Magnetic fluid lubrication of porous pivoted slider bearing with slip and squeeze velocity

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Abstract

In this paper the problem on "Magnetic fluid lubrication of porous-pivoted slider bearing with slip velocity by Ahmad et.al. (N. Ahmad, J. P. Singh, Magnetic fluid lubrication of porous-pivoted slider bearing with slip velocity, Journal of Engineering Tribology, 2007)" has been recapitulated using Jenkin’s model (J. T. Jenkins, A Theory of magnetic fluids, Archive for Rational Mechanics and Analysis, 1972) with the additional effect of squeeze velocity of the above plate. It is found that while discussing the above problem, (N. Ahmad, J. P. Singh, Magnetic fluid lubrication of porous-pivoted slider bearing with slip velocity, Journal of Engineering Tribology, 2007) has stated but ignored the term

\[ \frac{2}{r} \left( M \times M^* \right), \]

where

\[ M = \frac{D M}{Dt} + \frac{1}{2} (\nabla \times q) \times M, \]

in their study (Refer equation (2.2)). This paper reconsiders the above neglected term with

\[ M^* = \frac{1}{2} (\nabla \times q) \times M, \]

where

\[ M = \mu H. \]

Since \( M \) is the corotational derivative of magnetization vector, so it has an impact on the performance of the problem (P. Ram, P. D. S. Verma, Ferrofluid lubrication in porous inclined slider bearing, Indian Journal of Pure and Applied Mathematics, 1999). With the addition of the above term and under an oblique magnetic field, it is found that the dimensionless load carrying capacity can be improved substantially with and without squeeze effect. The paper also studied in detail about the effects of squeeze velocity and sliding velocity. It is observed that dimensionless load carrying capacity increases when squeeze velocity increases and sliding velocity decreases.

Keywords: Magnetic fluid; Slider bearing; Slip velocity; Squeeze velocity; Jenkin’s model.

1 Introduction

Magnetic fluids or Ferrofluids [15] are stable colloidal suspensions containing fine ferromagnetic particles which are dispersing in a liquid, called carrier liquid (in our case water), in which a surfactant is added to generate a coating layer preventing the flocculation of the particles. When an external magnetic field is applied, ferrofluids experience magnetic body forces which depend upon the magnetization of ferromagnetic particles. Owing to these features ferrofluids are useful in many applications, for example [5]. Agrawal [16] studied magnetic fluid based porous inclined slider bearing using Neuringer-Rosensweig’s model. Shah and Bhat in [13, 14] considered respectively squeeze film and slider bearing in their study using Neuringer-Rosensweig’s model. Recently Ahmad et. al. [7] studied "Magnetic fluid lubrication of porous-pivoted slider bearing with slip velocity" and they have ignored the term

\[ \rho \alpha^2 \nabla \times \left( \frac{M}{M^*} \times M^* \right), \]

where

\[ M^* = \frac{D M}{Dt} + \frac{1}{2} (\nabla \times q) \times M, \]

in the governing system of equations. In this paper we have recapitulated...
the above problem [7] including the ignored term which is given by Jenkin’s [4] and worked on by Ram and Verma [9], Shah and Bhat [11] in their study from different viewpoint. With the addition of the above term and under an oblique magnetic field, it is found that the dimensionless load carrying capacity can be improved substantially with and without squeeze effect. The paper also studied in detail about the effects of squeeze velocity and sliding velocity. It is observed that dimensionless load carrying capacity increases when squeeze velocity increases and sliding velocity decreases.

2 The Mathematical Model

The configuration of the porous-pivoted slider bearing with squeeze velocity is displayed in Figure 1 consists of a slider having a convex pad surface of length $A$(metres) with central thickness $H_c$(metres) and moving with uniform velocity $U$(ms$^{-1}$) in the $x$-direction. The stator has a porous matrix with uniform thickness $l_2$(metres) backed by a solid wall. The porous flat lower plate is normally approached by the upper plate with a uniform velocity $\dot{h}=d h/dt$, where $h$(metres) is the central film thickness and $t$ is time in second. The expression for the central film thickness $h$(metres) is given by [7, 11]

$$h = H_c \left\{ A \left( \frac{x}{A} - \frac{1}{2} \right)^2 - 1 \right\} + h_1 \left\{ a - \frac{a}{A} x + \frac{x}{A} \right\},$$

(2.1)

with $a = \frac{h_2}{h_1}$; $h_2$(metres) and $h_1$(metres) are maximum and minimum film thickness respectively.

The above bearing is lubricated with water based ferrofluid and the equations governing the flow of ferrofluid by Jenkin’s model [4, 7, 11] are

$$\rho \left\{ \frac{\partial \mathbf{q}}{\partial t} + (\mathbf{q} \cdot \nabla)\mathbf{q} \right\} = -\nabla p + \eta \nabla^2 \mathbf{q}$$

$$+ \mu_0(\mathbf{M} \cdot \nabla)\mathbf{H} + \rho \alpha^2 \nabla \times \left( \frac{\mathbf{M}}{M} \times \mathbf{M}^* \right),$$

(2.2)

$$\nabla \cdot \mathbf{q} = 0,$$  

(2.3)

$$\nabla \times \mathbf{H} = 0,$$  

(2.4)

$$\nabla \cdot (\mathbf{H} + 4\pi \mathbf{M}) = 0,$$  

(2.5)

$$\gamma \frac{D^2 \mathbf{M}}{D t^2} = -4\pi \rho \frac{M_s}{\mu_0} \frac{\mathbf{M}}{M} - \frac{2\alpha^2}{M} \mathbf{M}^* + \mathbf{H},$$

(2.6)

with

$$\mathbf{M}^* = \frac{D \mathbf{M}}{D t} + \frac{1}{2}(\nabla \times \mathbf{q}) \times \mathbf{M}.$$  

(2.7)

where $\rho$, $p$, $\eta$, $\mathbf{q}$, $\mu_0$, $\mathbf{M}$, $\mathbf{H}$, $M$, $M^*$, $\alpha^2$, $\mu_0$, $M_s$, $\gamma$ are fluid density, film pressure, fluid viscosity, fluid velocity, free space permeability, the magnetization vector, magnetic field vector, magnitude of magnetization vector, corotational derivative of $\mathbf{M}$, material constant, initial susceptibility of fluid, the saturation magnetization and another material constant of Jenkin’s model respectively.

In the present discussion, equation (2.6) is replaced by

$$\mathbf{M} = \mu \mathbf{H} \ (\mu \text{ is magnetic susceptibility}),$$  

(2.8)

as suggested by Maugin [1] and

$$\mathbf{M}^* = \frac{1}{2}(\nabla \times \mathbf{q}) \times \mathbf{M}.$$  

(2.9)

The lubricant is ferrofluid, so a magnetic field vector $\mathbf{H}$ is applied such that it is inclined at an angle $\phi$ as shown in Figure 1 with the stator and vanishes at the ends of the bearing. The angle
\( \phi \) is determined in Shah and Bhat [11] and the magnitude \( H \) of magnetic field is given by
\[
H^2 = Kx(A - x), \tag{2.10}
\]
where \( K \) being a quantity chosen to suit the dimensions of both sides of equation (2.10).

The equation of continuity in the film region is
\[
\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0, \tag{2.11}
\]
where \( u \) and \( w \) are components of film fluid velocity in \( x \)-direction and \( z \)-direction respectively.

The equation of continuity in porous region is
\[
\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{w}}{\partial z} = 0, \tag{2.12}
\]
where \( \bar{u} \) and \( \bar{w} \) are components of fluid velocity in the porous region in \( x \)-direction and \( z \)-direction respectively.

Referring the work of Agrawal [16] and Shah et. al. [11], using equations (2.2) to (2.9), one obtains
\[
\frac{\partial^2 u}{\partial z^2} = \frac{1}{\eta} \left( 1 - \frac{\rho \sigma^2 \phi H}{2\eta} \right) \frac{\partial}{\partial x} \left( p - \frac{1}{2} \mu_0 \phi H^2 \right). \tag{2.13}
\]

The velocity components of fluid in the porous region are
\[
\bar{u} = -\frac{\varphi}{\eta} \left\{ \frac{\partial}{\partial x} \left( P - \frac{1}{2} \mu_0 \phi H^2 \right) + \frac{\rho \sigma^2}{2} \phi \frac{\partial}{\partial z} \left( H \frac{\partial u}{\partial z} \right) \right\}, \tag{2.14}
\]
\[
\bar{w} = -\frac{\varphi}{\eta} \left\{ \frac{\partial}{\partial z} \left( P - \frac{1}{2} \mu_0 \phi H^2 \right) - \frac{\rho \sigma^2}{2} \phi \frac{\partial}{\partial x} \left( H \frac{\partial u}{\partial z} \right) \right\}, \tag{2.15}
\]
where \( \varphi \) and \( P \) are permeability and fluid pressure in the porous region respectively.

Substituting equations (2.14) and (2.15) into equation (2.12), one obtains
\[
\frac{\partial^2}{\partial z^2} \left( P - \frac{1}{2} \mu_0 \phi H^2 \right) + \frac{\partial^2}{\partial z^2} \left( P - \frac{1}{2} \mu_0 \phi H^2 \right) = 0, \tag{2.16}
\]
which on integration with respect to \( z \) across the porous region \((-l_2, 0)\), yields
\[
\frac{\partial}{\partial z} \left( P - \frac{1}{2} \mu_0 \phi H^2 \right) \bigg|_{z=0} = -l_2 \frac{\partial^2}{\partial x^2} \left( P - \frac{1}{2} \mu_0 \phi H^2 \right), \tag{2.17}
\]
using Morgan-Cameron approximation [3, 11, 12] and that the surface \( z = -l_2 \) is non-porous.

The relevant boundary conditions for the velocity field [2] in the lubricant region is
\[
u = \frac{1}{s} \frac{\partial u}{\partial z} \text{ at } z = 0, \tag{2.18}
\]
and
\[
u = U \text{ at } z = h, \tag{2.19}
\]
where \( \frac{1}{s} = \frac{\sqrt{2} \rho \sigma}{\phi} \); \( s \) is slip parameter and \( k \) is slip coefficient, which depends on the structure of the porous material.

Solving equation (2.13) with boundary conditions (2.18) and (2.19), one obtains
\[
u = \frac{1}{2 \eta} \left( 1 - \frac{\rho \sigma^2 \phi H}{2\eta} \right) \left\{ \frac{z^2}{(sh + 1)} - \frac{sh^2 z}{(sh + 1)^2} \right\} - \frac{h^2}{(sh + 1)} \frac{\partial}{\partial x} \left( p - \frac{1}{2} \mu_0 \phi H^2 \right) + \frac{U(sz + 1)}{(sh + 1)}. \tag{2.20}
\]
Integrating continuity equation (2.11) in film region over \((0, h)\), one obtains
\[
\frac{\partial}{\partial x} \int_0^h u \, dz + w_h - w_0 = 0. \tag{2.21}
\]
Using \( w_h = V = -h \) because of squeeze velocity is in the downward direction and \( w_0 = w|_{z=0} = \bar{w}|_{z=0} \) because of continuity of velocity component at \( z = 0 \) of film region and porous region respectively, equations (2.17), (2.20), (2.21), gives
\[
\frac{d}{dx} \left\{ g \frac{d}{dx} \left( p - \frac{1}{2} \mu_0 \phi H^2 \right) \right\} = \frac{df}{dx}, \tag{2.22}
\]
where
\[
g = 12 \phi l_2 + \frac{h^3 (sh + 4) - \left( \frac{3 \rho \sigma^2 \phi \varphi s}{\eta} h \right)}{(sh + 1) \left( 1 - \frac{\rho \sigma^2 \phi H}{2\eta} \right)} \tag{2.23}
\]
and
\[
f = \frac{6 \eta U h (sh + 2) - 6 \eta \rho \sigma^2 \phi \varphi s H}{(sh + 1)} + 12 \eta V x. \tag{2.24}
\]
Equation (2.22) is known as Reynolds’s equation.

Introducing following dimensionless quantities

\[
X = \frac{x}{A}, \quad \tilde{h} = h, \quad \tilde{s} = sh_1, \quad \tilde{p} = \frac{ph_1^2}{\eta AU},
\]

\[
\mu^* = \frac{\mu_0 \tilde{h}^2 AK}{\eta U}, \quad \beta^2 = \frac{\rho \alpha^2 \tilde{h} \sqrt{K}}{2}\eta,
\]

\[
\varphi = \frac{12 \epsilon l_1^2}{h_1^3}, \quad S = \frac{2V A}{U h_1}, \quad \gamma^* = \frac{6 \varphi}{h_1^2},
\]

the dimensionless form of equation (2.22) is

\[
\frac{d}{dX} \left\{ G \frac{d}{dX} \left( \tilde{p} - \frac{1}{2} \mu^* X (1 - X) \right) \right\} = \frac{dE}{dX},
\]

where

\[
G = \varphi + \tilde{h}^3 (\tilde{s} \tilde{h} + 4) - \beta^2 \gamma^* \tilde{h}^2 \sqrt{X (1 - X)}
\]

\[
= \frac{6 \tilde{h} (\tilde{s} \tilde{h} + 2) - 2\beta^2 \gamma^* \tilde{h} \sqrt{X (1 - X)}}{\tilde{s} \tilde{h} + 1} - 6SX,
\]

which is known as dimensionless form of Reynolds’s equation.

Solving equation (2.24) for pressure under the appropriate boundary conditions

\[
\tilde{p} = 0 \quad \text{at} \quad X = 0, 1,
\]

yields

\[
\tilde{p} = \frac{1}{2} \mu^* X (1 - X) + \int_0^X \left( \frac{E - Q}{G} \right) dX, \quad (2.25)
\]

where

\[
Q = \int_0^1 \frac{\tilde{h} dX}{\tilde{h}^2 dX}.
\]

The dimensionless form of equation (2.1) is

\[
\tilde{h} = l X^2 + m X + n, \quad (2.26)
\]

where

\[
l = 4 \delta, \quad m = -(4 \delta + a - 1), \quad n = a; \quad \delta = \frac{H_c}{h_1},
\]

The dimensionless form of load carrying capacity using (2.25) can be obtained as

\[
W = \int_0^1 \tilde{p} dX = \frac{\mu^*}{12} - \int_0^1 \left( \frac{E - Q}{G} \right) X dX. \quad (2.28)
\]

3 Results and Discussion

The problem on ”Magnetic fluid lubrication of porous-pivoted slider bearing with slip velocity by [7]” is recapitulated here for its optimum performance.

During the course of investigation it is observed from equation (2.13) that a constant magnetic field does not enhance the bearing characteristics in Rosensweig’s model as well as in Jenkin’s model of ferrofluid flow.

The values of the dimensionless load carrying capacity \(W\) has been calculated for the following values [6] of the parameters using Simpson’s 1/3 rule with step size 0.1.

\[
\begin{align*}
&h_1 = 0.000005(m), \quad h_2 = 0.00001(m), \\
&\tilde{\mu} = 0.05, \quad A = 0.02(m), \quad k = 0.1, \\
&\eta = 0.012(Kgm^{-1}s^{-1}), \quad \rho = 1400(Kgm^{-3}), \\
&\mu_0 = 4\pi \times 10^{-7}(Kgms^{-2}A^{-2}), \\
&H_c = 0.0000015(m), \quad l_2 = 0.0001(m).
\end{align*}
\]

The ferrofluid used here is water based. The magnetic field considered here is oblique to the stator and its strength is of order of magnetic field does not enhance the bearing characteristics in Rosensweig’s model as well as in Jenkin’s model of ferrofluid flow. The calculated values of \(W\) are presented graphically as shown in Figures 2 to 10 for various cases.

Figure 2 and 3 indicates the study of the effect of squeeze velocity (\(h \neq 0\)) when \(\alpha^2 \neq 0\) (Jenkin’s model) and \(\alpha^2 = 0\) (Rosensweig’s model) respectively with respect to order of magnetic field.
strength ($H$ is obtained from $K$ as per above calculation).

From Figure 2, it is observed that, for $\alpha^2 \neq 0$, $W$ increases considerably in the presence of squeeze velocity. Also, as $K$ increases (that is, as order of magnetic field strength increases), $W$ increases. From Figure 3, it is observed that, for $\alpha^2 = 0$, again $W$ increases considerably in the presence of squeeze velocity, but it does not affect much when the order of magnetic field strength increases.

Figure 4 and 5 shows the comparative study of Jenkin’s model and Rosensweig’s model when $\dot{h} \neq 0$ and $\dot{h} = 0$ respectively with respect to permeability $\varphi$ of the porous medium. From both the figures it is observed that, $W$ increases with the decrease of permeability $\varphi$. Also, when $\dot{h} \neq 0$, $W$ is increases more as compared to $\dot{h} = 0$. The same behavior of $\dot{W}$ can be observed from Figure 5 when $\dot{h} = 0$, that is, when there is no squeeze velocity.

Figure 6 and 7 shows the study of effect of squeeze velocity ($\dot{h} \neq 0$) when $\alpha^2 \neq 0$ (Jenkin’s model) and $\alpha^2 = 0$ (Rosensweig’s model) respectively with respect to order of magnetic field strength. From both the figures it is observed that, $\dot{W}$ increases with the decrease of permeability $\varphi$. Also, when $\dot{h} \neq 0$, $\dot{W}$ is increases more as compared to $\dot{h} = 0$. The same behavior of $\dot{W}$ can be observed from Figure 5 when $\dot{h} = 0$, that is, when there is no squeeze velocity.
Jenkin’s model and Rosensweig’s model when \( \dot{h} \neq 0 \) and \( \dot{h} = 0 \) respectively with respect to permeability \( \varphi \). From both the figures it is observed that, \( \bar{W} \) increases with the decrease of permeability \( \varphi \).

Figure 10 displays values of \( \bar{W} \) for various values of \( \dot{h} \) and \( U \) for \( \alpha^2 \neq 0 \), and from it the following observations can be made:

1. \( \bar{W} \) increases with the increase of \( \dot{h} \).
2. \( \bar{W} \) increases with the decrease of \( U \).

From Figures 2 to 9, it is observed that the values of \( \bar{W} \) increases substantially in the case of Jenkin’s model; that is, with the consideration of the ignored term of Ahmad et. al. [7] as \( \alpha^2 \nabla \times \left( \frac{\mathbf{M}}{M} \times \mathbf{M} \right) \) with \( \mathbf{M} = \frac{1}{2}(\nabla \times \mathbf{q}) \times \mathbf{M} \) and \( \mathbf{M} = \mu \mathbf{H} \) for \( \dot{h} = 0 \) and \( \dot{h} \neq 0 \) rather than Rosensweig’s case (Ahmad et. al. [7] for \( \dot{h} = 0 \)).

4 Conclusions

The problem on "Magnetic fluid lubrication of porous-pivoted slider bearing with slip velocity by [7]" is recapitulated here for its optimum performance with the inclusion of the ignored term \( \alpha^2 \nabla \times \left( \frac{\mathbf{M}}{M} \times \mathbf{M} \right) \) with \( \mathbf{M} = \frac{1}{2}(\nabla \times \mathbf{q}) \times \mathbf{M} \) and \( \mathbf{M} = \mu \mathbf{H} \). The ferrofluid used here is water based and magnetic field strength considered is of as shown in figures in order to get maximum magnetic field at \( x = A/2 \).

The design of the pivoted slider bearing can be made with the considerations of the following ob-
Figure 10: Values of $W$ for various values of $h$ and $U$ when $K = 10^{10}$, $\varphi = 10^{-12}(m^2)$ and $\alpha^2 = 0.0001(m^3A^{-1}s^{-1})$.

Observations:
Under an oblique magnetic field to the stator, the dimensionless load carrying capacity can be improved substantially by considering following features:

1. Ferrofluid flow behavior given by Jenkin’s model
2. Presence of the squeeze velocity
3. Smaller values of permeability parameter $\varphi$
4. Increasing values of $H^2$ up to $O(10^5)$ as per [8]

It should be noted from equation (2.13) that a constant magnetic field does not enhance the bearing characteristics in Rosensweig’s model of ferrofluid flow.

References


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