Available online at http://ijim.srbiau.ac.ir/
Int. J. Industrial Mathematics (ISSN 2008-5621)
Vol. 6, No. 3, 2014 Article ID IJIM-00467, 14 pages
Research Article

A multi-product vehicle routing scheduling model with time window constraints for cross docking system under uncertainty: A fuzzy possibilistic-stochastic programming

B. Vahdani * †, Sh. Sadigh Behzadi ‡

Abstract

Mathematical modeling of supply chain operations has proven to be one of the most complex tasks in the field of operations management and operations research. Despite the abundance of several modeling proposals in the literature; for vast majority of them, no effective universal application is conceived. This issue renders the proposed mathematical models inapplicable due largely to the fact that real-life supply chain problems are set forth in restrained terms or represented less strikingly than they would bear out. This paper is triggered to bridge this gap by proposing a universal mixed integer linear programming (MILP) framework which to large extent simulates many realistic considerations in vehicle routing scheduling problems in cross-docking systems which might have separately been attempted by other researchers. The developed model is pioneer in excogitating the vehicle routing scheduling problem with the following assumptions: a) multiple products are transported between pick-up and delivery nodes, b) delivery time-intervals are imposed on each delivery node, c) multiple types of vehicles operate in the system, d) capacity constraints exists for each vehicle type, and finally e) vehicles arrives simultaneously at cross-docking location. Moreover, to solve the model a hybrid solution methodology is presented by combining fuzzy possibilistic programming and stochastic programming. Finally, in order to demonstrate the accuracy and efficiency of the proposed model, an extensive sensitivity analysis is performed to scrutinize its parameters' demeanors.

Keywords : Cross docking; Vehicle routing scheduling; Fuzzy possibilistic programming; Stochastic programming.

1 Literature review

Rohrer [22] presents one of the earliest technical papers outlining modeling methods and issues in cross-docking systems. In his study, he is more focused on describing theoretical aspects of cross-docking than its practical implementation. Mosheiov [12] proposed a mathematical model along with two heuristic algorithms for vehicle routing problem consisting of pickup and delivery processes aiming at minimizing transportation costs and maximizing vehicle efficiency. Afterwards, a heuristic algorithm based on a neighbourhood algorithm and a Tabu Search algorithm was proposed for the optimization of transportation planning with one delivery center [10]. Apte and Viswanathan [18] proposed a framework for understanding and designing cross-docking systems, including techniques for improving the overall efficiencies of logistics and distribution networks. Thus, they present technical

*Corresponding author. b.vahdani@gmail.com
†Young Researchers and Elite Club, Qazvin Branch, Islamic Azad University, Qazvin, Iran.
‡Department of Mathematics, Islamic Azad University, Qazvin Branch, Qazvin, Iran.

issues associated to the network structures used for warehousing, the design of physical and information flows in cross-docking, and analysis and management systems. A new Tabu Search was developed on a classical traveling salesman problem [10]. They showed that their algorithm is able to find optimal solutions in a relatively short computational time. Another version of Tabu Search algorithm was proposed by Lau et al. [14] to minimize the transportation costs for vehicle routing during specified time windows and for a finite number of vehicles. Jia et al. [13] presented a modified GA, which is capable of solving traditional scheduling problems as well as distributed scheduling problems. The capability of the modified GA was also evaluated for solving the distributed scheduling problems. Li et al. [31] designed and implemented two heuristics to study a central problem for cross-docking, namely, eliminating or minimizing storage and order picking activity. They used JIT scheduling and thereby solved the NP-hard problem for real-time applications. In two studies, Lim et al. [2, 3] consider truck dock assignment problems with time windows and capacity constraints in the transshipment network. They first formulate an integer programming model and propose a Tabu Search and a genetic algorithm, and then, to minimize the operational cost of the cargo shipments and the total number of unfulfilled shipments, they formulate another integer programming model and propose a different genetic algorithm that uses integer programming constraints. A study on cross-docking scheduling problems according to total completion time for JIT logistics was conducted [8]. They developed a branch and bound algorithm heuristics on the basis of the different characteristics of the problem. Another similar research is studied by [7] on the cross-docking scheduling problem with total completion time, but this time with a dynamic programming method. An integrated cross-docking with the pickup and delivery process is studied by [32]. To modelize foregoing problem, they introduced a mathematical model to determine an optimal vehicle routing schedule. Since the problem was NP-hard they developed a tabu search algorithm for that problem. Waller et al. [23] developed several models to predict managerially important changes in a retailer's system wide inventory levels due to cross-docking. Chen and Song [9] considered a two-stage hybrid cross-docking scheduling problem. In their paper, they point out that the job in the second stage can be executed only when its precedent jobs in the first stage are done, and at least one stage must contain more than one machine. They solved their problem by presenting a mixed integer programming model for small-sized instances. They also presented four heuristics to investigate the performance of their algorithms in large-size instances. In contrast, a cross-docking system with a temporary storage area in front of the shipping dock was suggested by Yu and Egbelu [30]. In their problem, they determine product assignments from inbound trucks to outbound trucks simultaneously according to the trucks' docking sequences. Thus, they develop two solution techniques: a mathematical model to minimize makespan in small-sized problems but it fails to solve large-sized instances, and heuristic algorithms to enhance the solution efficiency. Wang and Regan [16] use real-time information about freight transferring within a cross-docking system to schedule waiting inbound trailers and in order to reduce the time freight spends in the cross-dock. Their dynamic simulation models enable them to compare the performance of several rule-based simulations. Their simulation results indicate that the time-based algorithms save more time than the first-come, first-served or look-ahead policies. Boysen et al. [24] propose a base model for scheduling trucks at cross-docking terminals. Their model is a building block solution procedure that helps solve more complex, real-world, truck scheduling problems. Dong et al. [11] deals with a vehicle routing and scheduling problem taking place in Flight Ticket Sales Companies for the service of free pickup and delivery of airline passengers to the airport. They modelize this problem under the framework of Vehicle Routing Problem with Time Windows (VRPTW), aims at minimizing the total operational costs, i.e., fixed start-up costs and variable traveling costs. They propose a mixed integer programming model in which service quality is factored in constraints by introducing passenger satisfaction degree functions that limit time deviations between actual and desired delivery times. A novel hybrid genetic algorithm (HGA) is proposed by Wang and Lu [6] for solving a capacitated vehicle routing problem (CVRP). The proposed HGA is mainly used to solve practical problems. Boysen [25] considers a new truck scheduling problem, which synchronizes inbound and outbound flows of goods at the zero-inventory cross docking terminals of the food industry. He presents a new multi-objective formulation for this problem which is solved by dynamic programming and simulated annealing approaches. Tang and Yan [28] propose two new application of a cross-docking system in which two approaches are implemented: (1) pre-distribution cross-docking operations (Pre-C) and (2) post-distribution cross-docking operations (Post-C). Because, each of these approaches has its own advantages and disadvantages, decision makers are encountered with a difficult option in that the operations costs at the cross-dock in Pre-C are lower than the Post-C, while the transshipment's quantity in Pre-C is higher than the Post-C. Vahdani and Zandieh [5] apply five meta-heuristic algorithms: genetic algorithm (GA), tabu search (TS), simulated annealing (SA), electromagnetism-like algorithm (EMA) and variable neighbourhood search (VNS) to schedule the trucks in cross-dock systems such that the total operation time is minimized when there is a temporary storage buffer located at the shipping dock. As was pointed out, few researches have been conducted regarding vehicle routing scheduling in the literature. Among those few researches, all of them have assumed that the considered cross-docking system is single product. This issue obviously violates the multi-product nature of cross-docking systems and it is not sensible at all. Another issue that has extensively been researched in vehicle routing scheduling is the time constraints for transportation activities in delivery nodes. The foregoing assumption is considered in the proposed model. Considering the nature of vehicles' actitives in delivery nodes and inasmuch as timely delivery is very critical problem for retailers; interval time constraints have been included in the proposed model on top of the other assumptions. The authors of this research believe that the simultaneous consideration of the foregoing assumptions make the problem considered in this paper more realistic.

Contributions of this paper to the location-allocation area of research are articulated as follows:

■ Incorporation of time window constraints in cross-docking vehicle routing scheduling problem.

■ Consideration of multi-products in pick-up and delivery processes in cross-docking system.

■ Including different vehicles with different capacities to solve the problem more realistically.

■ Investigation of time constraints for vehicles' activities in each node.

■ Proposal of a new mixed integer linear programming model to realistically solve vehicle routing scheduling problems in cross-docking system through embracing above-cited assumptions.

2 Mathematical formulation

In this paper, assume we have multiple products in our pick-up and delivery processes. A set of different vehicles are used to transport products from suppliers to retailers through a cross-dock. Supplies and demands are taken as deliveries and pickups within duration constraints on vehicle routes. Supplies are taken as deliveries within time windows. Split deliveries and pickup are not allowed. Additionally, we consider unlimited capacity for cross-dock center in our problem. Each supplier and retailer can be visited only once and the total quantity of products in a vehicle must be less than its capacity. Another assumption is that the total demanded quantity in each delivery node must be equal or less than total transported quantity in delivery process. The objective of developing such a mathematical model is to minimize both total transportation cost and fixed operational cost of vehicles in pickup and delivery process. In order to solve our problem, we develop a comprehensive mathematical model with following notations.

2.1 Notations

S: Set of suppliers in the pickup process ($i = 1, 2, , n$).

R: Set of retailers in the delivery process ($i^{'} = 1, 2, , m$).

G: Number of product type ($g = 1, 2, , G$).

K: Vehicle types are labeled by $k(k = 1, 2, , K)$ in the pickup process and the number of vehicles of type k is noted L_k.

$K^{'}$: Vehicle types are labeled by $k^{'}(k^{'} = 1, 2, , K^{'})$ in the delivery process and the number of vehicles of type $k^{'}$ is noted $L_{k^{'}}$.

0: Cross docking center.

n: Number of nodes in pick-up process.

m: Number of nodes in delivery process.

2.2 Parameters

p_{ig}: Loaded amount of product type g in node i in pick-up process.

$d_{i'g}$: Unloaded amount of product type g in node i' in delivery process.

c_{ijk}: Transportation cost for vehicle type k from node i to node j in pick-up process.

$\widetilde{c}_{i'j'k'}$: Transportation cost for vehicle type k' from node i' to node j' in delivery process.

$\widetilde{c}_k$: Operational cost of the vehicle k in pick-up process.

$\widetilde{c}'_k$: Operational cost of the vehicle k' in delivery process.

$\widetilde{t}^k_{ij}$: Time for the vehicle k to move from node i to node j in pick-up process.

$\widetilde{t}^{k'}_{i'j'}$: Time for the vehicle k' to move from node i' to node j' in delivery process.

$\widetilde{T}^s_k$: Maximum working time of vehicle k in pick-up process.

$\widetilde{T}^s_{k'}$ Maximum working time of vehicle k' in delivery process.

$\widetilde{l}_{i'}$, $\widetilde{u}_{i'}$: Minimum and maximum times defining the time horizon in delivery process.

$\widetilde{CA}_k$: Volume capacity of vehicle k in pick-up process.

$\widetilde{CA}_{k'}$ Volume capacity of vehicle k' in delivery process.

$\widetilde{v}_g$: Unit volume of product g.

2.3 Decision variables

x_{ijkl} : 1 if the lth vehicle of type k travels directly from node i to node j $(i \neq j)$ in pickup process and equal 0 otherwise;

$x_{i'j'k'l'}$: 1 if the l'th vehicle of type k' travels directly from node i' to node j' $(i' \neq j')$ in delivery process and equal 0 otherwise;

y_{ikl}: 1 if node i is visited by the lth vehicle of type k in pickup process and equal 0 otherwise;

$y_{i'k'l'}$: 1 if node i' is visited by the l'th vehicle of type k' in delivery process and equal 0 otherwise;

$z_{k,l}$: 1 if the lth vehicle of type k is used in pickup process and 0 otherwise;

$z_{k',l'}$: 1 if the l'th vehicle of type k' is used in delivery process and 0 otherwise;

w_{ik}: Volume of products to be collected by the vehicle of type k upon arriving at i in pickup process.

$w_{i'k'}$: Volume of products remaining to be delivered by the vehicle of type k' upon arriving at i' in delivery process.

at^k_i : Arrival time of vehicle k at node i in pickup process.

$at^{k'}_{i'}$: Arrival time of vehicle k' at node i' in delivery process.

2.4 Model

$$\min z = \sum_{i=0}^{n}\sum_{j=0}^{n}\sum_{k=1}^{K}\sum_{l=1}^{L_k} \widetilde{c}ijk\ x_{ijkl} + \sum_{i'=0}^{m}\sum_{j'=0}^{m}\sum_{k'=0}^{K'} \widetilde{c}i'j'k'\ x_{i'j'k'l'} + \sum_{k=1}^{K}\sum_{l=1}^{L_K} \widetilde{c}_k\ z_{k,l} + \sum_{k'=1}^{K'}\sum_{l'=1}^{L_{K'}} \widetilde{c}_{k'}\ z_{k',l'} \tag{2.1}$$

$$\sum_{k=1}^{K}\sum_{l=1}^{L_k} y_{ikl} = 1, \quad i \in (1,2,...,n). \tag{2.2}$$

$$\sum_{k'=1}^{K'}\sum_{l'=1}^{L_{k'}} y_{i'k'l'} = 1, \quad i' \in (1,2,...,m). \tag{2.3}$$

$$\sum_{i=0}^{n} x_{irkl} = \sum_{j=0}^{n} x_{rjkl}, \quad r \in (1,2,...,n), k \in (1,2,...,K), \quad l \in (1,2,...,L_k). \tag{2.4}$$

$$\sum_{i'=0}^{m} x_{i'r'k'l'} = \sum_{j'=0}^{m} x_{r'j'k'l'}, \quad r' \in (1,2,...,m), k' \in (1,2,...,K'), \quad l' \in (1,2,...,L_{k'}). \tag{2.5}$$

$$\sum_{j=0}^{n} x_{ijkl} = y_{ikl}, \quad i \in (1,2,...,n), k \in (1,2,...,K), \quad l \in (1,2,...,L_k). \tag{2.6}$$

$$\sum_{j'=0}^{m} x_{i'j'k'l'} = y_{i'k'l'}, \quad i' \in (1,2,...,m), k' \in (1,2,...,K'), \quad l' \in (1,2,...,L_{k'}). \tag{2.7}$$

$$\sum_{j=1}^{n} x_{0jkl} \leq 1, \quad k \in (1,2,...,K),\ l \in (1,2,...,L_k). \tag{2.8}$$

$$\sum_{i=1}^{n} x_{i0kl} \leq 1, \quad k \in (1,2,...,K),\ l \in (1,2,...,L_k). \tag{2.9}$$

$$\sum_{j'=1}^{n} x_{0j'k'l'} \leq 1, \quad k' \in (1,2,...,K'),\ l' \in (1,2,...,L_{k'}). \tag{2.10}$$

$$\sum_{i'=1}^{n} x_{i'0k'l'} \leq 1, \quad k' \in (1,2,...,K'),\ l \in (1,2,...,L_{k'}). \tag{2.11}$$

$$\sum_{i=1}^{n} y_{ikl} \leq M z_{kl}, \quad k \in (1,2,...,K),\ l \in (1,2,...,L_k). \tag{2.12}$$

$$\sum_{i'=1}^{m} y_{i'k'l'} \leq M z_{k'l'},\ k' \in (1,2,...,K'),\ l' \in (1,2,...,L_{k'}). \tag{2.13}$$

$$\sum_{i=0}^{n} \sum_{j=0}^{n} \tilde{t}_{ij}^{k} x_{ijkl} \leq \tilde{T}_k^s, \quad k \in (1,2,...,K),\ l \in (1,2,...,L_k). \tag{2.14}$$

$$\sum_{i'=0}^{m} \sum_{j'=0}^{m} \tilde{t}_{i'j'}^{k'} x_{ijkl} \leq \tilde{T}_{k'}^s, \quad k' \in (1,2,...,K'),\ l' \in (1,2,...,L_{k'}). \tag{2.15}$$

$$at_{i'}^{k'} \geq \tilde{t}_{0i'}^{k'},\ k' \in (1,2,...,K'),\ i' \in (1,2,...,m). \tag{2.16}$$

$$at_i^k \geq \tilde{t}_{0i}^k,\ k \in (1,2,...,K),\ i \in (1,2,...,n). \tag{2.17}$$

$$at_{i'}^{k'} + \tilde{t}_{i'j'}^{k'} - at_{j'}^{k'} \leq (1 - \sum_{l'=1}^{L_{k'}} x_{i'j'k'l'})\ \tilde{T}_{k'}^s, \quad i', j' \in (1,2,...,m),\ k' \in (1,2,...,K'). \tag{2.18}$$

$$at_i^k + \tilde{t}_{ij}^k - at^k \leq (1 - \sum_{l=1}^{L_k} x_{ijkl})\ \tilde{T}_k^s, \quad i,j \in (1,2,...,n),\ k \in (1,2,...,K). \tag{2.19}$$

$$\tilde{l}_{i'} \leq at_{i'}^{k'} \leq \tilde{u}_{i'}, \quad i' \in (1,2,...,m),\ k' \in (1,2,...,K'). \tag{2.20}$$

$$w_{0k} \leq \widetilde{CA}_k,\ k \in (1,2,...,K). \tag{2.21}$$

$$w_{ik} + \sum_{g=1}^{G} \tilde{v}_g p_{ig} - w_{jk} \leq (1 - \sum_{l=1}^{L_k} x_{ijkl})\widetilde{CA}_k,\ i \in (0,1,...,n),\ j \in (1,2,...,n),\ k \in (1,2,...,K). \tag{2.22}$$

$$w_{0k'} \leq \widetilde{CA}_{k'},\ k' \in (1,2,...,K'). \tag{2.23}$$

$$w_{i'k'} + \sum_{g=1}^{G} \tilde{v}_g p_{i'g} - w_{j'k'} \leq (1 - \sum_{l'=1}^{L_{k'}} x_{i'j'k'l'})\widetilde{CA}_{k'},\ i' \in (0,1,...,m),\ j' \in (1,2,...,m),\ k' \in (1,2,...,K'). \tag{2.24}$$

$$at_0^k = at_0^{k''},\ k \neq k''. \tag{2.25}$$

$$at_0^{k'} = at_0^{k'''},\ k' \neq k'''. \tag{2.26}$$

$$x_{ijkl}, x_{i'j'k'l'}, z_{kl}, z_{k'l'}, y_{ikl}, y_{i'k'l'} \in \{0,1\}. \tag{2.27}$$

$$w_{ik}, w_{i'k'}, at_i^k, at_{i'}^{k'} \geq 0. \tag{2.28}$$

Objective function Eq.(2.1) minimizes the total costs including transportation costs and operational costs. constraints Eq.(2.2) and Eq.(2.3) specify that each supplier and retailer is visited exactly once in pickup and delivery process , respectively and while constraints Eq.(2.4) and Eq.(2.5) are flow conservation equations. Constraints Eq.(2.6) and Eq.(2.7) express the y_{ikl} and $y_{i'k'l'}$ variables in terms of the x_{ijkl} and $x_{i'j'k'l'}$ variables, respectively. Whether or not a vehicle arrives at and leaves a cross-dock is shown in Eq.(2.8), Eq.(2.9), Eq.(2.10) and Eq.(2.11). Constraints Eq.(2.12) mean that no supplier can be visited by the lth vehicle of type k if this vehicle is not used in pickup process. Constraints Eq.(2.13) mean that no retailer can be visited by the l'th vehicle of type k' if this vehicle is not used in delivery process. Constraints Eq.(2.14) and Eq.(2.15) specify the duration constraints on vehicle routes in pickup and delivery process, respectively. Constraints Eq.(2.16), Eq.(2.17), Eq.(2.18), Eq.(2.19) and Eq.(2.20) ensure that all time windows are respected. Moreover, these constraints ensure that every supplier or retailer is on a route connected to the set of cross docks. Constraints Eq.(2.21), Eq.(2.22), Eq.(2.23) and Eq.(2.24) expresses that the quantity of loaded and unloaded products in a certain vehicle cannot exceed the maximum capacity of the vehicle in pickup and delivery process. Constraints for simultaneous arrival to a cross-dock are given in Eq.(2.25) and Eq.(2.26). Finally, Constraints Eq.(2.27) and Eq.(2.28) enforce the binary and non-negativity restrictions on decision variables.

3 Solution Methodology

The proposed mathematical model is a fuzzy possibilistic-stochastic programming problem. To solve the model, a hybrid solution methodology is developed based on fuzzy possibilistic programming and stochastic programming. For this purpose, the original model under uncertainty is transformed into an equivalent auxiliary crisp model by utilizing an efficient fuzzy possibilistic-stochastic solution approach resulted from the hybridization of the recent effective methods presented by Liu et al. [17], Jimenez et al. [19] and Pishvaee and Torabi [21]:(a) fuzzy possibilistic programming and (b) chance-constraint programming. The proposed solution methodology is employed to find the final preferred compromise solution under uncertainty [26].

3.1 Hybrid solution methodology

In order to write equivalent auxiliary crisp model, the chance-constrained programming is integrated within the fuzzy possibilistic framework for taking account of distribution information of the model's right-hand sides. Also, appropriate possibility distributions of the parameters in the objective function and constraints are determined according to the definition of the expected interval (EI) and expected value (EV) of fuzzy numbers. Finally, it results in a hybrid FPSP model as follows:

$$\begin{aligned} &\min f = \widetilde{C}X \\ &S.t: \end{aligned} \tag{3.29}$$

$$\widetilde{A}X \geq \widetilde{B}, \tag{3.30}$$

$$\widetilde{D}X = \widetilde{F}. \tag{3.31}$$

With

$$\widetilde{B} = (b_1, b_2, ..., b_{m_1}, b_{m_1+1}^{(p_1)}, b_{m_1+2}^{(p_2)}, ..., b_m^{(p_m - m_1)}),$$

$$x_j \geq 0,\ x_j \in X,\ j = 1, 2, ..., n.$$

where that $\widetilde{C}$ is a triangular fuzzy number, Eq.(3.32) can be expressed as the membership function of $\widetilde{C}$:

$$\tilde{\mu}_C(x) = \begin{cases} f_c(x) = \frac{x - c^p}{c^m - c^p} & if c^p \leq x \leq c^m \\ 1 & if x = c^m \\ g_c(x) = \frac{c^o - x}{c^o - c^m} & if c^m \leq x \leq c^o \\ 0 & if x \leq c^p \ or\ x \geq c^o \end{cases} \tag{3.32}$$

The EI and EV of triangular fuzzy number $\widetilde{C}$ can be obtained as follows [19, 21]:

$$\begin{aligned} &EI(\widetilde{C}) \\ &= [E_1^c, E_2^c] \\ &= [\textstyle\int_0^1 f_c^{-1}(x)\ dx, \int_0^1 g_c^{-1}(x)\ dx] \\ &= [\frac{1}{2}(c^p + c^m), \frac{1}{2}(c^m + c^o)], \end{aligned}$$

and

$$\begin{aligned} &EV(\widetilde{C}) = \frac{E_1^c + E_2^c}{2} = \frac{c^p + 2c^m + c^o}{4}, \\ &E_1^D = \frac{1}{2}(D^p + D^m),\ E_2^D = \frac{1}{2}(D^m + D^o), \\ &E_1^F = \frac{1}{2}(F^p + F^m), \\ &E_2^F = \frac{1}{2}(F^m + F^o). \end{aligned}$$

For details on the method, the reader can refer to [21]. Constraints in Eq.(3.30)have fuzzy left hand-side and right hand-side coefficients. In addition, some of the right-hand sides in Eq.(3.30), $(i.e., b_{m_1+1}^{(p_1)}, b_{m_1+2}^{(p_2)}, ..., b_m^{(p_m - m_1)})$ are presented as probability distributions. Hence, if below conditions hold in terms of level sets,

$$\begin{aligned} &\{\mu_{A_{ij}}(a_{ij}) \mid a_{ij} \in [0, 1]\} = \{\alpha_{i1}, \alpha_{i2}, ..., \alpha_{ik}\}, \\ &0 \leq \alpha_{i1} \leq \alpha_{i2} \leq ... \leq \alpha_{ik} \leq 1, \quad i = 1, 2, ...m. \end{aligned} \tag{3.33}$$

Then, fuzzy constraints in Eq.(3.30) can be replaced by the following $2k$ precise inequalities, in which k indicates k levels of α-cut.

$$\overline{A}^l X \leq \overline{B}^l, \; l = 1, 2, ..., k, \tag{3.34}$$

$$\underline{A}^l X \leq \underline{A}^l, \; l = 1, 2, ..., k, \tag{3.35}$$

where

$$\begin{aligned} \overline{A}^l &= \sup(A^l), \\ \overline{B}^l &= \sup(B^l), \\ \underline{A}^l &= \inf(A^l), \\ \underline{B}^l &= (B^l). \end{aligned}$$

Eq.(3.29) can be transformed into a conventional linear programming problem based on [19, 21, 26] as follows:

$$\min f = EV(\widetilde{C}X \tag{3.36}$$

$$\begin{aligned} &S.t: \\ &\textstyle\sum_{j=1}^{n}(\widetilde{a}_{ij}^s) \leq \widetilde{B}_i^s, \end{aligned} \tag{3.37}$$

with

$$\widetilde{B}_i^s = \begin{cases} \overline{b}_i^s & \forall i = 1, 2, ..., k_1, \\ \overline{b}^{s(p_i)})_i & \forall i = m_1 + 1, m_1 + 2, ..., m; \\ s = k_1 + 1, k_1 + 2, ..., k. \end{cases}$$

$$\sum_{j=1}^{n}(\underline{a}_{ij}^s X) \geq \underline{B}_i^s, \tag{3.38}$$

with

$$\underline{B}_i^s = \begin{cases} \underline{b}_i^s & \forall i = 1, 2, ..., k_1, \\ \underline{b}^{s(p_i)})_i & \forall i = m_1 + 1, m_1 + 2, ..., m; \\ s = k_1 + 1, k_1 + 2, ..., k. \end{cases}$$

$$\begin{aligned} &[(1 - \tfrac{\alpha}{2})E_2^D + \tfrac{\alpha}{2}E_1^D]x \geq \tfrac{\alpha}{2}E_2^F \\ &+(1 - \tfrac{\alpha}{2})E_1^F \end{aligned} \tag{3.39}$$

$$\begin{aligned} &[\tfrac{\alpha}{2}E_2^D + (1 - \tfrac{\alpha}{2})E_1^D]x \geq (1 - \tfrac{\alpha}{2})E_2^F \\ &+\tfrac{\alpha}{2}E_1^F, \end{aligned} \tag{3.40}$$

$$x_j \geq 0, \; j = 1, 2, ..., n. \tag{3.41}$$

For the right-hand side of constraint, boundaries of its fuzzy intervals under any α -cut levels have random characteristics. They can be presented as normal distributions as follows:

$$p[\underline{b}_2(s)] = \tfrac{1}{\sqrt{2\pi\underline{\sigma}}} \exp\{-\tfrac{[\underline{b}_2(s) - \underline{\mu}]^2}{2\underline{\sigma}^2}\}, \tag{3.42}$$

and

$$p[\overline{b}_2(s)] = \tfrac{1}{\sqrt{2\pi\overline{\sigma}}} \exp\{-\tfrac{[\overline{b}_2(s) - \overline{\mu}]^2}{2\overline{\sigma}^2}\}, \tag{3.43}$$

where $\underline{\mu}$ and $\overline{\mu}$ are expected values of $\underline{b}_2(s)$ and $\overline{b}_2(s)$, respectively; Also, $\underline{\sigma}^2$ and $\underline{\sigma}^2$ are the relevant variances [17].

3.2 The equivalent auxiliary crisp model

According to above descriptions, the equivalent auxiliary crisp model can be formulated as follows:

$$\begin{aligned} &\min z = \\ &\textstyle\sum_{i=0}^{n}\sum_{j=0}^{n}\sum_{k=1}^{K}\sum_{l=1}^{L_k}(\frac{c_{ijk}^p + 2c_{ijk}^m + c_{ijk}^o}{4})x_{ijkl} \\ &+\textstyle\sum_{i'=0}^{m}\sum_{j'=0}^{m}\sum_{k'=1}^{K'}\sum_{l'=1}^{L_{k'}} \\ &(\tfrac{c_{i'j'k'}^p + 2c_{i'j'k'}^m + c_{i'j'k'}^o}{4})x_{i'j'k'l'} \\ &+\textstyle\sum_{k=1}^{K}\sum_{l=1}^{L_k}(\frac{c_k^p + 2c_k^m + c_k^o}{4})z_{kl} \\ &+\textstyle\sum_{k'=1}^{K'}\sum_{l'=1}^{L_{k'}}(\frac{c_{k'}^p + 2c_{k'}^m + c_{k'}^o}{4})z_{k'l'} \end{aligned} \tag{3.44}$$

$$\textstyle\sum_{k=1}^{K}\sum_{l=1}^{L_k} y_{ikl} = 1, \quad i \in (1, 2, ..., n). \tag{3.45}$$

$$\textstyle\sum_{k'=1}^{K'}\sum_{l'=1}^{L_{k'}} y_{i'k'l'} = 1, \quad i' \in (1, 2, ..., m). \tag{3.46}$$

$$\begin{aligned} &\textstyle\sum_{i=0}^{n} x_{irkl} = \sum_{j=0}^{n} x_{rjkl}, \\ &\quad r \in (1, 2, ..., n), k \in (1, 2, ..., K), \\ &\quad l \in (1, 2, ..., L_k). \end{aligned} \tag{3.47}$$

$$\begin{aligned} &\textstyle\sum_{i'=0}^{m} x_{i'r'k'l'} = \sum_{j'=0}^{m} x_{r'j'k'l'}, \\ &\quad r' \in (1, 2, ..., m), k' \in (1, 2, ..., K'), \\ &\quad l' \in (1, 2, ..., L_{k'}). \end{aligned} \tag{3.48}$$

$$\begin{aligned} &\textstyle\sum_{j=0}^{n} x_{ijkl} = y_{ikl}, \\ &\quad i \in (1, 2, ..., n), k \in (1, 2, ..., K), \\ &\quad l \in (1, 2, ..., L_k). \end{aligned} \tag{3.49}$$

$$\sum_{j'=0}^{m} x_{i'j'k'l'} = y_{i'k'l'}, \quad i' \in (1,2,...,m), k' \in (1,2,...,K'), \quad l' \in (1,2,...,L_{k'}). \tag{3.50}$$

$$\sum_{j=1}^{n} x_{0jkl} \leq 1,\ k \in (1,2,...,K), \quad l \in (1,2,...,L_k). \tag{3.51}$$

$$\sum_{i=1}^{n} x_{i0kl} \leq 1,\ k \in (1,2,...,K), \quad l \in (1,2,...,L_k). \tag{3.52}$$

$$\sum_{j'=1}^{n} x_{0j'k'l'} \leq 1,\ k' \in (1,2,...,K'), \quad l' \in (1,2,...,L_{k'}). \tag{3.53}$$

$$\sum_{i'=1}^{n} x_{i'0k'l'} \leq 1,\ k' \in (1,2,...,K'), \quad l \in (1,2,...,L_{k'}). \tag{3.54}$$

$$\sum_{i=1}^{n} y_{ikl} \leq M z_{kl},\ k \in (1,2,...,K), \quad l \in (1,2,...,L_k). \tag{3.55}$$

$$\sum_{i'=1}^{m} y_{i'k'l'} \leq M z_{k'l'}, \quad k' \in (1,2,...,K'), \quad l' \in (1,2,...,L_{k'}). \tag{3.56}$$

$$\sum_{i=0}^{n}\sum_{j=0}^{n}(\sup\{(t_{ij}^{ko} - \alpha(t_{ij}^{ko} - t_{ij}^{km})), (t_{ij}^{kp} + \alpha(t_{ij}^{km} - t_{ij}^{kp}))\})x_{ijkl} \leq \sup\{(T^{o_k} - \alpha(T_k^0 - T_k^m)), (T^{p_k} + \alpha(T^{m_k} - T_k^p))\}^{p_k}, \in (1,2,...,K),\ l \in (1,2,...,L_k). \tag{3.57}$$

$$\sum_{i=0}^{n}\sum_{j=0}^{n}(\inf\{(t_{ij}^{ko} - \alpha(t_{ij}^{ko} - t_{ij}^{km})), (t_{ij}^{kp} + \alpha(t_{ij}^{km} - t_{ij}^{kp}))\})x_{ijkl} \geq \inf\{(T^{o_k} - \alpha(T_k^0 - T_k^m)), (T^{p_k} + \alpha(T^{m_k} - T_k^p))\}^{p_k}, \quad k \in (1,2,...,K),\ l \in (1,2,...,L_k). \tag{3.58}$$

$$\sum_{i'=0}^{m}\sum_{j'=0}^{m}(\sup\{(t_{i'j'}^{k'o} - \alpha(t_{i'j'}^{k'o} - t_{i'j'}^{k'm})), (t_{i'j'}^{k'p} + \alpha(t_{i'j'}^{k'm} - t_{i'j'}^{k'p}))\})x_{i'j'k'l'} \leq \sup\{(T^{o_{k'}} - \alpha(T_{k'}^0 - T_{k'}^m)), (T^{p_{k'}} + \alpha(T^{m_{k'}} - T_{k'}^p))\}^{p_{k'}}, \quad k' \in (1,2,...,K'),\ l \in (1,2,...,L_{k'}). \tag{3.59}$$

$$\sum_{i'=0}^{m}\sum_{j'=0}^{m}(\inf\{(t_{i'j'}^{k'o} - \alpha(t_{i'j'}^{k'o} - t_{i'j'}^{k'm})), (t_{i'j'}^{k'p} + \alpha(t_{i'j'}^{k'm} - t_{i'j'}^{k'p}))\})x_{i'j'k'l'} \geq \inf\{(T^{o_{k'}} - \alpha(T_{k'}^0 - T_{k'}^m)), (T^{p_{k'}} + \alpha(T^{m_{k'}} - T_{k'}^p))\}^{p_{k'}}, \quad k' \in (1,2,...,K'),\ l \in (1,2,...,L_{k'}). \tag{3.60}$$

$$at_{i'}^{k'} \geq \alpha(\tfrac{t_{oi'}^{m'} + t_{oi'}^{ok'}}{2}) + (1-\alpha)(\tfrac{t_{oi'}^{pk'} + t_{0i'}^{mk'}}{2}), \quad k' \in (1,2,...,K'),\ i' \in (1,2,...,m). \tag{3.61}$$

$$at_i^k \geq \alpha(\tfrac{t_{oi}^m + t_{oi}^{ok}}{2}) + (1-\alpha)(\tfrac{t_{oi}^{pk} + t_{0i}^{mk}}{2}), \quad k \in (1,2,...,K),\ i \in (1,2,...,n). \tag{3.62}$$

$$at_{j'}^{k'} + (\sup[(t_{i'j'}^{k'o} - \alpha(t_{i'j'}^{k'd} - t_{i'j'}^{k'm})), (t_{i'j'}^{k'p} + \alpha(t_{i'j'}^{k'm}) - t_{i'j'}^{k'p}))]) - at_{i'}^{k'} \leq (1 - \sum_{l'}^{L_{k'}} x_{i'j'k'l'})(\sup[(T_{k'}^o - \alpha(T_{k'}^o - T_{k'}^m)), (T_{k'}^p + \alpha(T_{k'}^m - T_{k'}^p))]^{p_{k'}}), \quad \forall i', j' = 1,2,...,m,\ k' = 1,2,...,K'. \tag{3.63}$$

$$\begin{aligned}
& at_{j'}^{k'} + (\inf[(t_{i'j'}^{k'o} - \alpha(t_{i'j'}^{k'd} - t_{i'j'}^{k'm})), \\
& (t_{i'j'}^{k'p} + \alpha(t_{i'j'}^{k'm}) - t_{i'j'}^{k'p}))]) - at_{i'}^{k'} \\
& \geq (1 - \textstyle\sum_{l'}^{L_{k'}} x_{i'j'k'l'})(\inf[(T_{k'}^{o} - \alpha(T_{k'}^{o} - T_{k'}^{m})), \\
& (T_{k'}^{p} + \alpha(T_{k'}^{m} - T_{k'}^{p}))]^{p_{k'}}), \\
& \forall i', j' = 1, 2, ..., m,\ k' = 1, 2, ..., K'.
\end{aligned} \tag{3.64}$$

$$\begin{aligned}
& at_{j}^{k} + (\sup[(t_{ij}^{ko} - \alpha(t_{ij}^{kd} - t_{ij}^{km})), \\
& (t_{ij}^{kp} + \alpha(t_{ij}^{km}) - t_{ij}^{kp}))]) - at_{i}^{k} \\
& \leq (1 - \textstyle\sum_{l}^{L_k} x_{ijkl})(\sup[(T_{k}^{o} - \alpha(T_{k}^{o} - T_{k}^{m})), \\
& (T_{k}^{p} + \alpha(T_{k}^{m} - T_{k}^{p}))]^{p_k}), \\
& \forall i, j = 1, 2, ..., m,\ k = 1, 2, ..., K.
\end{aligned} \tag{3.65}$$

$$\begin{aligned}
& at_{j}^{k} + (\inf[(t_{ij}^{ko} - \alpha(t_{ij}^{kd} - t_{ij}^{km})), \\
& (t_{ij}^{kp} + \alpha(t_{ij}^{km}) - t_{ij}^{kp}))]) - at_{i}^{k} \\
& \geq (1 - \textstyle\sum_{l}^{L_k} x_{ijkl})(\inf[(T_{k}^{o} - \alpha(T_{k}^{o} - T_{k}^{m})), \\
& (T_{k}^{p} + \alpha(T_{k}^{m} - T_{k}^{p}))]^{p_k}), \\
& \forall i, j = 1, 2, ..., m,\ k = 1, 2, ..., K.
\end{aligned} \tag{3.66}$$

$$\begin{aligned}
& \alpha(\tfrac{l_i^m + l_i^o}{2}) + (1-\alpha)(\tfrac{l_i^p + l_i^m}{2}) \leq at_{i'}^{k'} \\
& \leq (1-\alpha)(\tfrac{u_i^m + u_i^o}{2}) + \alpha(\tfrac{u_i^p + u_i^m}{2}), \\
& \forall i' = 1, 2, ..., m,\ k' = 1, 2, ..., K'.
\end{aligned} \tag{3.67}$$

$$\begin{aligned}
& w_{0k} \leq (1-\alpha)(\tfrac{CA_k^m + CA_k^o)}{2})+ \\
& \alpha(\tfrac{CA_k^p + CA_k^m}{2}), \forall k = 1, 2, ..., K.
\end{aligned} \tag{3.68}$$

$$\begin{aligned}
& w_{0k'} \leq (1-\alpha)(\tfrac{CA_{k'}^m + CA_{k'}^o)}{2})+ \\
& \alpha(\tfrac{CA_{k'}^p + CA_{k'}^m}{2}), \forall k' = 1, 2, ..., K'.
\end{aligned} \tag{3.69}$$

$$\begin{aligned}
& w_{ik} + \textstyle\sum_{g=1}^{G}[\alpha(\tfrac{v_g^m + v_g^o}{2})+ \\
& (1-\alpha)(\tfrac{v_g^p + v_g^m}{2})]p_{ig} - \\
& w_{jk} \leq (1 - \textstyle\sum_{l=1}^{L_k} x_{ijkl})[(1-\alpha) \\
& (\tfrac{CA_k^m + CA_k^o}{2}) + \alpha(\tfrac{CA_k^p + CA_k^m}{2})], \\
& \forall i = 0, 1, ..., n,\ j = 1, 2, ..., n,\ k = 1, 2, ..., K.
\end{aligned} \tag{3.70}$$

$$\begin{aligned}
& w_{i'k'} + \textstyle\sum_{g=1}^{G}[\alpha(\tfrac{v_g^m + v_g^o}{2}) \\
& +(1-\alpha)(\tfrac{v_g^p + v_g^m}{2})]d_{i'g} - \\
& w_{j'k'} \leq (1 - \textstyle\sum_{l'=1}^{L_{k'}} x_{i'j'k'l'})[(1-\alpha) \\
& (\tfrac{CA_{k'}^m + CA_{k'}^o}{2}) + \alpha(\tfrac{CA_{k'}^p + CA_{k'}^m}{2})], \\
& \forall i' = 0, 1, ..., m,\ j' = 1, 2, ..., m, \\
& k' = 1, 2, ..., K'.
\end{aligned} \tag{3.71}$$

$$at_0^k = at_0^{k''},\ k \neq k''. \tag{3.72}$$

$$at_0^{k'} = at_0^{k'''},\ k' \neq k'''. \tag{3.73}$$

$$\begin{aligned}
& x_{ijkl}, x_{i'j'k'l'}, z_{kl}, z_{k'l'}, y_{ikl}, \\
& y_{i'k'l'} \in \{0, 1\}.
\end{aligned} \tag{3.74}$$

$$w_{ik}, w_{i'k'}, at_i^k, at_{i'}^{k'} \geq 0. \tag{3.75}$$

4 Numerical Example

To illustrate the validity and applicability of the proposed mathematical model, several numerical experiments are considered and the related results are provided in this section. For this purpose, five test problems are designed that their sizes are given in Table 1. According to Table 2, the proposed fuzzy possibilistic-stochastic MILP model is solved and reported by GAMS optimization software. The numerical experiments for each size are calculated under three α-cut levels ($\alpha = 0.2, 0.4, 0.6$). It is pointed out that the

Table 1: Parameters values.

Parameters	Test problem 1	Test problem 2
n	4	5
m	6	7
G	3	3
K	4	4
K'	4	4
$\widetilde{T}_s^k, \widetilde{T}_s^{k'}$	$Uniform(800, 1300)$	$Uniform(800, 1300)$
p_{ig}	$Uniform(8, 30)$	$Uniform(15, 35)$
$d_{i'g}$	$Uniform(5, 30)$	$Uniform(5, 30)$
$\widetilde{c}_{ij}$	$Uniform(300, 500)$	$Uniform(250, 500)$
$\widetilde{c}_{i'j'}$	$Uniform(110, 530)$	$Uniform(100, 530)$
$\widetilde{c}_k$	$Uniform(600, 800)$	$Uniform(600, 800)$
$\widetilde{c}_{k'}$	$Uniform(600, 800)$	$Uniform(600, 800)$
$\widetilde{t}_{ij}^k$	$Uniform(6, 16)$	$Uniform(4, 16)$
$\widetilde{t}_{i'j'}^k$	$Uniform(9, 20)$	$Uniform(10, 20)$
$\widetilde{CA}_k, \widetilde{CA}_{k'}$	$Uniform(900, 1200)$	$Uniform(9, 1200)$
$\widetilde{v}_j$	$Uniform(1.2, 2.3)$	$Uniform(1.2, 2.3)$
$\widetilde{l}_{i'}$	$Uniform(3, 5)$	$Uniform(3, 5)$
$\widetilde{u}_{i'}$	$Uniform(20, 35)$	$Uniform(20, 35)$

(Continue Table 1).

Test problem3	Test problem 4	Test problem 5
6	7	8
8	9	10
3	4	4
4	5	6
4	5	6
$Uniform(800, 1300)$	$Uniform(800, 1300)$	$Uniform(800, 1300)$
$Uniform(10, 35)$	$Uniform(8, 35)$	$Uniform(5, 35)$
$Uniform(5, 35)$	$Uniform(5, 25)$	$Uniform(5, 20)$
$Uniform(90, 500)$	$Uniform(150, 500)$	$Uniform(110, 500)$
$Uniform(90, 500)$	$Uniform(80, 50)$	$Uniform(90, 490)$
$Uniform(600, 800)$	$Uniform(600, 800)$	$Uniform(600, 800)$
$Uniform(600, 800)$	$Uniform(600, 800)$	$Uniform(600, 800)$
$Uniform(4, 16)$	$Uniform(4, 16)$	$Uniform(4, 18)$
$Uniform(8, 20)$	$Uniform(6, 20)$	$Uniform(7, 200)$
$Uniform(1200)$	$Uniform(900)$	$Uniform(1200)$
$Uniform(1.2, 2.3)$	$Uniform(1.2, 2.3)$	$Uniform(1.2, 2.3)$
$Uniform(3, 5)$	$Uniform(3, 5)$	$Uniform(3, 5)$
$Uniform(20, 35)$	$Uniform(20, 35)$	$Uniform(20, 35)$

boundaries of fuzzy intervals for the right-hand side of constraint under α-cut levels have random characteristics, and they can be presented as normal distributions. Also, the values of the probability $(p_k, p_{k'})$ set to 0.1 and 0.3 in the five test problems. Finally, the computational results for the proposed model are reported in Table 2. In order to demonstrate the accuracy and efficiency of the proposed model, an extensive sensitivity analysis is performed to scrutinize its parameters' demeanor. The parameters include the numbers of both pick-up and delivery nodes as well as transportation costs. Additionally, the sensitivity analysis has been done on the final solution of both single-product and multi-product cross-docking systems.

Table 2: Computational results under different combination of α and p_k, $p-k'$

Test problem	Probability values	Objective function under different values α-cut levels		
		0.2	0.4	0.6
$ProblemNo.1$	$p_k = p_{k'} = 0.1$	5715.7	5512.5	53109.1
$ProblemNo.1$	$p_k = p_{k'} = 0.3$	5431.5	5365.3	5201.8
$ProblemNo.2$	$p_k = p_{k'} = 0.1$	6232.7	5365.3	5201.8
$ProblemNo.2$	$p_k = p_{k'} = 0.3$	6039.9	6012.6	5879.2
$ProblemNo.3$	$p_k = p_{k'} = 0.1$	6980.6	5943.5	5633.9
$ProblemNo.3$	$p_k = p_{k'} = 0.3$	6706.2	6608.6	6451.4
$ProblemNo.4$	$p_k = p_{k'} = 0.1$	7550.7	7214.8	7087.3
$ProblemNo.4$	$p_k = p_{k'} = 0.3$	7335.6	7146.9	6819.2
$ProblemNo.5$	$p_k = p_{k'} = 0.1$	8420	8166.5	7931.7
$ProblemNo.5$	$p_k = p_{k'} = 0.3$	8124.9	7866.1	7544.7

4.1 Increase in the numbers of pick-up nodes:

As can be seen in Figure 1, the value of objective function consistently increases as the numbers of pick-up nodes increase. Possibly, this is because of the facts that as the numbers of pick-up nodes increase, higher numbers of vehicles are required for pick-up activities. Consequently, the costs of renting or buying new vehicles are added to the supply chain cost. On the other hand, it is natural that as the numbers of pick-up nodes increase, the distance that each vehicle has to traverse to cover the added nodes increases. This issue directly contributes to transportation cost increase.

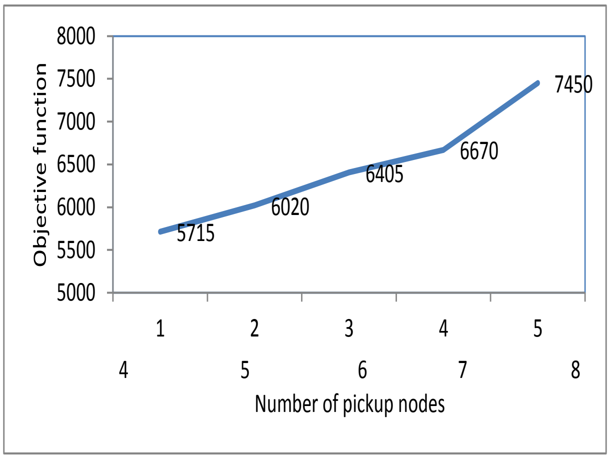

Figure 1: Objective function and number of pickup nodes.

4.2 Increase in the numbers of delivery nodes:

As can be seen in Figure 2, the value of objective function consistently increases as the numbers of delivery nodes increase. Most likely, this is because of the facts that as the numbers of delivery nodes increase, higher numbers of vehicles are required for delivery activities. Consequently, the costs of renting or buying new vehicles are added to the supply chain cost. On the other hand, it is natural that as the numbers of delivery nodes increase, the distance that each vehicle has to traverse to cover the added nodes increases. This issue also directly contributes to transportation cost increase. Figure 2 obviously demonstrates that increase in the numbers of delivery nodes causes the objective function value to increase by a higher coefficient as opposed to the case in which the numbers of pick-up nodes increase. This issue is perfectly normal in our case because no time constraints have been imposed on pick-up nodes, whereas in delivery nodes, vehicles are subject to time constraints for delivering their products within their pre-specified delivery time. This issue mandates decision makers to dispatch more vehicles to fulfill the delivery requirements of a node if the delivery time exceeds the pre-specified delivery time-interval for that node. Thus, the transportation cost associated with delivery nodes are substantially higher than the time the pick-up activities are done.

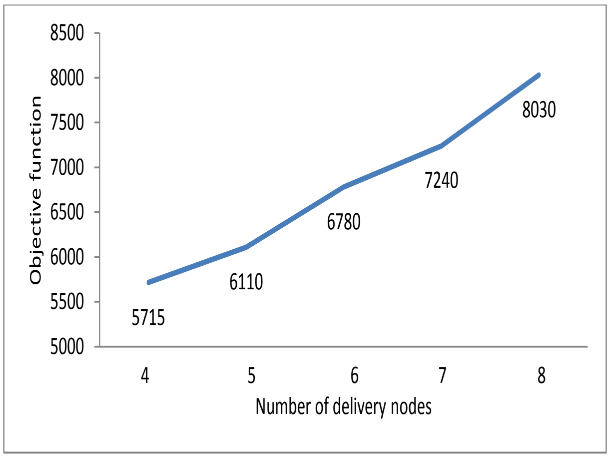

Figure 2: Objective function and number of delivery nodes.

4.3 Increase in the transportation costs:

As can be seen in Figure 3, the value of objective function proportionally increases as the transportation costs rise. Therefore, it can be concluded that transportation costs play substantial role in increasing the objective function value.

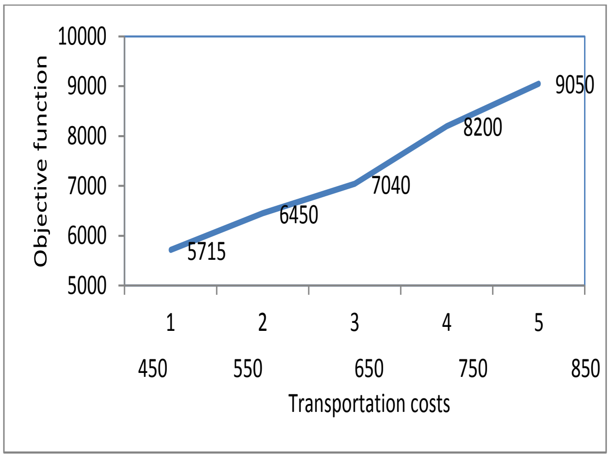

Figure 3: Objective function and transportation costs.

4.4 Solutions of the model in single and multi-product cases:

As can be seen in Figure 4, the value of objective function is markedly higher than the one in single product case. The argument that can be made for this difference in objective function is that since the capacity of vehicles are limited, they usually accommodate for single product case. But in the case of multi-products, more vehicle capacity is needed and consequently we require more vehicles to meet the pick-up and delivery requirements. As a consequence, both the transportation cost and rental or purchases of new vehicles increase.

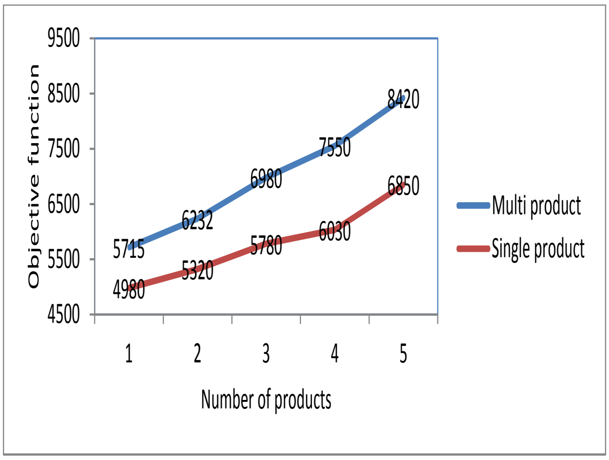

Figure 4: Objective function and number product.

5 Conclusion

In this paper, a new Mixed Integer Linear Programming formulation was developed and applied for the problem of vehicle routing scheduling. The proposed model remedies the shortcomings of previously developed models in the field. Several applicable assumptions for the first time were incorporated in the proposed formulation: a) existence of multiple products that are transported between pick-up and delivery nodes, b) delivery time-intervals are imposed on each delivery node, c) multiple types of vehicles operate in the system, d) capacity constraints exists for each vehicle type, and finally e) vehicles arrives simultaneously at cross-docking location. Moreover, we utilized a hybrid solution methodology by combining fuzzy possibilistic programming and stochastic programming. Additionally, comprehensive sensitivity analyses were performed on the parameter of the model to investigate how the demeanor of the model changes with respect to possible changes in the supply chain network. The sensitivity analyses help adopt appropriate decisions to better manage the distribution centers in supply chains. As a direction for future research, the current study can be extended to incorporate the existence of multiple cross-docking locations in the supply chain.

Acknowledgement

The authors are grateful for the partially financial support from the Young Researchers and Elite Club, Islamic Azad University, Qazvin Branch, Qazvin, Iran.

References

[1] A. Landrieu, Y. Mati, Z. Binder, *A tabu search heuristic for the single vehicle pickup and delivery problem with time windows*, Journal of Intelligent Manufacturing 12 (2001) 5-6.

[2] A. Lim, H. Ma, Z. Miao, *Truck dock assignment problem with operational time constraint within crossdocks. In M. Ali and R. Dapoigny (Eds.)*, Advances in applied artificial intelligence, Berlin/Heidelberg: Springer, 4031 (2006) 262-271.

[3] A. Lim, H. Ma, Z. Miao, *Truck dock assignment problem with time windows and capacity constraint in transshipment network through crossdocks, In M. Gavrilova (Ed.)*, Computational science and its applications-ICCSA, Berlin/Heidelberg: Springer 3982 (2006) 688-697.

[4] B. Schaffer, *Cross docking can increase efficiency*, Automatic I.D. News 14 (1998) 34-37.

[5] B. Vahdani, M. Zandieh, *Scheduling trucks in cross-docking systems: Robust meta-heuristics*, Computers and Industrial Engineering 58 (2010) 12-24.

[6] C. H. Wang, J. Z. Lu, *An effective evolutionary algorithm for the practical capacitated vehicle routing problems*, Journal of Intelligent Manufacturing 21 (2010) 363-375.

[7] D. Y. Ma, F. Chen, *Dynamic programming algorithm on two machines cross docking scheduling*, Journal of Shanghai Jiao Tong University 41 (2007) 852-856.

[8] F. Chen, K. L. Song, *Cross docking logistics scheduling problem and its approximation and exact algorithms*, Industry Engineering and Management 6 (2006) 53-58.

[9] F. Chen, K. L. Song, *Minimizing makespan in two stage hybrid cross docking scheduling problem*, Computers and Operations Research 36 (2009) 2066-2073.

[10] G. Barbarosoglu, D. Ozgur, *A Tabu search algorithm for the vehicle routing problem*, Computer and Operational Research, 26 (1999) 255-270.

[11] G. Dong, J. Tang, K. K. Lai, Y. Kong, *An exact algorithm for vehicle routing and scheduling problem of free pickup and delivery service in flight ticket sales companies based on set partitioning model*, Journal of Intelligent Manufacturing 22 (2011) 789-799.

[12] G. Mosheiov, *Vehicle routing with pick-up and delivery: tour-partitioning heuristics*, Computers and Industrial Engineering 34 (1998) 669-684.

[13] H. Z. Jia, A. Y. C. Nee, J. Y. H. Fuh, Y. F. Zhang, *A modified genetic algorithm for distributed scheduling*, Journal of Intelligent Manufacturing 14 (2003) 3-4.

[14] H. C. Lau, M. Sim, K. M. Teo, *Vehicle routing problem with time windows and a limited number of vehicles*, European Journal of Operational Research 148 (2003) 559-569.

[15] H. Yan, S. L. Tang, *Pre-distribution and post-distribution cross-docking operations*, Transportation Research Part E 45 (2009) 843-859.

[16] J. F. Wang, A. Regan, *Real-time trailer scheduling for cross dock operations*, Transportation Journal 47 (2008) 5-20.

[17] L. Liu, G.H. Huang, Y. Liu, G.A. Fuller, G.M. Zeng, *A fuzzy-stochastic robust programming model for regional air quality management under uncertainty*, Eng. Optim 35 (2003) 177-199.

[18] M. U. Apte, S. Viswanathan, *Effective cross docking for improving distribution efficiencies*, International Journal of Logistics: Research and Applications 3 (2000) 291-302.

[19] M. Jimenez, M. Arenas, A. Bilbao, M.V. Rodriguez, *Linear programming with fuzzy parameters: an interactive method resolution*, Eur. J. Oper. Res 177 (2007) 1599-1609.

[20] M. Otadi, *Solving fully fuzzy linear programming*, International Journal of Industrial Mathematics 6 (2014)19-26.

[21] M.S. Pishvaee, S.A. Torabi, *A possibilistic programming approach for closed-loop supply chain network design under uncertainty*, Fuzzy Sets Syst 161 (2010) 2668-2683.

[22] M. Rohrer, *Simulation and cross docking*, In Proceedings of the 1995 winter simulation conference (1995) 846-849.

[23] M. A. Waller, C. C. Richard, J. Ozment, *Impact of cross-docking on inventory in a decentralized retail supply chain*, Transportation Research Part E 42 (2006) 359-382.

[24] N. Boysen, M. Fliedner, A. Scholl, *Scheduling inbound and outbound trucks at cross docking terminals*, OR Spectrum (2008) 1-27.

[25] N. Boysen, *Truck scheduling at zero-inventory cross docking terminals*, Computers and Operations Research 37 (2010) 32-41.

[26] S. M. Mousavi, B. Vahdani, R. Tavakkoli-Moghaddam, H. Hashemi, *Location of cross-docking centers and vehicle routing scheduling under uncertainty: A fuzzy possibilistic-stochastic programming model*, Applied Mathematical Modelling (2013) http://dx.doi.org/10.1016/j.apm.2013.10.029.

[27] S. H. Nasseri , E. Behmanesh, F. Taleshian, M. Abdolalipoor, N. A. Taghi-Nezhad, *Fully fuzzy linear programming with inequality constraints*, International Journal of Industrial Mathematics 5 (2013) 309-316.

[28] S. L. Tang, H. Yan, *Pre-distributionvs.post-distribution for cross-docking with transshipments*, Omega 38 (2010) 192-202.

[29] T. Moore, C. Roy, *Manage inventory in a real-time environment*, Transportation and Distribution, 2 (1998)68-73.

[30] W. Yu, P. J. Egbelu, *Scheduling of inbound and outbound trucks in cross docking systems with temporary storage*, European Journal of Operational Research 184 (2008) 377-396.

[31] Y. Li, A. Lim, B. Rodrigues, *Crossdocking-JIT scheduling with time windows*, Journal of the Operational Research Society 55 (2004) 1342-1351.

[32] Y. H. Lee, W. J. Jung, K. M. Lee, *Vehicle routing scheduling for cross-docking in the supply chain*, Computers and Industrial Engineering 51 (2006) 247-256.

B. Vahdani received Ph.D. degree from the Department of Industrial Engineering at University of Tehran. He also obtained B.S. and M.S. degrees in Industrial Engineering. He is the member of Irans National Elite Foundation. His current research interests include: Supply chain network design, facility location and design, logistics planning and scheduling, multi-criteria decision making, uncertain programming, artificial neural networks, meta-heuristics algorithms and operations research applications. He has published several papers and book chapters in the aforementioned areas.

Shadan Sadigh Behzadi she has born in Tehran- Iran in 1983. Got B.Sc and M.Sc degrees in applied mathematics, numerical analysis field from Islamic Azad University, Central Tehran Branch and PHD degree in applied mathematics, here numerical analysis field from Science and Research Branch, Islamic Azad University. Main research interest includ numerical solution of nonlinear Fredholm- Volterra integro-differential equation systems, ordinary differential equation, partial differential equation, fuzzy system and some equations in fluid dynamics.