order to find an optimal solution for FCTPs. Generally, the solving methods of the FCTP can be classified as follows: exact, heuristic and meta-heuristic methods.

The most known methods for solving the FCTP in the exact methods class are as follows: an algorithm to find exact solution of the FCTP by decomposing itinto a master integer program and a series of transportation sub-programs [10], the cutting planes method, the ranking-the-extreme-points method, and the branch-and-bound method. These methods are generally inefficient and computationally expensive.

Many researchers attempted to solve the small size FCTP using heuristic methods such as [1, 2, 3, 13]. Although heuristic methods are usually computationally efficient, the major disadvantage of heuristic methods is the possibility of terminating at a local optimum that is far distant from the global optimum. However, the meta-heuristic methods were proposed to solve such hard opti-

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mization problems [7].

All of the literature noted above briefly introduce the FCTP concept in an effort to familiarize the reader with the underlying theory and then present the approaches with precise data to solve the FCTP. In fact, for each possible transportation pattern in the real world, some or all the parameters are not only well-defined, precise data, but also vague or fuzzy data. Since the fuzzy set theory was proposed by Zadeh in 1965 [17], we have been able to handle such data. The role of fuzzy sets in decision making processes is best described in the original statements of Bellman and Zadeh [4]. Thus, decision making processes are better described and solved by using fuzzy set theory, rather than precise approaches [14]. Moreover, this was one motivation for the mathematical programming problems, and in an early attempt, Zimmermann [18, 19] have proposed the application of the fuzzy set theory to the linear programming and multi-criteria decision-making problem. Chanas et al. [5] presented a fuzzy linear programming model to solve TP with fuzzy supply and demand values. Chanas and Kuchta [6] developed an algorithm to obtain the optimal solution based on type of TPs with fuzzy coefficients. Recently, Safi and Razmjoo [16] solved FCTP in which all of the parameters are interval data. They considered two different order relations for interval numbers, and developed two solution procedures in order to obtain an optimal solution for interval fixed FCTP.

With regard to solving the fuzzy fixed-charge transportation problem (FFCTP), a research has hardly been conducted. Therefore, any method, which provides a good solution for it, will be distinguished. To this end, a new method is proposed to find an approximation solution close to the optimal solution to the FFCTP such that all of the parameters (transportation cost, fixed cost, demand and supply) are triangular fuzzy numbers (TFNs). The present paper, first, tries to convert the FFCTP into the fuzzy transportation problem (FTP) by using the development of Balinski's formula. This becomes a linear version of the FFCTP for the next stage, and then, tries to obtain the fuzzy optimal solution the linear version of the FFCTP. The proposed method obtains a lower and upper bounds both on the fuzzy optimal value of the FFCTP can be easily obtained by using the approximation solution. Since the proposed method is a direct extension of classical method, then the proposed method is very easy to understand and to apply on real-life problems for the decision makers.

The rest of this paper is organized as follows: in Section 2, some basic definitions and arithmetic operations between two TFNs are reviewed. In Section 4, formulation of the FCTP is reviewed. The FFCTP is presented in Section 5. In the next Section, we develop the approximation method to solve FFCTP. To explain the method, a numerical example is solved in Section 6. Finally conclusions are pointed out in the last Section.

2 Preliminaries

In this section, we briefly will state some fundamental definitions and basic notation of fuzzy set theory which is crucial for the remainder of this paper.

2.1 Basic definitions

Definition 2.1 *If X is a collection of objects denoted generically by x, then a fuzzy set in X is a set of ordered pairs $\tilde{A} = \{(x, \tilde{A}(x))|x \in X\}$ where$\tilde{A}(x)$ is called the membership function which associates with each $x \in X$ a number in [0, 1] indicating to what degree x is a number.*

Definition 2.2 *A fuzzy set $\tilde{A}$ on $\Re$ is a fuzzy number if the following conditions hold:*

(i) *Its membership function is piecewise continuous function.*

(ii) *There exist three intervals [a,b], [b,c] and [c,d] such that $\tilde{A}$ is strictly increasing on [a,b], equal to 1 on [b,c], strictly decreasing on [c,d] and equal to 0 elsewhere.*

Definition 2.3 *A fuzzy number $\tilde{A} = (a, b, c)$ is said to be a triangular fuzzy number (TFN) if its membership function is given by*

$$\tilde{A}(x) = \begin{cases} \frac{x-a}{b-a}, & a \le x \le b, \\ \frac{x-c}{b-c}, & b \le x \le c, \\ 0 & otherwise. \end{cases}$$

Definition 2.4 *A triangular fuzzy number $\tilde{A} = (a, b, c)$ is said to be non-negative if $a \geq 0$.*

Definition 2.5 *Two triangular fuzzy numbers $\tilde{A} = (a_1, a_2, a_3)$ and $\tilde{B} = (b_1, b_2, b_3)$ are said to be equal if and only if $a_i = b_i$ for $i = 1, 2, 3$.*

2.2 Arithmetic operations

In this subsection, we reviewed arithmetic operations between two TFNs [14]. Let $\tilde{A} = (a_1, a_2, a_3)$ and $\tilde{B} = (b_1, b_2, b_3)$ be two TFNs. Define,

(1) $\tilde{A} \oplus \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$.

(2) $\tilde{A} \ominus \tilde{B} = (a_1 - b_3, a_2 - b_2, a_3 - b_1)$.

(3)

$$\lambda \tilde{A} = \begin{cases} (\lambda a_1, \lambda a_2, \lambda a_3), & \lambda > 0 \\ (\lambda a_3, \lambda a_2, \lambda a_1), & \lambda < 0 \end{cases}$$

(4) Let $\tilde{D} = (d_1, d_2, d_3)$ be any TFN and $\tilde{E} = (e_1, e_2, e_3)$ be a non-negative TFN, then

$$\tilde{D} \otimes \tilde{E} \simeq \begin{cases} (d_1e_1, d_2e_2, d_3e_3), & d_1 \geq 0, \\ (d_1e_3, d_2e_2, d_3e_3), & d_1 < 0, d_3 \geq 0, \\ (d_1e_3, d_2e_2, d_3e_1), & d_3 < 0, \end{cases}$$

and

$$\frac{\tilde{D}}{\tilde{E}} \simeq (\frac{d_1}{e_3}, \frac{d_2}{e_2}, \frac{d_3}{e_1})$$

2.3 Ranking function

A ranking function is suited to compare the fuzzy numbers. A ranking function is defined as, $\Re : F(\mathbb{R}) \to \mathbb{R}$ where $F(\Re)$ is a set of fuzzy numbers, that is, a mapping which mapseach fuzzy number into the real line. Let $\tilde{A} = (a_1, a_2, a_3)$ be a TFN, then, $\Re(\tilde{A}) = \frac{a_1+2a_2+a_3}{4}$ [16].
Now, suppose that $\tilde{A}$ and $\tilde{B}$ be two TFNs. Therefore,

(1) $\Re(\tilde{A}) > \Re(\tilde{B})$ iff $\tilde{A} > \tilde{B}$ i.e., minimum $\{\tilde{A}, \tilde{B}\} = \tilde{B}$,

(2) $\Re(\tilde{A}) < \Re(\tilde{B})$ iff $\tilde{A} < \tilde{B}$ i.e., minimum $\{\tilde{A}, \tilde{B}\} = \tilde{A}$,

(3) $\Re(\tilde{A}) = \Re(\tilde{B})$ iff $\tilde{A} = \tilde{B}$ i.e., minimum $\{\tilde{A}, \tilde{B}\} = \tilde{B} = \tilde{A}$.

3 Fixed-charge transportation problem

Consider a TP with m sources and n destinations. Each of the source $i = 1, 2, \cdots, m$ has S_i units of supply, and each the destination $j = 1, 2, \cdots, n$ has a demand of D_j units and also, each of the m source can ship to any of the n destinations at a shipping cost per unit c_{ij} plus a fixed cost f_{ij} assumed for opening this route *(i,j)*. Let x_{ij} denote the number of units to be shipped from the source i to the destination j. We need to determine which routes are to be opened and the size of the shipment on those routes, so that the total cost of meeting demand, given the supply constraints, is minimized. Then, the FCTP is the following mixed integer programming problem [3]:

$$\begin{aligned} \min \quad & \sum_{i=1}^{m}\sum_{j=1}^{n}(c_{ij}\ x_{ij} + f_{ij}\ y_{ij}) \\ s.t. \quad & \\ & \sum_{j=1}^{n} x_{ij} = S_i,\ i = 1, 2, \cdots, m, \\ & \sum_{i=1}^{m} x_{ij} = D_j,\ j = 1, 2, \cdots, n, \\ & x_{ij} \geq 0, \quad \forall i, j \\ & y_{ij} = \begin{cases} 1, & x_{ij} \geq 0, \\ 0, & otherwise, \end{cases} \end{aligned} \tag{3.1}$$

where c_{ij} and f_{ij} are the real numbers.
Without losing generality, we assume that $\sum_{i=1}^{m} S_i = \sum_{j=1}^{n} D_j$, that is, the TP is balanced. Let TP be unbalanced, then by introducing a dummy source or a dummy destination it can be converted to a balanced TP.

Despite its similarity to the conventional TP, the FCTP is significantly harder to solve because of the discontinuity in the objective function introduced by the fixed costs.

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$$
\begin{aligned}
\min \quad & \sum_{i=1}^{m}\sum_{j=1}^{n}(c_{ij}\ x_{ij} + f_{ij}\ y_{ij}) \\
s.t. \quad & \\
& \sum_{j=1}^{n} x_{ij} = S_i,\ i = 1, 2, \cdots, m, \\
& \sum_{i=1}^{m} x_{ij} = D_j,\ j = 1, 2, \cdots, n, \\
& x_{ij} \geq 0, \quad \forall i, j \\
& y_{ij} = \begin{cases} 1, & x_{ij} \geq 0, \\ 0, & otherwise, \end{cases}
\end{aligned} \tag{4.2}
$$

where c_{ij} and f_{ij} are the real numbers.
Without losing generality, we assume that $\sum_{i=1}^{m} S_i = \sum_{j=1}^{n} D_j$, that is, the TP is balanced. Let TP be unbalanced, then by introducing a dummy source or a dummy destination it can be converted to a balanced TP.

Despite its similarity to the conventional TP, the FCTP is significantly harder to solve because of the discontinuity in the objective function introduced by the fixed costs.

5 Fuzzy fixed-charge transportation problem

In conventional FCTPs, it is assumed that the decision-maker is sure about the precise values of the transportation cost, the fixed cost, the supply and the demand of the product. Since this is not a realistic assumption, it seems necessary to use the fuzzy numbers and fuzzy variables further in order to make the FCTP more applicable in the real world. Here, we assume that all of the parameters (the transportation cost from the i th source to the j th destination, the fixed cost to open a route *(i,j)*, the supply of the product at i th source and the demand of the product at j th destination) are not deterministic numbers, but they are the TFNs, so, total transportation costs become fuzzy as well. The fuzzy fixed-charge transportation problem (FFCTP) is the following mathematical form:

$$
\begin{aligned}
\min \quad & \sum_{i=1}^{m}\sum_{j=1}^{n}(\tilde{c}_{ij} \otimes \tilde{x}_{ij} \oplus \tilde{f}_{ij} \otimes \tilde{y}_{ij}) \\
s.t. \quad & \\
& \sum_{j=1}^{n} \tilde{x}_{ij} = \tilde{S}_i,\ i = 1, 2, \cdots, m, \\
& \sum_{i=1}^{m} \tilde{x}_{ij} = \tilde{D}_j,\ j = 1, 2, \cdots, n, \\
& \tilde{x}_{ij} \geq \tilde{0}, \quad \forall i, j \\
& \tilde{y}_{ij} = \begin{cases} \tilde{1}, & \tilde{x}_{ij} \geq \tilde{0}, \\ \tilde{0}, & otherwise, \end{cases}
\end{aligned} \tag{5.3}
$$

where, $\tilde{c}_{ij}, \tilde{f}_{ij}, \tilde{x}_{ij}, \tilde{D}_j$ and $\tilde{S}_i$ are the TFNs, and $\tilde{x}_{ij}$ is the number of approximate units of the product that should be transported from the i th source to j th destination or fuzzy decision variables.

Balinski [3] proposed an approximation solution with heuristic method for the FCTP. This paper tries to develop the Balinski's heuristic method for the FFCTP. To do so, first, suppose that all of the parameters are TFNs, and then the Balinski matrix is obtained by formulating a linear version of the FFCTP by relaxing the fuzzy integer restriction on $\tilde{y}_{ij}$ in the objective function of model (4.2) as follows:

$\tilde{y}_{ij} = \frac{\tilde{x}_{ij}}{\tilde{M}_{ij}}$, where $\tilde{M}_{ij} = min\{\tilde{S}_i, \tilde{D}_j\}$.

The linear version of the FFCTP can be represented as follows:

$$
\begin{aligned}
\min \quad & \sum_{i=1}^{m}\sum_{j=1}^{n}(\tilde{c}_{ij} \oplus \frac{\tilde{f}_{ij}}{\tilde{M}_{ij}}) \otimes \tilde{x}_{ij} \\
s.t. \quad & \\
& \sum_{j=1}^{n} \tilde{x}_{ij} = \tilde{S}_i,\ i = 1, 2, \cdots, m, \\
& \sum_{i=1}^{m} \tilde{x}_{ij} = \tilde{D}_j,\ j = 1, 2, \cdots, n, \\
& \tilde{x}_{ij} \geq \tilde{0}, \quad \forall i, j
\end{aligned} \tag{5.4}
$$

We call this "the Approximation Fuzzy Transportation Problem (AFTP)" in which the unit transportation cost is recalculated according to:

$$\tilde{C}_{ij} = \tilde{c}_{ij} \oplus \frac{\tilde{f}_{ij}}{\tilde{M}_{ij}}.$$

The AFTP is the classical FTP with the fuzzy transportation costs.
Assume that, $\{x^*_{ij}\}$ is the fuzzy optimal solution of the AFTP. It can be easily modified into a fuzzy feasible solution $\{\tilde{x}'_{ij}, \tilde{y}'_{ij}\}$ of (4.2) as follows:

$\tilde{x}'_{ij} = \tilde{y}'_{ij} = \tilde{0}$ if $\tilde{x}^*_{ij} = \tilde{0}$ and $\tilde{x}'_{ij} = \tilde{x}^*_{ij}$ and $\tilde{y}'_{ij} = \tilde{1}$ if $\tilde{x}^*_{ij} > 0$.

Theorem 5.1 *The optimal value of the AFTP provides a lower bound to the optimal objective value of problem (4.2).*

proof. Let $\{\tilde{x}^*_{ij}\}$ be an arbitrary fuzzy optimal solution of the AFTP, and $\{\bar{\tilde{x}}_{ij}, \bar{\tilde{y}}_{ij}\}$ be an fuzzy optimal solution for (4.2), where $\bar{\tilde{y}}_{ij} = \tilde{1}$ if $\bar{\tilde{x}}_{ij} > \tilde{0}$. Since $\{\bar{\tilde{x}}_{ij}\}$ is a fuzzy feasible solution of the AFTP, therefore,

$$\sum_{i=1}^{m}\sum_{j=1}^{n}(\tilde{c}_{ij} \oplus \frac{\tilde{f}_{ij}}{\tilde{M}_{ij}}) \otimes \tilde{x}^*_{ij} \leq$$
$$\sum_{i=1}^{m}\sum_{j=1}^{n}(\tilde{c}_{ij} \oplus \frac{\tilde{f}_{ij}}{\tilde{M}_{ij}}) \otimes \bar{\tilde{x}}_{ij} \leq$$
$$\sum_{i=1}^{m}\sum_{j=1}^{n}(\tilde{c}_{ij} \otimes \bar{\tilde{x}}_{ij} \oplus \frac{\tilde{f}_{ij}}{\bar{\tilde{x}}_{ij}} \otimes \bar{\tilde{x}}_{ij}) =$$
$$\sum_{i=1}^{m}\sum_{j=1}^{n}(\tilde{c}_{ij} \otimes \bar{\tilde{x}}_{ij} \oplus \tilde{f}_{ij} \otimes \bar{\tilde{y}}_{ij}).$$

Theorem 5.2 *Suppose that $(\tilde{x}'_{ij}, \tilde{y}'_{ij})$ is an arbitrary fuzzy feasible solution of the FFCTP. Then, the objective value of $(\tilde{x}'_{ij}, \tilde{y}'_{ij})$ of (4.2) provides an upper bound to the fuzzy optimal value of (4.2).*

proof. Its Proof is straightforward.

Corollary 5.1 *According to the above theorems, the fuzzy optimal value of the FFCTP $\tilde{Z}^*_{FFCTP}$ is between the fuzzy optimal value of the AFTP $\tilde{Z}^*_{AFTP}$ and the fuzzy objective value of an arbitrary fuzzy feasible solution of the FFCTP $\tilde{Z}_{FFCTP}$. That is,*

$$\tilde{Z}^*_L = \tilde{Z}^*_{AFTP} \leq \tilde{Z}^*_{FFCTP} = \tilde{Z}_{FFCTP} = \tilde{Z}^*_U.$$

Corollary 5.2 *Let $\{\tilde{x}'_{ij}, \tilde{y}'_{ij}\}$ be a fuzzy feasible solution of (4.2), and using this solution $\tilde{Z}^*_L = \tilde{Z}^*_U$. Then, $\{\tilde{x}'_{ij}, \tilde{y}'_{ij}\}$ is a fuzzy optimal solution of (4.2) and $\tilde{Z}^*_L = \tilde{Z}^*_{FFCTP} = \tilde{Z}^*_U$.*

6 The approximation solution for the FFCTP

In this Section, a new method is proposed to find both lower and upper bounds for the fuzzy optimal value of the FFCTP. The steps of the approximation solution method are as follows:

Step 1. Convert the FFCTP into the FTP as the AFTP by using the following formula:

$$\tilde{C}_{ij} = \tilde{c}_{ij} \oplus \frac{\tilde{f}_{ij}}{\tilde{M}_{ij}}$$

where, $\tilde{M}_{ij} = \min\{\tilde{S}_i, \tilde{D}_j\}$.

Step 2. Substituting $\tilde{C}_{ij}$, obtained in Step 1, the model (4.2) is as follows:

$$\begin{aligned} \min \quad & \sum_{i=1}^{m}\sum_{j=1}^{n}\tilde{C}_{ij} \otimes \tilde{x}_{ij} \\ s.t. \quad & \\ & \sum_{j=1}^{n}\tilde{x}_{ij} = \tilde{S}_i, \; i = 1, 2, \cdots, m, \\ & \sum_{i=1}^{m}\tilde{x}_{ij} = \tilde{D}_j, \; j = 1, 2, \cdots, n, \\ & \tilde{x}_{ij} \geq \tilde{0}, \quad \forall i, j \end{aligned} \tag{6.5}$$

Step 3. Let all parameters $\tilde{C}_{ij}$, $\tilde{x}_{ij}$, $\tilde{S}_i$ and $\tilde{D}_j$ such that they are represented by TFNs, $(c^l_{ij}, c^m_{ij}, c^u_{ij}), (x^l_{ij}, x^m_{ij}, x^u_{ij}), (s^l_i, s^m_i, s^u_i)$ and (d^l_j, d^m_j, d^u_j) respectively, and then the FTP, obtained in step 2 is written as follows:

$$\begin{aligned} \min \quad & \sum_{i=1}^{m}\sum_{j=1}^{n}(c^l_{ij}, c^m_{ij}, c^u_{ij}) \otimes (x^l_{ij}, x^m_{ij}, x^u_{ij}) \\ s.t. \quad & \\ & \sum_{j=1}^{n}(x^l_{ij}, x^m_{ij}, x^u_{ij}) = \tilde{S}_i, \; \forall i, \\ & \sum_{i=1}^{m}(x^l_{ij}, x^m_{ij}, x^u_{ij}) = \tilde{D}_j, \; \forall j, \\ & (x^l_{ij}, x^m_{ij}, x^u_{ij}) \geq \tilde{0}, \quad \forall i, j \end{aligned} \tag{6.6}$$

Step 4. Use the arithmetic operations, defined in Section 2.2 and Definition 2.4, and then convert the FTP, obtained in step 3, into the following classical linear programming problem:

$$
\begin{aligned}
\min \quad & \Re(\tilde{Z}) \\
s.t. \quad & \\
& \sum_{j=1}^{n} x_{ij}^{l} = s_i^{l} && \forall i, \\
& \sum_{j=1}^{n} x_{ij}^{m} = s_i^{m} && \forall i, \\
& \sum_{j=1}^{n} x_{ij}^{u} = s_i^{u} && \forall i, \\
& \sum_{i=1}^{n} x_{ij}^{l} = d_j^{l} && \forall j, \\
& \sum_{i=1}^{n} x_{ij}^{m} = d_j^{m} && \forall j, \\
& \sum_{i=1}^{n} x_{ij}^{u} = d_j^{u} && \forall j, \\
& x_{ij}^{m} - x_{ij}^{l} \geq 0, && \forall i, j, \\
& x_{ij}^{u} - x_{ij}^{m} \geq 0, \ x_{ij}^{l} \geq 0, && \forall i, j.
\end{aligned}
\tag{6.7}
$$

where $\tilde{Z} = \left(\sum_{i=1}^{m} \sum_{j=1}^{n} (c_{ij}^{l}, c_{ij}^{m}, c_{ij}^{u}) \otimes (x_{ij}^{l}, x_{ij}^{m}, x_{ij}^{u}) \right)$.

Step 5. Find the optimal solution x_{ij}^{l}, x_{ij}^{m} and x_{ij}^{u} by solving the classical linear programming problem obtained in step 4.

Step 6. Find the fuzzy optimal solution of problem (6.7) (and hence problem (6.5)) by putting the value of x_{ij}^{l}, x_{ij}^{m} and x_{ij}^{u} in $\tilde{x}_{ij} = (x_{ij}^{l}, x_{ij}^{m}, x_{ij}^{u})$.

Step 7. Provide a fuzzy lower bound ($\tilde{Z}_L^*$) on the fuzzy optimal value of the FFCTP ($\tilde{Z}_{FFCTP}^*$) according to the theorem 1, by calculating the fuzzy optimal value of the AFTP.

Step8. Provide a fuzzy upper bound ($\tilde{Z}_U^*$) on the fuzzy optimal value of the FFCTP ($\tilde{Z}_{FFCTP}^*$) according to Theorem 5.2, by calculating the objective value of an arbitrary fuzzy feasible solution of the FFCTP.

7 Numerical Example

In this section the approximation solution method is illustrated with the help of numerical example.

Suppose that a company has three factories in three different cities of [1, 2, 3]. The goods of these factories are assembled and sent to the major markets in the three other cities. Also, assume that there is a fuzzy fixed cost in this transportation problem. Namely, the cost of sending no units along route (i, j) is zero; but any positive shipment incurs a fixed cost plus a cost proportional to the number of units transported. The transportation cost, the fixed cost, the demand values, the supply values and the number of units of the product that transported for each route (i, j) are TFNs and are presented in Table 7.8 and Table 7.8. The company wants to determine the fuzzy quantity of the product that should be transported from each of the sources to each destination so that the total fuzzy transportation cost would minimize.

	(5,7,10)	(4,8,13)	(6,10,15)
(6,10,14)	(1,4,9)	(1,2,5)	(2,5,8)
(3,8,16)	(8,9,12)	(3,5,8)	(7,9,13)
(6,7,8)	(11,12,20)	(1,5,10)	(4,5,8)

Table 7.8. The fuzzy transportation costs

	(5,7,10)	(4,8,13)	(6,10,15)
(6,10,14)	(3,5,8)	(7,9,13)	(3,8,18)
(3,8,16)	(2,5,9)	(8,13,17)	(6,18,25)
(6,7,8)	(1,3,8)	(5,7,18)	(7,17,20)

Table 7.8. The fuzzy fixed costs

The above problem is balanced, because, $\sum_{i=1}^{m} \tilde{S}_i = \sum_{j=1}^{n} \tilde{D}_j = (15, 25, 38)$.
A lower and an upper bound both for the fuzzy optimal value of the FFCTP in the given example by using the approximation solution method proposed in Section 6, can be obtained. The fuzzy

transportation costs by converting the FFCTP into the FTP, mentioned in step 1, are as follows:

$$\tilde{c}_{11} = (1.3, 4.71, 10.6), \tilde{c}_{12} = (1.54, 3.12, 8.25)$$

$$\tilde{c}_{13} = (2.21, 5.8, 10.17), \tilde{c}_{21} = (8.2, 9.71, 13.8)$$

$$\tilde{c}_{22} = (3.62, 6.62, 2.25), \tilde{c}_{23} = (7.38, 11.25, 21)$$

$$\tilde{c}_{31} = (11.12, 2.43, 21.33), \tilde{c}_{32} = (1.62, 6, 13)$$

$\tilde{c}_{33} = (4.88, 7.43, 11.33)$.

The fuzzy optimal solution of the FTP by using Steps 3, 4, 5 and 6, is obtained as follows:

$\tilde{x}_{11} = (5, 6, 6)$, $\tilde{x}_{12} = (1, 1, 1)$,
$\tilde{x}_{13} = (0, 3, 7)$, $\tilde{x}_{21} = (0, 1, 4)$,
$\tilde{x}_{22} = (3, 7, 12)$, $\tilde{x}_{23} = (0, 0, 0)$,
$\tilde{x}_{31} = (0, 0, 0)$, $\tilde{x}_{32} = (0, 0, 0)$,
$\tilde{x}_{33} = (6, 7, 8)$.

The lower bound $\tilde{Z}^*_L$ on the fuzzy optimal value of the FFCTP $\tilde{Z}^*_{FFCTP}$ by calculating the fuzzy optimal value of the AFTP (problem (6.5)) is as follows: $\tilde{Z}^*_L = (48.18, 156.84, 435.88)$. The upper bound $\tilde{Z}^*_U$ on the fuzzy optimal value of the FFCTP $\tilde{Z}^*_{FFCTP}$ by calculating the fuzzy objective value of the feasible solution of the FFCTP (problem (5.3)), is as follows: $\tilde{Z}^*_U = (69, 177, 403)$.
Therefore, the fuzzy optimal value of the FFCTP must be between $\tilde{Z}^*_L$ and $\tilde{Z}^*_U$ as follows:

$$(48.18, 156.84, 435.88) \leq \tilde{Z}^* \leq (69, 177, 403)$$

where $\tilde{Z}^* = \tilde{Z}^*_{FFCTP}$. Now, using the ranking function in Section , we have:

$$199.435 \leq \Re(\tilde{Z}^*_{FFCTP}) \leq 206.5.$$

8 Conclusion

This paper has proposed a new method to find an approximation solution close to the optimal solution for the fixed charge transportation problem in which all of the parameters (transportation cost, fixed cost, decision variables, demand and supply) are triangular fuzzy numbers. The lower and upper bounds on the fuzzy optimal value of the fuzzy fixed charge transportation problem can be easily obtained by using the approximation solution method, which is the main advantage of the proposed method. We have elaborated further on the proposed results by reporting the numerical example.

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