

On the relative efficiency in general network structures

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Abstract

Data Envelopment Analysis (DEA) is an efficiency measurement tool for evaluation of similar Decision Making Units (DMUs). In DEA, weights are assigned to inputs and outputs and the absolute efficiency score is obtained by the ratio of weighted sum of outputs to weighted sum of inputs. In traditional DEA models, this measure is also equivalent with relative efficiency score which evaluates DMUs in compare with the most efficient DMU. Recently network DEA models are appeared in the literature, which try to assess DMUs regarding their internal production divisions and intermediate products. In this paper we compare absolute and relative efficiency scores in network framework. Since in network DEA models, an efficient DMU does not exist necessarily, the relative efficiency model helps us to have at least one efficient DMU in our assessments.

Keywords : Network DEA; Absolute efficiency; Relative efficiency; Divisional efficiency; Overall efficiency.

1 Introduction

DEA is a tool for measuring efficiency scores of similar DMUs. All DMUs receive similar inputs to produce similar outputs, but within different amounts. In recent years, this method has been developed to the Network DEA models [1]. Network DEA models not only consider to inputs and outputs as the external production factors, but also consider to internal structure of the DMUs and the internal production factors [2]. In fact, these models do not see DMUs just as black boxes which convert main inputs into main outputs. Network DEA models are now widely applied in different applications such as supply chain evaluations [3, 4, 5], banking industry [6],

transportation network evaluation [7] and so on. They contribute internal divisions in their assessments and have more precise results. Network DEA was first introduced by Fare and Grosskopf [8]. Different models were appeared due to different network structures such as two-stage network structures [9, 10] multistage [11], shared resource structures [12] series and parallel structures [13] in classifications in the literature. In different type of structures categories, different models are represented in multiplier and envelopment form which are not dual equivalent pairs. For instance see the models in [14, 15] and [16] for some instances in two-stage models.

Moreover, the models which are explicitly formulated for general network structures are stated in envelopment form [17, 18]. Fukuyama and Mirdahghan [19] and Chen et al. [20] reviewed some problems in efficiency scores in Tone and Tsutsuis model. Also Boloori et al. [21] investigated some pitfalls in Lozano's model [17]. They explicitly formulated a multiplier model for general network

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structure by following Kao [14] which considered a unique weight for a factor whatever it is used (either as an input or as an output). This multiplier formulation is used in this paper in order to investigate on relative and absolute efficiency concepts in network DEA.

In DEA literature, the Absolute efficiency score for a DMU is defined by the ratio of its weighted sum of outputs to the weighted sum of inputs. This measure is commonly used in multiplier form of conventional DEA models. On the other hand, the relative efficiency score was first represented by Cooper et al. [22] and Thompson and Thrall [23]. The relative efficiency measure was defined in order to choose an optimal set of weights for a DMU which represents the assessed DMU in the best light in comparison to the other DMUs. Podinovski [24] mentioned that, in traditional DEA models [25] and in the absence of weight restrictions, the relative efficiency score coincides with the absolute efficiency score.

In this paper these efficiency scores are investigated in network DEA framework. Since one does not have an efficient DMU in network DEA models necessarily, then the relative efficiency measure helps us to have at least one 100 % efficient DMU in relative efficiency sense in our assessments.

The structure of the paper is organized as follows. Section 2 discusses on multiplier form of general network structures and its formulation represented in [21]. In Section 3 the absolute and relative efficiency scores in network structures will be represented. Finally Section 4 deals with an illustrative example which clarifies the abilities of this model.

2 Multiplier Model for General Network Structures

Based on the definition of general network structure discussed in [21], factors can simultaneously be the main or intermediate one and also they may be shared too. For more description consider Fig. 1 which depicts a network structure. Here an input or output factor may be shared among divisions. For instance input factor q_3 is shared among division 2 and division 3. Also output factor q_4 is produced from division 4 and division 5 cooperatively. In this figure, q_2 is a main factor only (as well as q_5). But a main factor can be

used / produced as an intermediate factor too. For instance, factor q_4 is an intermediate factor, produced in division 5 for the use in division 6. It is also a main output factor which is shared among division 4 and 5. Similarly factor q_3 is also main input and intermediate factor simultaneously.

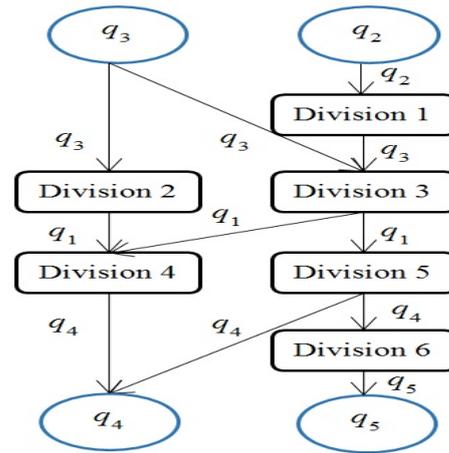


Figure 1: Example of a network structure

In order to formulating a general network structure, let $q = 1, \dots, Q$ types of factors are flowing in the network either as main input/outputs or as intermediate ones. Also in division k , let M_k, N_k be the set of main outputs and main inputs and also \tilde{M}_k, \tilde{N}_k to be the set of intermediate inputs and intermediate outputs of division k , respectively. Since a division may have inputs which are supplied both internally and externally, it is possible that $N_k \cap \tilde{N}_k \neq \emptyset$ for some division k . Also some outputs may be produced for internal use in another division of the network structure and also for external use as a final output. So, it is possible to have $M_k \cap \tilde{M}_k \neq \emptyset$ for some division k . In fact, intermediate factors are not distinct from main factors necessarily and a factor in a division may be used / produced both internally and externally. Therefore the index set of factors are denoted by $M_k, N_k, \tilde{M}_k, \tilde{N}_k$ as the subsets of $\{1, \dots, Q\}$ which some of them may be empty or possibly have intersection with each other. These

sets of factors can be defined by:

$$\tilde{M}_k = \{q|\exists h : k \xrightarrow{q} h\} \tag{a}$$

$$\tilde{N}_k = \{q|\exists h : h \xrightarrow{q} k\} \tag{b}$$

$$M_k = \{q|q \text{ is a main output in div. } k\} \tag{c}$$

$$N_k = \{q|q \text{ is a main input in div. } k\} \tag{d}$$

$$(2.1)$$

Assume that there are $j = 1, \dots, J$ DMUs and each consists of $k = 1, \dots, K$ production divisions. The flow amount of intermediate product q from division k to division h is denoted by $z_{qj}^{(k,h)}$. We may use the symbol $k \xrightarrow{q} h$ to say that q flows from division k to division h , where needed. Also x_{qj}^k is the value of main input q devoted to division k in DMU j and y_{qj}^k is the value of main output q produced by division k of DMU $_j$ as the final output. Fig. 2 depicts a schema of a division in any general network structure. For brief notation, the amounts of intermediate factor q in division k which is used or produced are respectively denoted by:

$$a_{qj}^k = \sum_{h:h \xrightarrow{q} k} z_{qj}^{(h,k)} \quad \forall q \in \tilde{N}_k \tag{2.2}$$

$$b_{qj}^k = \sum_{h:k \xrightarrow{q} h} z_{qj}^{(k,h)} \quad \forall q \in \tilde{M}_k$$

As we see them in Fig. 2.

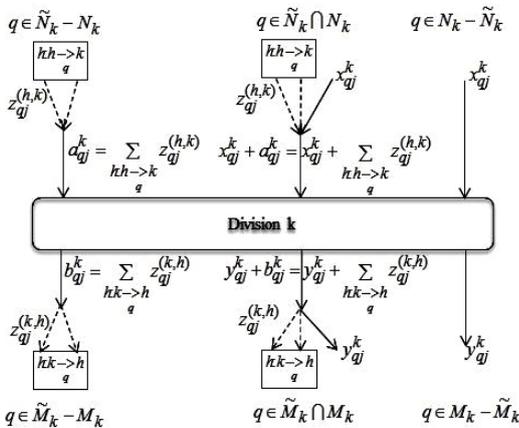


Figure 2: A division’s schema in a general network structure

Following Kao [13], a unique weight is considered for each factor whatever it is used. So w_q is used corresponding to q -th factor wherever it exists. So the divisional efficiency for general structures can be defined as below:

Definition 2.1 Absolute divisional efficiency score of division k in DMU j is defined by the ratio of weighted sum of outputs to the weighted sum of inputs (both main and intermediate factors):

$$AE.Div_k.DMU_j = \theta(j, k, \mathbf{w})$$

$$= \frac{\sum_{q \in M_k} w_q y_{qj}^k + \sum_{q \in \tilde{M}_k} w_q b_{qj}^k}{\sum_{q \in N_k} w_q x_{qj}^k + \sum_{q \in \tilde{N}_k} w_q a_{qj}^k}$$

$$= \frac{\sum_{q \in M_k} w_q y_{qj}^k + \sum_{q \in \tilde{M}_k} \sum_{h:k \xrightarrow{q} h} w_q z_{qj}^{(k,h)}}{\sum_{q \in N_k} w_q x_{qj}^k + \sum_{q \in \tilde{N}_k} \sum_{h:h \xrightarrow{q} k} w_q z_{qj}^{(h,k)}} \tag{2.3}$$

We denote $\theta(j, k, \mathbf{w})$ by $AE.Div_k.DMU_j$ where needed. Also the overall efficiency as a measure which deals with minimizing main input and maximizing main outputs can be defined as below:

Definition 2.2 Absolute overall efficiency score of DMU j is defined by the ratio of weighted summation of its main outputs to the weighted summation of its main inputs:

$$AE.DMU_j = \theta(j, \mathbf{w})$$

$$= \frac{\sum_{q \in M} \sum_{k:q \in M_k} w_q y_{qj}^k}{\sum_{q \in N} \sum_{k:q \in N_k} w_q x_{qj}^k} \tag{2.4}$$

where $N = \cup_k N_k$ and $M = \cup_k M_k$ are the sets of main inputs and main outputs respectively. Similarly $AE.DMU_j$ denotes to $\theta(j, \mathbf{w})$ where needed.

Note that since a main input or main output may be shared among division, their total amount for main DMU are $\sum_{k:q \in N_k} x_{qj}^k$ or $\sum_{k:q \in M_k} y_{qj}^k$ respectively. That's why these values are applied in the overall efficiency measure θ . Based on

the above definitions, the maximum absolute efficiency score of the under assessment DMU is assessed regarding s assigned to factors in divisions. So the model would be:

$$\max\{\theta(o, \mathbf{w})|\theta(j, k, \mathbf{w}) \leq 1 \ \forall j, k\} \quad (2.5)$$

If the model becomes linear by Charnes-Cooper transformation [25], the below model is obtained:

$$\begin{aligned} \theta_o^* &= \max \sum_{q \in M} \sum_{k:q \in M_k} w_q y_{qo}^k \\ s.t. \sum_{q \in N} \sum_{k:q \in N_k} w_q x_{qo}^k &= 1 \\ \sum_{q \in M_k} w_q y_{qj}^k + \sum_{q \in \tilde{M}_k} \sum_{h:k \xrightarrow{q} h} w_q z_{qj}^{(k,h)} \\ - \sum_{q \in N_k} w_q x_{qj}^k - \sum_{q \in \tilde{N}_k} \sum_{h:h \xrightarrow{q} k} w_q z_{qj}^{(h,k)} &\leq 0 \ \forall j, k \end{aligned} \quad (2.6)$$

Boloori et al. [21] introduced a unified dual pair of models for general network structures based on the above model. Since this multiplier model works for general network structures, we will use it as the base in this paper and by comparing the absolute and relative efficiency concepts; we try to ensure having at least one efficient DMU.

3 Relative Efficiency In Network DEA

As mentioned in the introduction, in network DEA models we may have not any 100% efficient DMU in contrast with conventional DEA models. Therefore we use the relative efficiency concept as well as its definition in traditional DEA models[23]. Podinovski [24] mentioned that the relative efficiency and absolute efficiency scores coincide on each other in traditional DEA model in the absence of any weight restriction constraint. This causes to have at least one efficient DMU in conventional DEA models. Here we define similar definitions for relative efficiency of a DMU in network structures. Then we will see that the relative efficiency score introduces the at

least one %100 efficient DMU, while the absolute efficiency score do not do so necessarily.

Definition 3.1 *Relative overall efficiency score of DMU j is defined by :*

$$RE.DMU_j = \frac{\theta(j, \mathbf{w})}{\max_l \theta(l, \mathbf{w})} \quad (3.7)$$

By the use of the definitions 2.1, 2.2 and 3.1 we could have two types of models for evaluating the overall efficiency of a DMU as below which the first one measures the absolute overall efficiency score and the second one measures the relative overall efficiency score:

$$\begin{aligned} \max \quad & AE.DMU_o \\ s.t. \quad & AE.Div_k.DMU_j \leq 1 \ \forall j, k \end{aligned} \quad (3.8)$$

$$\begin{aligned} \max \quad & RE.DMU_o \\ s.t. \quad & AE.Div_k.DMU_j \leq 1 \ \forall j, k \end{aligned} \quad (3.9)$$

Note that we can define the relative divisional efficiency score similarly:

Definition 3.2 *Relative divisional efficiency score of division k in DMU j is defined by:*

$$RE.Div_k.DMU_j = \frac{\theta(j, k, \mathbf{w})}{\max_l \theta(l, k, \mathbf{w})} \quad (3.10)$$

Due to the definition of the relative divisional efficiency score, $RE.Div_k.DMU_j \leq 1$ are evident constraints. So we did not add these inequalities as the constraints in models (3.8) and (3.9). Model (3.8) is the same as model (2.6) stated before. In model (3.9) we should maximize the relative overall efficiency of the under assessment DMU. In order to this, consider that $\frac{1}{\beta} = \max_l \theta(l, \mathbf{w})$. So the objective function is equal to $\beta\theta(o, \mathbf{w})$. Also by the definition of $\frac{1}{\beta}$, we have $\theta(j, \mathbf{w}) \leq \frac{1}{\beta} \ \forall j$. Consequently, by maximizing $\beta\theta(o, \mathbf{w})$ in the objective function and also including $\beta\theta(j, \mathbf{w}) \leq 1 \ \forall j$ constraints, model (3.9) becomes as below:

$$\begin{aligned} & \max_{\mathbf{w}} \beta \theta(o, \mathbf{w}) \\ & s.t : \beta \theta(j, \mathbf{w}) \leq 1 \end{aligned} \tag{3.11}$$

$$\begin{aligned} & \theta(j, k, \mathbf{w}) \leq 1 \quad \forall j, k \\ & \beta \geq \epsilon \end{aligned}$$

Here $\frac{1}{\beta}$ is considered as the minimum upper bound for $\theta(j, \mathbf{w})$'s and it is positive. Equivalently β is greater than or equal to an Archimedean value ϵ . Substituting definitions 2.1, 2.2 and 3.1 in the above model and using Charnes-Cooper transformation yield to a NLP model as is brought in (3.12). So we have the following theorem:

Theorem 3.1 *The relative overall efficiency score of the under assessment DMU in a network structure obtained by model (3.9) is equivalently obtained by model (3.12):*

$$\begin{aligned} & \max \beta \sum_{q \in M} \sum_{k:q \in M_k} w_q y_{qo}^k \\ & s.t. \sum_{q \in N} \sum_{k:q \in N_k} w_q x_{qo}^k = 1 \\ & \beta \sum_{q \in M} \sum_{k:q \in M_k} w_q y_{qj}^k \leq \sum_{q \in N} \sum_{k:q \in N_k} w_q x_{qj}^k \quad \forall j \\ & \sum_{q \in M_k} w_q y_{qj}^k + \sum_{q \in \tilde{M}_k} \sum_{h:k \xrightarrow{q} h} w_q z_{qj}^{(k,h)} \\ & - \sum_{q \in N_k} w_q x_{qj}^k - \sum_{q \in \tilde{N}_k} \sum_{h:h \xrightarrow{q} k} w_q z_{qj}^{(h,k)} \leq 0 \quad \forall j, k \\ & w_q \geq 0, \beta \geq \epsilon \end{aligned} \tag{3.12}$$

Model (3.8) or equivalently (2.6) has different results in compare with model (3.9) or equivalently (3.12) as the following Lemma indicates:

Lemma 3.1 *The relative efficiency score in model (3.12) (or equivalently 3.9) is greater than or equal to the absolute efficiency score in model (2.6) (or equivalently 3.8).*

Proof. It is enough to prove that the constraints $\theta(k, j, \mathbf{w}) \leq 1 \quad \forall j, k$ which are common in both models imply that $\theta(j, \mathbf{w}) \leq 1 \quad \forall j$. Then, since $\frac{1}{\beta} = \max_j \theta(j, \mathbf{w})$ and $\theta(j, \mathbf{w}) \leq$

$1 \quad \forall j$ we would have $\frac{1}{\beta} \leq 1$ and consequently $\frac{\theta(j, \mathbf{w})}{1/\beta} \leq \theta(j, \mathbf{w})$ which clearly indicates that $RE.DMU_j \geq AE.DMU_j$. Now to see that constraints $\theta(k, j, \mathbf{w}) \leq 1 \quad \forall j, k$ imply $\theta(j, \mathbf{w}) \leq 1 \quad \forall j$, consider their equivalent constraints:

$$\begin{aligned} & \sum_{q \in M_k} w_q y_{qj}^k + \sum_{q \in \tilde{M}_k} \sum_{h:k \xrightarrow{q} h} w_q z_{qj}^{(k,h)} \\ & - \sum_{q \in N_k} w_q x_{qj}^k - \sum_{q \in \tilde{N}_k} \sum_{h:h \xrightarrow{q} k} w_q z_{qj}^{(h,k)} \leq 0 \quad \forall j, k \end{aligned}$$

By the summation of these inequalities for all divisions, we will have the following inequality:

$$\begin{aligned} & \sum_k \sum_{q \in M_k} w_q y_{qj}^k - \sum_k \sum_{q \in N_k} w_q x_{qj}^k \\ & + \left[\sum_k \sum_{q \in \tilde{M}_k} \sum_{h:k \xrightarrow{q} h} w_q z_{qj}^{(k,h)} \right. \\ & \left. - \sum_k \sum_{q \in \tilde{N}_k} \sum_{h:h \xrightarrow{q} k} w_q z_{qj}^{(h,k)} \right] \leq 0 \quad \forall j \end{aligned}$$

The value in the bracket demonstrates the production of factor q as intermediate factor in all divisions minus its consumption in all division (multiplied by its price). So the bracket value clearly equals to zero. The remaining items imply that:

$$\frac{\sum_k \sum_{q \in M_k} w_q y_{qj}^k}{\sum_k \sum_{q \in N_k} w_q x_{qj}^k} \leq 1 \quad \forall j$$

This clearly indicates that $\theta(j, \mathbf{w}) \leq 1 \quad \forall j$ and by the descriptions stated at the beginning of the proof, we have $RE.DMU_j \geq AE.DMU_j$.

In fact, In contrast with conventional DEA models, the relative overall efficiency score and the absolute efficiency score for a DMU are different in network structures. Clearly, the definition of the relative efficiency ensures the existence of at least one efficient DMU. The relative efficiency score in model (3.9) (or 3.12) also may change the ranking of DMUs in compare with model (2.6) (or 3.8) too, as we will see in the next section.

4 Illustrative Example

In this section an example of five DMUs discussed in [13] are considered. We see that if we use model (3.8) (or equivalently 2.6), we do not have

any efficient DMU. But by the use of model (3.9) (or equivalently 3.12) we have at least one efficient DMU (in this example we have 2 efficient DMUs). Also we see that, the relative efficiency scores in model (3.9) are greater than or equal to the absolute efficiency scores obtained in model (3.8) which verifies again the lemma proved before. Consider five DMUs in a network structure as below:

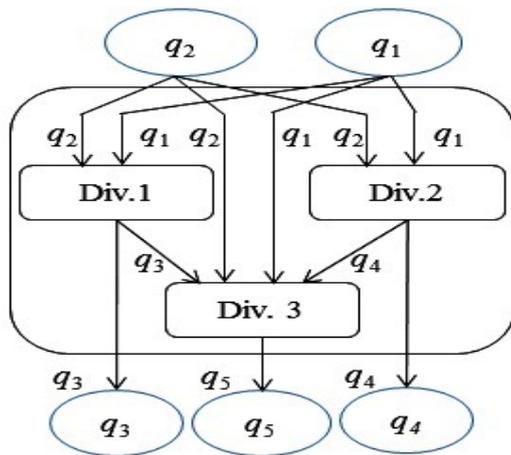


Figure 3: Network structure used in a hypothetical example in [13]

The data set related to this network structure is as Table 1.

We applied models (3.12) and (2.6) and obtained the relative and absolute efficiency scores for these five DMUs. The efficiency measures and ranking of DMUs in each efficiency measure are reported in Table 2.

As discussed later, the relative efficiency score and absolute efficiency scores are different in network framework in contrast with conventional DEA models. In model (3.12) there exists at least one efficient DMU (i.e. Relative overall efficient DMU). Also ranking orders of DMUs are changed as reported in above table. The relative efficiency scores are greater than or equal to absolute efficiency scores.

5 Conclusion

In network DEA models, there exist some pitfalls. We do not have efficient DMUs as well as conventional DEA models. Recently Boloori et al. [21] represented a dual pair of network DEA models to remove the duality pitfall in network DEA. In this paper, relative efficiency model was

investigated based on that dual pair of network DEA models. This model introduces at least one efficient DMU. The Lagrangian dual form of this model is represented for future studies.

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Table 1: Data set related to 5 DMUs in network structure of Fig. 3

	J_1	J_2	J_3	J_4	J_5
$div_1.q_1$	3	2	3	4	5
$div_1.q_2$	5	3	4	6	6
$div_2.q_1$	4	2	5	5	5
$div_2.q_2$	3	1	3	5	4
$div_3.q_1$	4	3	3	5	4
$div_3.q_2$	6	3	7	3	5
$div_1.div_3.q_3$	2	1	1	1	1
$div_2.div_3.q_4$	1	1	1	1	2
$div_1.q_3$	2	1	1	2	3
$div_2.q_4$	2	2	1	3	2
$div_3.q_5$	1	1	2	1	3

Table 2: Relative and Absolute efficiency scores and DMUs' ranking

DMU	J_1	J_2	J_3	J_4	J_5
$RE.DMU_j$	0.94	1	0.85	0.89	1
$RankingRE.DMU_j$	2	1	4	3	1
$AE.DMU_j$	0.44	0.6	0.5	0.41	0.76
$RankingAE.DMU_j$	4	2	3	5	1

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