Available online at http://ijim.srbiau.ac.ir/
Int. J. Industrial Mathematics (ISSN 2008-5621)
Vol. 6, No. 1, 2014 Article ID IJIM-00438, 9 pages
Research Article

FGP approach to multi objective quadratic fractional programming problem

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Abstract

Multi objective quadratic fractional programming (MOQFP) problem involves optimization of several objective functions in the form of a ratio of numerator and denominator functions which involve both contains linear and quadratic forms with the assumption that the set of feasible solutions is a convex polyhedral with a finite number of extreme points and the denominator part of each of the objective functions is non-zero in the constraint set. In this paper, we extend the procedure as suggested by Lachhwani (Proc. Nat. Acad. Sci. India, 82(4), 317-322) based on fuzzy goal programming approach for the solution of multi objective quadratic fractional programming (MOQFP) problem. The proposed technique is simple, efficient and requires less computational work. In the proposed FGP model formulation, corresponding objectives of equivalent multi objective programming problem are transformed into fuzzy goals (membership functions) by means of assigning an aspiration level to each of them and suitable membership function is defined for each objectives. Then achievement of the highest membership value of each of fuzzy goals is formulated by minimizing the sum of negative deviational variables. The proposed methodology is illustrated with numerical example in order to support the proposed methodology.

Keywords : Multiple objective quadratic fractional programming; Fuzzy Goal programming; membership function; Negative deviational variable.

1 Introduction

Multiobjective programming models involve optimization of several conflicting objective functions with constraints. Wallenius [23], Zimmermann [28, 29], Yager [25], Hanan [9], Narasimhan [17], Rubin and Narasimhan [20], Ying-Yung [26], Chanas [5], Rommelfanger [21], Gupta and Chakraborty [6, 8] and many researchers used and modified the concept of multiobjective decision making problems and also discussed different approaches to tackle the multiobjective programming problem. Balbas and Galperin et al. [3] gave a sensitivity analysis in multi objective optimization. Jain [12] suggested a new form of gauss elimination technique for inequalities for solving separable non linear programming problem. Ansari and Zhiani [2] presented a new method for solving multi-objective linear bilevel multi follower programming prob-

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lem. Yan and Wei et al. [27] constructed an efficient solution structure of multi objective linear programming. Jain and Lachhwani [10] considered multiobjective programming problem with fuzzy relational equations. Afterwards, Jain and Lachhwani [11] obtained the solution of multi objective linear fractional programming problem by converting it into fuzzy programming problem.

Numerous methods for multi objective optimization problems have been suggested in the literature. Each method appears to have some advantages as well as disadvantages. In the context of each application, some of the methods seem more appropriate than others. However, the issue of choosing a proper method in a given context is still a subject of active research. A number of researchers have worked for fuzzy mathematical programming problem using goal programming approach like Pal and Moitra et al. [18] suggested a goal programming procedure for fuzzy multiobjective linear fractional programming problem. Shahroudi and Soltani [22] proposed use of goal programming for setting in Irans autoindustry. Chao-Fang et al. [7] proposed a generalized varying domain optimization method using fuzzy goal programming for multi objective optimization problem with priorities. Pramanik and Roy [19] gave a procedure for solving multi level programming problems in a large hierarchical decentralized organization throu- gh linear fuzzy goal programming approach.
Ibrahim A. Baky [4] presented fuzzy goal programming (FGP) algorithm for solving decentralised bi-level multi objective (DBL-MOP) problems with a single decision maker at the upper level and multiple decision makers at the lower level. Li and Hu [13] proposed a satisfying optimization method based on goal programming for fuzzy multi objective optimization problem with the aim of achieving the higher desirable satisfying degree. But a very few of the researchers have considered multiobjective quadratic fractional programming problem specifically. This situation inspired us to propose FGP approach for the solution of multi-objective quadratic fractional programming (MOQFP) problem.

A multi objective quadratic fractional programming (MOQFP) problem seeks to optimize more than one objective function in the form of a ratio in which denominator and numerator both contains linear and quadratic forms. We assume that the set of feasible solutions is a convex polyhedral with a finite number of extreme points and the denominator of the objective functions is non-zero in the constraint set. However, Wolfe [24] gave a simplex method to solve quadratic programming problem with single objective function with the assumption that the set of feasible solutions is a convex polyhedral with a finite number of extreme points. Mishra and Ghosh [16] proposed an interactive fuzzy programming method for obtaining a satisfactory solution to a bi-level quadratic fractional programming problem. Recently, Ammar [1] considered multiobjective quadratic programming problem having fuzzy random coefficients matrix in the objectives and constraints and the decision vector are fuzzy pseudo random variables. Lachhwani [14] suggested fuzzy goal programming approach to solve multi objective quadratic programming problem. The aim of this paper is to extend FGP approach suggested by Lachhwani [14] to solve MOQFP problem and present simple, efficient method which requires less computational works. In the proposed FGP model formulation, firstly MOQFPP is transformed into another equivalent multi objective non linear programming problem and equivalence between them is established. Secondly the objectives are transformed into fuzzy goals (membership functions) by means of assigning an aspiration level to each of them and suitable membership function is defined for each objectives. Then achievement of the highest membership value of each of fuzzy goals is formulated by minimizing the sum of negative deviational variables. The paper is organised as follows: In Section 2, we discuss formulation of MOQFPP and related notations. Its equivalence with multi objective non linear programming problem in context of compromise optimal solution is also established in the same section. In next Section, we discuss proposed FGP approach to tackle MOQFPP and formulate mathematical models related to it. To illustrate the proposed methodology, numerical example is considered in Section 4. Concluding remarks are given in the last section.

2 Problem Formulation

The general mathematical format of multi objective quadratic fractional programming (MOQFP) problem can be stated as:

$$Max.\{Z_1(X), Z_2(X), ..., Z_k(X)\} \tag{2.1}$$

where

$$Z_1(X) = \frac{(C_iX + \alpha_i)(\lambda_i C_i X + \beta_i)}{(D_iX + \gamma_i)(\mu_i D_i X + \delta_i)}$$

$$\forall i = 1, ..., k$$

Subject to,

$$X \in S = \left\{ X \in R^n \mid AX \begin{pmatrix} \leq \\ = \\ \geq \end{pmatrix} b, \right.$$

$$\left. X \geq 0, \lambda_i, \mu_i \geq 0, b \in R^m, \forall i = 1, ..., k \right\}$$

Here C_i and $D_i (i = 1, ..., k)$ are row vectors with n-components, α_i, β_i, γ_i, δ_i, are scalars, X and b are column vectors with n and m components respectively. It is assumed that $D_iX + \gamma_i > 0$ and $\mu_i D_i X + \delta_i > 0$ $(i = 1, ..., k)$ for all $X \in S$. Now we consider another equivalent multiobjective non linear programming problem as follows:

$$Max.\{Z_1(X), Z_2(X), ..., Z_k(X)\} \tag{2.2}$$

$$where \quad Z_1(X) = (C_iX + \alpha_i)(\lambda_i C_i X + \beta_i) Y_1$$

$$\forall i = 1, ..., k$$

Subject to,

$$X \in S = \left\{ X \in R^n \mid AX \begin{pmatrix} \leq \\ = \\ \geq \end{pmatrix} b, \right.$$

$$\left. X \geq 0, b \in R^m \right\},$$

$$(D_iX + \gamma_i)(\mu_i D_i X + \delta_i) Y_i = 1$$

$$and \quad \lambda_i, \mu_i \geq 0, Y_i > 0 \quad \forall i = 1, ..., k$$

Where problem (2.2) is obtained from (2.1) by the transformation $Y_i = 1/(D_iX + \gamma_i)$ $(\mu_i D_i X + \delta_i)$ with the equality constraints $Y_i(D_iX + \gamma_i)(\mu_i D_i X + \delta_i) = 1$. Now we prove the equivalence between MOQFPP (2.1) and multiobjective non-linear programming problem (2.2) in continuation of following related definitions as:

Definition 2.1 *An ideal solution (ideal point) X^* is the finite optimal solution to the single objective programming problem i.e.*

$$Maximize Z$$

$$X \in S = \left\{ X \in R^n \mid AX \begin{pmatrix} \leq \\ = \\ \geq \end{pmatrix} b, \right.$$

$$\left. X \geq 0, b \in R^m \right\},$$

Definition 2.2 *$X^0 \in S$ is an efficient solution to problem (2.1) - (2.2) if and only if there exists no other $X \in S$ such that $Z_i \geq Z_i^0$ for all i=1,2,...,k and $Z_i > Z_i^0$ for at least one i. For our purpose, we define ideal solution (ideal point) of single objective and compromise efficient solution for multi objective programming problems.*

Definition 2.3 *For problem (2.1), a compromise optimal solution [14] is an efficient solution selected by the decision maker (DM) as being the best solution where the selection is based on the DM's explicit or implicit criteria.*

Zeleny [30] as well as most authors describes the act of finding a compromise optimal solution to problem as "...an effort or emulate the ideal solution as closely as possible". Our FGP model for determining compromise optimal (efficient) solution based on the finding of the totality or subset of efficient solutions with the DM, then choosing one best solution on some explicit or implicit algorithm.

Theorem 2.1 *If (2.1) reaches at a compromise optimal solution $\overline{X} = \overline{X}^*$. Then (2.2) also reaches at same compromise optimal solution $\overline{X}^*, \overline{Y_i}^* \in S$ and the values of objective functions at these points are equal.*

Proof: *Let $\overline{X}^*$ be a compromise optimal solution of problem (2.1). It follows that corresponding values of $\overline{Y_i}^* = 1/(D_i\ \overline{X}^* + \gamma_i)\ \ (\mu_i D_i\ \overline{X}^* + \delta_i)$ can be obtained using values of $\overline{X}^*$ in new introduced constraints. This implies that (2.2) also reaches at some compromise optimal solution $(\overline{X}^*, \overline{Y_i}^*)$ and the values of the objective functions at problem (2.1) and (2.2) are equal as:*

$$Z_i(\overline{X}^*) = \frac{(C_i\ \overline{X}^* + \alpha_i)(\lambda_i C_i\ \overline{X}^* + \beta_i)}{(D_i\ \overline{X}^* + \gamma_i)(\mu_i D_i\ \overline{X}^* + \delta_i)}$$

$$= (C_i\ \overline{X}^* + \alpha_i)(\lambda_i C_i\ \overline{X}^* + \beta_i)\ \overline{Y_i}^*$$

Now, in the field of fuzzy programming, the fuzzy goals are characterized by their associated membership functions. The membership function for the i th fuzzy goal can be defined according to Gupta and Chakraborty [8] as:

$$\mu_i(d_i(X)) = \begin{cases} 0 & if \quad d_i(X) \geq p \\ \frac{p - d_i(X)}{p} & if \quad 0 < d_i(X) < p \\ 1 & if \quad d_i(X) \leq 0 \end{cases} \tag{2.3}$$

Where the distance function d_i with unit weight as:

$$d_i(X) = \mid \overline{Z_i} - Z_i(X) \mid, \quad \forall i = 1, ..., k$$

This distance depends upon X. At $X = \overline{X}$ (ideal point in X-space), $d_i = 0$ and at $X = \underline{X}$ (nadir point in X-space), $Z_i(X) = \underline{Z_i}$, we get the maximum value of $d_i(X)$ as:

$$\overline{d_i} = \mid \overline{Z_i} - \underline{Z_i} \mid \quad \forall i = 1, ..., k \tag{2.4}$$

$$And \quad p = sup \quad \{\overline{d_i}\} \quad \forall i = 1, ..., k \tag{2.5}$$

Also in the fuzzy decision making environment, the achievement of the objective goals to their aspired levels to the extend possible is actually represented by the possible achievement of their respective membership values to the highest degree. Regarding the presently available procedures, a FGP approach seems to be most appropriate for multi objective programming problem.

3 Fuzzy Goal Programming Formulation

In fuzzy goal programming approaches, the highest degree of each of membership functions is 1. So, as in Mohamed [15] for the defined membership function in (2.3), the flexible membership goals with the aspired level 1 can be expressed as:

$$\frac{p - d_i(X)}{p} + d_i^- - d_i^+ = 1$$

$$i.e. \quad -\overline{Z_i} + Z_i(X) + pd_i^- - pd_i^+ = 0,$$

$$\forall i = 1, ..., k \tag{3.6}$$

where $d_i^-(\geq 0)$ and $d_i^+(\geq 0)$ with $d_i^- d_i^+ = 0$ represent the under and over deviational variables respectively from the aspired levels. It can be easily realized that the membership goals in expression (3.6) are inherently non linear equation and this may reduce computational difficulties in the solution process. The i^{th} membership goal with aspired level 1 can be presented as:

$$-\overline{Z_i} + (C_i X + \alpha_i)(\lambda_i C_i X + \beta_i) Y$$

$$+ pd_i^- - pd_i^+ = 0, \tag{3.7}$$

It may be noted that any over deviation from a fuzzy goal indicate the full achievement of the membership value. However, for model simplification the expression (3) can be considered as a general form of goal expression of the above stated membership goals. It may be noted that when a membership goal is fully achieved, and when its achievement is zero, $d_i^+ = 1$ are found in the solution. Now, if the most widely used and simplest version of GP (i.e. minsum GP) is introduced to formulate the model of the problem (2.1) under consideration, then GP model formulation becomes:

Model I Find X so as to

$$Minimize \quad \chi = \sum_{i=1}^{k} w_i d_i^- \tag{3.8}$$

Subject to,

$$-\overline{Z_i} + (C_i X + \alpha_i)(\lambda_i C_i X + \beta_i) Y_i$$

$$+pd_i^- - pd_i^+ = 0$$

$$X \in S = \left\{ X \in R^n \mid AX \begin{pmatrix} \leq \\ = \\ \geq \end{pmatrix} b, \right.$$

$$\left. X \geq 0, Y > 0, b \in R^m \right\},$$

$$(D_iX + \gamma_i)(\mu_i D_i X + \delta_i) Y_i = 1$$

$$and \qquad \lambda_i, \mu_i \geq 0, d_i^-, d_i^+ \geq 0, \forall i = 1, ..., k$$

where χ represents the fuzzy achievement function consisting of the weighted under deviational variables where the numerical weights $w_i \geq 0, (\forall i = 1, ..., k)$ represent the relative importance of achieving the aspired level of the respective fuzzy goals subject to the constraints in the decision making situation. To assess the relative importance of the fuzzy goals properly, the non zero weighted scheme suggested by Mohamed [15] can be used to assign the values to $w_i(\geq 0), i = 1, ..., k$. In the present formulation w_i can be determined as:

$$w_i = \frac{1}{\overline{Z_i} - \underline{Z_i}}$$

The above model can also be rewritten as:

<u>Model II</u> Find X so as to

$$Minimize \quad \chi = \sum_{i=1}^{k} d_i^- \tag{3.9}$$

Subject to,

$$-\overline{Z_i} + (C_iX + \alpha_i)(\lambda_i C_i X + \beta_i) Y_i$$

$$+pd_i^- - pd_i^+ = 0$$

$$X \in S = \left\{ X \in R^n \mid AX \begin{pmatrix} \leq \\ = \\ \geq \end{pmatrix} b, \right.$$

$$\left. X \geq 0, Y > 0, b \in R^m \right\},$$

$$(D_iX + \gamma_i)(\mu_i D_i X + \delta_i) Y_i = 1$$

$$and \qquad \lambda_i, \mu_i \geq 0, d_i^-, d_i^+ \geq 0, \forall i = 1, ..., k$$

In model II the numerical weights are taken as unity.

<u>Model III</u> Find X so as to

$$Minimize \quad \chi = \sum_{i=1}^{k} d_i^- \tag{3.10}$$

Subject to,

$$-\overline{Z_i} + (C_iX + \alpha_i)(\lambda_i C_i X + \beta_i) Y_i + pd_i^- \geq 0$$

$$X \in S = \left\{ X \in R^n \mid AX \begin{pmatrix} \leq \\ = \\ \geq \end{pmatrix} b, \right.$$

$$\left. X \geq 0, Y > 0, b \in R^m \right\},$$

$$(D_iX + \gamma_i)(\mu_i D_i X + \delta_i) Y_i = 1$$

$$and \qquad \lambda_i, \mu_i \geq 0, d_i^-, d_i^+ \geq 0, \forall i = 1, ..., k$$

However, the above models involve constrains quadratic in nature but the model I, II and III can be easily solved using non linear techniques or software packages like LINGO, TORA etc. Following the above discussion, we can construct the proposed FGP algorithm for solution of MOQFPP problem as:

Step 1. Solve each Convert MOQFPP (2.1) into multiobjective non linear programming problem (2.2).

Step 2. Solve each objective function individually with given set of constraints and calculate maximum and minimum of each objective function under the given constraints.

Step 3. Calculate the distance

$$\overline{d_i} = \mid \overline{Z_i} - \underline{Z_i} \mid .$$

Step 4. Calculate $p = sup \{\overline{d_i}\}$

Step 5. Formulate the FGP model I, II or III according to (3.8), (3.9) and (3.10) respectively.

Step 6. Solve the FGP model.

Step 7. If solution is satisfactory, then STOP. Otherwise modify the values of weights of negative deviational variables.

4 Numerical Example

The following example is illustrated to show the proposed methodology for MOQFPP:

Table 1: Comaprison between FGP model II and III for Example 4.1

Numerical Example 4.1 Proposed FGP model	(x_1, x_2, y_1, y_2)	$\mu_1(d_1(X)), \mu_2(d_2(X))$
Model II	(0.7999, 0.0800,0.0059,0.0029)	(0.82224,0.73334)
Model III	(0.8000,0.0799,0.0059,0.0029)	(0.82217,0.73325)

Example 4.1

$$Maximize \quad \{Z_1(X), Z_2(X)\}$$

where

$$Z_1(X) = \frac{(2x_1 + 20x_2 + 12)(x_1 + 10x_2 + 17)}{(-2x_1 - 5x_2 + 15)(2x_1 + 5x_2 + 11)}$$

$$Z_2(X) = \frac{(2x_1 + 20x_2 + 12)(3x_1 + 30x_2 + 51)}{(-4x_1 - 10x_2 + 30)(2x_1 + 5x_2 + 11)}$$

$$subject\ to, \quad x_1 + 15x_2 \leq 2$$

$$3x_1 + 20x_2 \leq 4, \ and\ x_1, x_2 \geq 0$$

Step 1: Its equivalent multiobjective non linear programming problem can described as:

$$Maximize \quad \{Z_1(X, y_1), Z_2(X, y_2)\}$$

where

$$Z_1(X, y_1) = (2x_1 + 20x_2 + 12)(x_1 + 10x_2 + 17)y_1$$

$$Z_2(X, y_2) = (2x_1 + 20x_2 + 12)(3x_1 + 30x_2 + 51)y_2$$

subject to,

$$(-2x_1 - 5x_2 + 15)(2x_1 + 5x_2 + 11)y_1 = 1$$

$$(-4x_1 - 10x_2 + 30)(2x_1 + 5x_2 + 11)y_2 = 1$$

$$x_1 + 15x_2 \leq 2$$

$$3x_1 + 20x_2 \leq 4$$

$$and \qquad x_1, x_2 \geq 0 \quad y_1, y_2 > 0$$

Step 2: To formulate fuzzy membership function (2.3), we calculate the value of each objective function individually using non linear technique as: $\overline{Z_1} = 1.67289, \underline{Z_1} = 1.23636$, $Z_2 = 2.50934, \underline{Z_2} = 1.85454$,

Step 3: Calculating:

$$\overline{d_i} = \mid \overline{Z_i} - \underline{Z_i} \mid \quad \forall i = 1, ..., k$$

$$\overline{d_1} = \mid \overline{Z_1} - \underline{Z_1} \mid = 0.43653$$

$$\overline{d_2} = \mid \overline{Z_2} - \underline{Z_2} \mid = 0.6548$$

Step 4: $p = sup\ \{\overline{d_i}\} = 0.43653$

Step 5: Thus the equivalent FGP model formulations are obtained as:

Model II Find $X(x_1, x_2, y_1, y_2)$ so as to Minimize $\chi = (d_1^- + d_2^-)$

Subject to,

$$(2x_1 + 20x_2 + 12)(x_1 + 10x_2 + 17)y_1$$

$$+0.43653d_1^- - 0.43653d_1^+ - 1.67289 = 0$$

$$(2x_1 + 20x_2 + 12)(3x_1 + 30x_2 + 51)y_2$$

$$+0.43653d_2^- - 0.43653d_2^+ - 2.50934 = 0$$

$$(-2x_1 - 5x_2 + 15)(2x_1 + 5x_2 + 11)y_1 = 1$$

$$(-4x_1 - 10x_2 + 30)(2x_1 + 5x_2 + 11)y_2 = 1$$

$$x_1 + 15x_2 \leq 2$$

$$3x_1 + 20x_2 \leq 4$$

$$and \quad x_1, x_2, d_1^-, d_2^-, d_1^+, d_2^+ \geq 0, y_1, y_2 > 0$$

Solving the above problem using non linear techniques or software package (as shown in Figure (1) and Figure (2), the compromise optimal solution obtained as:

$$x_1 = 0.7999, x_2 = 0.08, y_1 = 0.005917,$$

$$y_2 = 0.002958, d_1^- = 0, d_2^- = 0, d_1^+ = 0, d_2^+ = 0,$$

with achieved value of membership functions:

$$\mu_1(d_1(X)) = 0.82224, \mu_2(d_2(X)) = 0.73334.$$

Model III Find $X(x_1, x_2, y_1, y_2)$ so as to Minimize $\chi = (d_1^- + d_2^-)$

Subject to,

$$(2x_1 + 20x_2 + 12)(x_1 + 10x_2 + 17)y_1$$

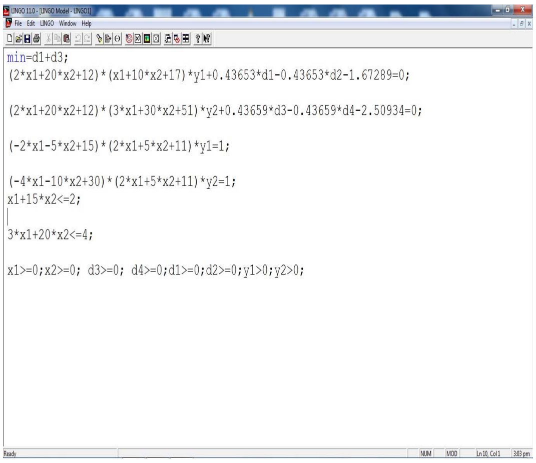

Figure 1: Description of FGP model II (for Example 4.1) in LINGO 11.0 (trial version)

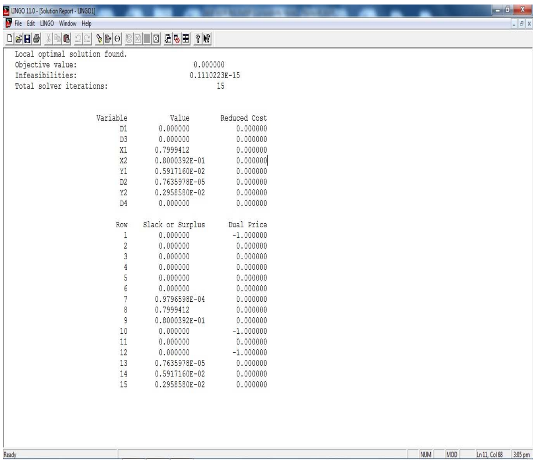

Figure 2: Solution of FGP model II (for Example 4.1) in LINGO 11.0 (trial version).

$$+0.43653d_1^- - 1.67289 \geq 0$$

$$(2x_1 + 20x_2 + 12)(3x_1 + 30x_2 + 51)y_2$$

$$+0.43653d_2^- - 2.50934 \geq 0$$

$$(-2x_1 - 5x_2 + 15)(2x_1 + 5x_2 + 11)y_1 = 1$$

$$(-4x_1 - 10x_2 + 30)(2x_1 + 5x_2 + 11)y_2 = 1$$

$$x_1 + 15x_2 \leq 2$$

$$3x_1 + 20x_2 \leq 4$$

$$and \quad x_1, x_2, d_1^-, d_2^- \geq 0, y_1, y_2 > 0$$

Solving the above problem using non linear techniques or software package (as shown in Figure (3) and Figure (4)), the compromise optimal solution obtained as:

$$x_1 = 0.8, x_2 = 0.07999, y_1 = 0.005917,$$

$$y_2 = 0.002958, d_1^- = 0, d_2^- = 0.$$

with achieved value of membership functions:

$$\mu_1(d_1(X)) = 0.82217, \mu_2(d_2(X)) = 0.73325.$$

Here a comparison table (in Table 1) between proposed FGP models is also given below which shows that the FGP models are close to one another in terms of achieved values of membership functions.

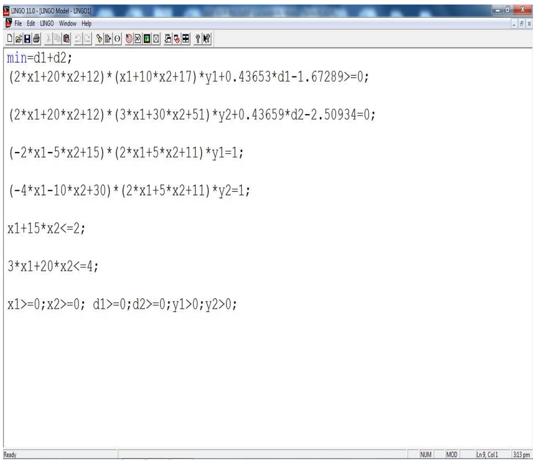

Figure 3: Description of FGP model III (for Example 4.1) in LINGO 11.0 (trial version).

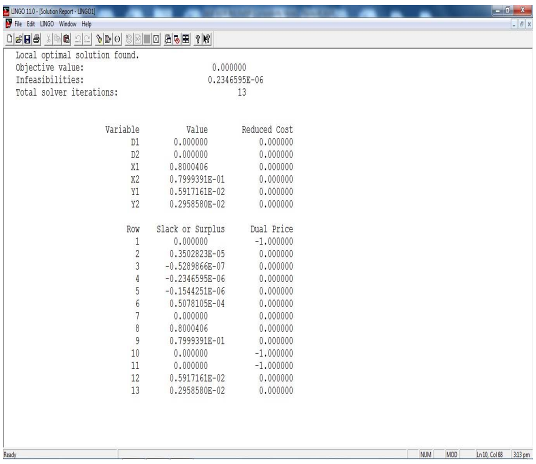

Figure 4: Solution of FGP model III (for Example 4.1) in LINGO 11.0 (trial version).

5 Conclusion

An effort has been made to extend FGP approach to solve MOQFP problem. The proposed methodology yields a compromise optimal solution of MOQFP problem with a higher degree of satisfaction. The main advantage of the proposed technique is that it is simple, efficient and requires less computational work as it finally converts MOQFP problem into non linear programming problem which can be easily solved using non linear techniques or software packages like LINGO, CPLEX etc. The proposed methodology can be further extended to solve multi objective integer programming problem as future research work.

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