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Research Article

On convergence of homotopy analysis method to solve the Schrodinger equation with a power law nonlinearity

M. A. Fariborzi Araghi *†, S. Naghshband ‡§

Abstract

In this paper, the homotopy analysis method (HAM) is considered to obtain the solution of the Schrodinger equation with a power law nonlinearity. For this purpose, a theorem is proved to show the convergence of the series solution obtained from the proposed method. Also, an example is solved to illustrate the efficiency of the mentioned algorithm and the h-curve is plotted to determine the region of convergence.

Keywords : Schrodinger equation; Power law nonlinearity; Homotopy analysis mehod (HAM); Convergence.

1 Introduction

The Schrodinger equation is one of the important partial differential equations with many applications in physics and chemistry. In recent years, some methods were introduced in order to find the explicit or approximate solution of this equation in linear or nonlinear case such as finite difference method [16], variational iteration method [17], Adomian decomposition method [5, 12], homotopy perturbation method [5, 6, 8, 12], differential transform method [11] and homotopy analysis method [4, 7]. One of the considerable cases of the nonlinear Schrodinger equations is power law nonlinearity which was studied by Wazwaz in [18] by using the tanh-coth method and was solved by variational iteration method in [17]. In this work, we consider Schrodinger equation with a power law nonlinearity of the form

$$i\frac{\partial w}{\partial t} + a\frac{\partial^2 w}{\partial x^2} + b\frac{\partial^4 w}{\partial x^4} + c|w|^{2n}w = 0, n \geq 2,$$

$$w(x,0) = f(x), i^2 = -1, \qquad (1.1)$$

where a, b, c are real constants and $w = w(x,t)$ is a complex unknown function [18].

The homotopy analysis method (HAM) which was introduced by Liao [9, 10] is an effective and powerful method to find the analytical or approximate solution of nonlinear problems with complicated nonlinearity [1, 2, 3, 13, 14, 15].

*Corresponding author. m_ fariborzi@iauctb.ac.ir
†Department of Mathematics, Islamic Azad University, Central Tehran Branch, P. O. Box 13185.768, Tehran, Iran.
‡Department of Mathematics, Islamic Azad University, Central Tehran Branch, P. O. Box 13185.768, Tehran, Iran.
§Department of Mathematics, Islamic Azad University, West Tehran Branch, Tehran, Iran.

In the present paper, we propose the convergence of HAM in order to obtain the explicit solution of Eq. 1.1. In this case, a theorem is proved which illustrates the convergence of the method. In Section 2, some preliminaries are given. In Section 3, the HAM is introduced to solve Eq. 1.1 and in Section 4, the convergence theorem is proved. Finally in Section 5, an example is solved to illustrate the importance of the method.

2 Preliminaries

We consider the following differential equation:

$$N[w(x,t)] = 0,$$

where N is a nonlinear operator, x and t denote the independent variables and w is an unknown function. In the HAM, the zeroth-order deformation equationis is constructed as:

$$(1-q)L[\Phi(x,t,q) - w_0(x,t)] = qhH(x,t)N[\Phi(x,t,q)], \tag{2.2}$$

where $q \in [0,1]$ is the embedding parameter, $h \neq 0$ is an auxiliary parameter, L is an auxliary linear oprator and $H(x,t)$ is an auxiliary function. $\Phi(x,t,q)$ is an unknown function and $w_0(x,t)$ is an initial guess of $w(x,t)$. It is clear, if $q=0$ and $q=1$ then:

$$\Phi(x,t,0) = w_0(x,t), \quad \Phi(x,t,1) = w(x,t),$$

respectively. Therefore, when q increases from 0 to 1, the solution $\Phi(x,t,q)$ varies from $w_0(x,t)$ to the exact solution $w(x,t)$. By Taylor's theorem, it can be expanded $\Phi(x,t,q)$ in a power series of the embedding parameter q as:

$$\Phi(x,t,q) = w_0(x,t) + \sum_{m=1}^{\infty} w_m(x,t)q^m, \tag{2.3}$$

where

$$w_m(x,t) = \frac{1}{m!}\frac{\partial^m \Phi(x,t,q)}{\partial q^m}|_{q=0}. \tag{2.4}$$

Let the initial guess $w_0(x,t)$, the auxiliaray linear operator L, the nonzero auxiliaray parameter h and the auxiliaray function $H(x,t)$ be properly chosen such that the power series 2.3 converges at $q=1$, then,

$$w(x,t) = w_0(x,t) + \sum_{m=1}^{\infty} w_m(x,t), \tag{2.5}$$

which must be the solution of the original nonlinear equation. Now, we define the following set of vectors:

$$\vec{w_n} = \{w_0(x,t), w_1(x,t), \ldots, w_n(x,t)\}. \tag{2.6}$$

By differentiating the zeroth order deformation Eq. 2.2 m times with respect to the embedding parameter q and then setting $q=0$ and finally dividing by $m!$, we will have the following mth order deformation equation:

$$L[w_m(x,t) - \chi_m w_{m-1}(x,t)] = hH(x,t)R_m(\vec{w}_{m-1}), \tag{2.7}$$

where

$$R_m(\vec{w}_{m-1}) = \frac{1}{(m-1)!}\frac{\partial^{m-1} N[\Phi(x,t,q)]}{\partial q^{m-1}}|_{q=0}, \tag{2.8}$$

and

$$\chi_m = \begin{cases} 0 & m \leq 1 \\ 1 & m > 1. \end{cases} \tag{2.9}$$

It shoud be mentioned that $w_m(x,t)$ for $m \geq 1$ is goverend by the linear Eq. 2.7 with boundary conditions that come from the original problem. For more details about HAM, we refer the reader to [9,10].

3 Main Idea

In this Section, we consider Eq. 1.1, as follows:

$$i\frac{\partial w}{\partial t} + a\frac{\partial^2 w}{\partial x^2} + b\frac{\partial^4 w}{\partial x^4} + cw^{2n+1}\bar{w}^{2n} = 0, \tag{3.10}$$

$$w(x,0) = f(x), i^2 = -1.$$

We consider:

$$L[\Phi(x,t,q)] = i\frac{\partial \Phi(x,t,q)}{\partial t}, \quad L(c) = 0, \tag{3.11}$$

where c is a real constant and

$$N[\Phi(x,t,q)] = i\frac{\partial\Phi(x,t,q)}{\partial t} + a\frac{\partial^2\Phi(x,t,q)}{\partial x^2}$$

$$+ b\frac{\partial^4\Phi(x,t,q)}{\partial x^4} + c\Phi^{2n+1}(x,t,q)\bar{\Phi}^{2n}(x,t,q). \quad (3.12)$$

Therefore,

$$R_m(w_{m-1}) =$$

$$i\frac{\partial w_{m-1}}{\partial t} + a\frac{\partial^2 w_{m-1}}{\partial x^2} + b\frac{\partial^4 w_{m-1}}{\partial x^4} +$$

$$c\sum_{k_1=0}^{m-1}\sum_{k_2=0}^{k_1}\sum_{k_3=0}^{k_1-k_2}\cdots$$

$$\sum_{k_{2n+1}=0}^{k_1-k_2-\ldots-k_{2n}}\sum_{\alpha_1=0}^{m-1-k_1}\sum_{\alpha_2=0}^{m-1-k_1-\alpha_1}\cdots$$

$$\sum_{\alpha_{2n-1}=0}^{m-1-k_1-\alpha_1-\ldots-\alpha_{2n-2}} w_{k_2}w_{k_3}\ldots$$

$$w_{k_{2n+1}}w_{(k_1-k_2-\ldots-k_{2n+1})}$$

$$\bar{w}_{\alpha_1}\bar{w}_{\alpha_2}\ldots\bar{w}_{\alpha_{2n-1}}\bar{w}_{(m-1-k_1-\alpha_1-\ldots-\alpha_{2n-1})}. \quad (3.13)$$

The number of summations in 3.13 will be $4n$. If $H(x,t) = 1$ then the m-th order deformation equation 2.7 is:

$$w_m = \chi_m w_{m-1} + \frac{h}{i}\int_0^t R_m(w_{m-1})dt + c,$$

$$m \geq 1, \quad (3.14)$$

with $w_0(x,t) = f(x)$,

$$R_1(w_0) = c \mid f(x) \mid^8 f(x) +$$

$$af^{(2)}(x) + bf^{(4)}(x),$$

and

$$w_1(x,t) = -hi(c \mid f(x) \mid^8 f(x) +$$

$$af^{(2)}(x) + bf^{(4)}(x))t.$$

4 Convergence of the HAM

In this Section, we prove the convergence of the series solution obtained from the HAM to the exact solution of the Eq. 3.10.

Theorem 4.1 *If the series solution $w_0(x,t) + w_1(x,t) + w_2(x,t) + \ldots$ obtained from the HAM is convergent, and also the series $\sum_{m=0}^{\infty}\frac{\partial w_m}{\partial t}, \sum_{m=0}^{\infty}\frac{\partial^2 w_m}{\partial x^2}, \sum_{m=0}^{\infty}\frac{\partial^4 w_m}{\partial x^4}$, are convergent then the series $\sum_{m=0}^{\infty} w_m(x,t)$ converges to the exact solution of Eq. 3.10.*

Proof. Let

$$w(x,t) = \sum_{m=0}^{\infty} w_m(x,t).$$

In this case,

$$\lim_{m\to\infty} w_m(x,t) = 0. \quad (4.15)$$

So

$$\sum_{m=1}^{n}[w_m(x,t) - \chi_m w_{m-1}(x,t)] = w_n(x,t) \quad (4.16)$$

By using Eq. 4.16,

$$\sum_{m=1}^{\infty}[w_m(x,t) - \chi_m w_{m-1}(x,t)] =$$

$$\lim_{n\to\infty} w_n(x,t) = 0, \quad (4.17)$$

and since the operator L is linear, from 4.17 we have:

$$\sum_{m=1}^{\infty} L[w_m(x,t) - \chi_m w_{m-1}(x,t)] =$$

$$L(\sum_{m=1}^{\infty}(w_m(x,t) - \chi_m w_{m-1}(x,t)) = 0. \quad (4.18)$$

By applying

$$L[w_m(x,t) - \chi_m w_{m-1}] = hH(x,t)R_m(w_{m-1}), \quad (4.19)$$

from 4.18 we get:

$$\sum_{m=1}^{\infty} L[w_m(x,t) - \chi_m w_{m-1}] = hH(x,t) \sum_{m=1}^{\infty} R_m(w_{m-1}). \quad (4.20)$$

Since $h, H(x,t) \neq 0$ then

$$\sum_{m=1}^{\infty} [R_m(w_{m-1})] = 0. \quad (4.21)$$

According to Eq. 3.13,

$$\sum_{m=1}^{\infty} [R_m(w_{m-1})] = i \sum_{m=1}^{\infty} \frac{\partial w_{m-1}}{\partial t} + a \sum_{m=1}^{\infty} \frac{\partial^2 w_{m-1}}{\partial x^2} + b \sum_{m=1}^{\infty} \frac{\partial^4 w_{m-1}}{\partial x^4} + c \sum_{m=1}^{\infty} \sum_{k_1=0}^{m-1} \sum_{k_2=0}^{k_1} \sum_{k_3=0}^{k_1-k_2} \cdots \sum_{k_{2n+1}=0}^{k_1-k_2-\ldots-k_{2n}} \sum_{\alpha_1=0}^{m-1-k_1} \sum_{\alpha_2=0}^{m-1-k_1-\alpha_1} \cdots \sum_{\alpha_{2n-1}=0}^{m-1-k_1-\alpha_1-\ldots-\alpha_{2n-2}} w_{k_2} w_{k_3} \ldots w_{k_{2n+1}} w_{(k_1-k_2-\ldots-k_{2n+1})} \bar{w}_{\alpha_1} \bar{w}_{\alpha_2} \ldots \bar{w}_{\alpha_{2n-1}} \bar{w}_{(m-1-k_1-\alpha_1-\ldots-\alpha_{2n-1})}.$$

So, we have

$$\sum_{m=1}^{\infty} [R_m(w_{m-1})] = i \sum_{m=0}^{\infty} \frac{\partial w_m}{\partial t} + a \sum_{m=0}^{\infty} \frac{\partial^2 w_m}{\partial x^2} + b \sum_{m=0}^{\infty} \frac{\partial^4 w_m}{\partial x^4} + c \sum_{k_1=0}^{\infty} \sum_{m=k_1+1}^{\infty} \sum_{k_2=0}^{k_1} \sum_{k_3=0}^{k_1-k_2} \cdots \sum_{k_{2n+1}=0}^{k_1-k_2-\ldots-k_{2n}} \sum_{\alpha_1=0}^{m-1-k_1} \sum_{\alpha_2=0}^{m-1-k_1-\alpha_1} \cdots \sum_{\alpha_{2n-1}=0}^{m-1-k_1-\alpha_1-\ldots-\alpha_{2n-2}} w_{k_2} w_{k_3} \ldots w_{k_{2n+1}} w_{(k_1-k_2-\ldots-k_{2n+1})} \bar{w}_{\alpha_1} \bar{w}_{\alpha_2} \ldots \bar{w}_{\alpha_{2n-1}} \bar{w}_{(m-1-k_1-\alpha_1-\ldots-\alpha_{2n-1})}$$

$$= i \sum_{m=0}^{\infty} \frac{\partial w_m}{\partial t} + a \sum_{m=0}^{\infty} \frac{\partial^2 w_m}{\partial x^2} + b \sum_{m=0}^{\infty} \frac{\partial^4 w_m}{\partial x^4} + c \sum_{k_1=0}^{\infty} \sum_{m=1}^{\infty} \sum_{k_2=0}^{k_1} \sum_{k_3=0}^{k_1-k_2} \cdots \sum_{k_{2n+1}=0}^{k_1-k_2-\ldots-k_{2n}} \sum_{\alpha_1=0}^{m-1} \sum_{\alpha_2=0}^{m-1-\alpha_1} \cdots \sum_{\alpha_{2n-1}=0}^{m-1-\alpha_1-\ldots-\alpha_{2n-2}} w_{k_2} w_{k_3} \ldots w_{k_{2n+1}} w_{(k_1-k_2-\ldots-k_{2n+1})} \bar{w}_{\alpha_1} \bar{w}_{\alpha_2} \ldots \bar{w}_{\alpha_{2n-1}} \bar{w}_{(m-1-\alpha_1-\ldots-\alpha_{2n-1})}$$

$$= i \sum_{m=0}^{\infty} \frac{\partial w_m}{\partial t} + a \sum_{m=0}^{\infty} \frac{\partial^2 w_m}{\partial x^2} + b \sum_{m=0}^{\infty} \frac{\partial^4 w_m}{\partial x^4} + c \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{k_1} \sum_{k_3=0}^{k_1-k_2} \cdots \sum_{k_{2n+1}=0}^{k_1-k_2-\ldots-k_{2n}} \sum_{\alpha_1=0}^{\infty} \sum_{m=\alpha_1+1}^{\infty} \sum_{\alpha_2=0}^{m-1-\alpha_1} \cdots \sum_{\alpha_{2n-1}=0}^{m-1-\alpha_1-\ldots-\alpha_{2n-2}} w_{k_2} w_{k_3} \ldots w_{k_{2n+1}} w_{(k_1-k_2-\ldots-k_{2n+1})} \bar{w}_{\alpha_1} \bar{w}_{\alpha_2} \ldots \bar{w}_{\alpha_{2n-1}} \bar{w}_{(m-1-\alpha_1-\ldots-\alpha_{2n-1})}$$

$$= i \sum_{m=0}^{\infty} \frac{\partial w_m}{\partial t} + a \sum_{m=0}^{\infty} \frac{\partial^2 w_m}{\partial x^2} + b \sum_{m=0}^{\infty} \frac{\partial^4 w_m}{\partial x^4} + c \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{k_1} \sum_{k_3=0}^{k_1-k_2} \cdots \sum_{k_{2n+1}=0}^{k_1-k_2-\ldots-k_{2n}} \sum_{\alpha_1=0}^{\infty} \sum_{\alpha_2=0}^{\infty} \cdots \sum_{\alpha_{2n-1}=0}^{\infty} \sum_{m=1+\alpha_1+\alpha_2+\ldots+\alpha_{2n-1}}^{\infty}$$

$$w_{k_2}w_{k_3}\dots w_{k_{2n+1}}$$
$$w_{(k_1-k_2-\dots-k_{2n+1})}\bar{w}_{\alpha_1}\bar{w}_{\alpha_2}\dots$$
$$\bar{w}_{\alpha_{2n-1}}\bar{w}_{(m-1-\alpha_1-\dots-\alpha_{2n-1})}.$$

$$= i\sum_{m=0}^{\infty}\frac{\partial w_m}{\partial t}+a\sum_{m=0}^{\infty}\frac{\partial^2 w_m}{\partial x^2}+b\sum_{m=0}^{\infty}\frac{\partial^4 w_m}{\partial x^4}+$$
$$c\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{k_1}\sum_{k_3=0}^{k_1-k_2}\dots\sum_{k_{2n+1}=0}^{k_1-k_2-\dots-k_{2n}}\sum_{\alpha_1=0}^{\infty}\sum_{\alpha_2=0}^{\infty}\dots$$
$$\sum_{\alpha_{2n-1}=0}^{\infty}\sum_{m=0}^{\infty}w_{k_2}w_{k_3}\dots w_{k_{2n+1}}w_{(k_1-k_2-\dots-k_{2n+1})}$$
$$\bar{w}_{\alpha_1}\bar{w}_{\alpha_2}\dots\bar{w}_{2n-1}\bar{w}_m$$

$$= i\sum_{m=0}^{\infty}\frac{\partial w_m}{\partial t}+a\sum_{m=0}^{\infty}\frac{\partial^2 w_m}{\partial x^2}+b\sum_{m=0}^{\infty}\frac{\partial^4 w_m}{\partial x^4}+$$
$$c\sum_{k_2=0}^{\infty}\sum_{k_1=k_2}^{\infty}\sum_{k_3=0}^{k_1-k_2}\dots\sum_{k_{2n+1}=0}^{k_1-k_2-\dots-k_{2n}}w_{k_2}\dots$$
$$w_{k_{2n+1}}w_{(k_1-k_2-\dots-k_{2n+1})}$$
$$\sum_{\alpha_1=0}^{\infty}\bar{w}_{\alpha_1}\sum_{\alpha_2=0}^{\infty}\bar{w}_{\alpha_2}\dots\sum_{\alpha_{2n-1}=0}^{\infty}\bar{w}_{\alpha_{2n-1}}\sum_{m=0}^{\infty}\bar{w}_m$$

$$= i\sum_{m=0}^{\infty}\frac{\partial w_m}{\partial t}+a\sum_{m=0}^{\infty}\frac{\partial^2 w_m}{\partial x^2}+b\sum_{m=0}^{\infty}\frac{\partial^4 w_m}{\partial x^4}+$$
$$c\sum_{k_2=0}^{\infty}\sum_{k_1=0}^{\infty}\sum_{k_3=0}^{k_1}\dots$$
$$\sum_{k_{2n+1}=0}^{k_1-k_3\dots-k_{2n}}w_{k_2}\dots w_{k_{2n+1}}w_{(k_1-k_3\dots-k_{2n+1})}$$
$$\sum_{\alpha_1=0}^{\infty}\bar{w}_{\alpha_1}\sum_{\alpha_2=0}^{\infty}\bar{w}_{\alpha_2}\dots\sum_{\alpha_{2n-1}=0}^{\infty}\bar{w}_{\alpha_{2n-1}}\sum_{m=0}^{\infty}\bar{w}_m$$

$$= i\sum_{m=0}^{\infty}\frac{\partial w_m}{\partial t}+a\sum_{m=0}^{\infty}\frac{\partial^2 w_m}{\partial x^2}+b\sum_{m=0}^{\infty}\frac{\partial^4 w_m}{\partial x^4}+$$
$$c\sum_{k_2=0}^{\infty}\sum_{k_3=0}^{\infty}\dots\sum_{k_{2n+1}=0}^{\infty}\sum_{k_1=0}^{\infty}w_{k_2}w_{k_3}\dots w_{2n+1}w_{k_1}.$$

Therefore, from the relations $w=\sum_{m=0}^{\infty}w_m$ and $\sum_{m=0}^{\infty}\bar{w}_m=\overline{\sum_{m=0}^{\infty}w_m}$ we obtain:

$$\sum_{m=1}^{\infty}[R_m(w_{m-1})]=$$
$$i\frac{\partial}{\partial t}\sum_{m=0}^{\infty}w_m+a\frac{\partial^2}{\partial t^2}\sum_{m=0}^{\infty}w_m+$$
$$b\frac{\partial^4}{\partial t^4}\sum_{m=0}^{\infty}w_m+c\sum_{k_2=0}^{\infty}w_{k_2}\sum_{k_3=0}^{\infty}w_{k_3}\sum_{k_4=0}^{\infty}w_{k_4}\dots$$
$$\sum_{k_{2n+1}=0}^{\infty}w_{k_{2n+1}}\sum_{k_1=0}^{\infty}w_{k_1}$$
$$\sum_{\alpha_1=0}^{\infty}\bar{w}_{\alpha_1}\sum_{\alpha_2=0}^{\infty}\bar{w}_{\alpha_2}\dots\sum_{\alpha_{2n-1}=0}^{\infty}\bar{w}_{\alpha_{2n-1}}\sum_{m=0}^{\infty}\bar{w}_m$$
$$=i\frac{\partial}{\partial t}\sum_{m=0}^{\infty}w_m+a\frac{\partial^2}{\partial t^2}\sum_{m=0}^{\infty}w_m+$$
$$b\frac{\partial^4}{\partial t^4}\sum_{m=0}^{\infty}w_m+cw^{2n+1}\bar{w}^{2n}. \tag{4.22}$$

Hence, From Eqs. 4.21 and 4.22 we conclude that $w(x,t)=\sum_{m=0}^{\infty}w_m(x,t)$ is the exact solution of Eq. 3.10 and the proof is completed.

5 A sample example

In this Section, we solve a Schrodinger equation with power law nonlinearity via HAM by applying Eq. 3.13 and repersent the numerical results. Also, we plot the h-curve to show the regoin of convergence. The results have been provided by Mathematica.

Example 5.1 *Consider the following Schrodinger equation:*

$$i\frac{\partial w}{\partial t}+\frac{\partial^2 w}{\partial x^2}-\frac{\partial^4 w}{\partial x^4}+w^5\bar{w}^4=0,$$
$$w(x,0)=1, i^2=-1.$$

Table 1: The results of the example with the errors at the point (2,1).

n	$w_n(x,t)$	$\vert w_n(x,t) - w(x,t)\vert$
2	0.500000000+1.000000000i	0.163571772
4	0.541666667+0.833333333i	0.008251233
5	0.541666667+0.841666667i	0.001378322
6	0.540277778+0.841666667i	0.000197213
7	0.540277778+0.841468254i	0.000024680
8	0.540302579+0.841468254i	2.74450100E-6
9	0.540302579+0.841471010i	2.74627387E-7
10	0.540302304+0.841471010i	2.49787194E-8

Then from 3.14, $w_0(x,t) = 1$, $R_1(w_0) = 1$ and

$$w_1(x,t) = \frac{h}{i}\int_0^t dt = -hit,$$

$$w_2(x,t) = w_1(x,t) - hi\int_0^t (h - hit)dt =$$

$$-hit - h^2it - h^2\frac{t^2}{2},$$

$$w_3(x,t) = w_2(x,t) - hi$$

$$\int_0^t (-hit - h^2it - h^2\frac{t^2}{2})dt =$$

$$1/6ht(-6i(1+h)^2 - 6h(1+h)t + ih^2t^2).$$

If $h = -1$ then

$$w_0(x,t) = 1,$$

$$w_1(x,t) = it,$$

$$w_2(x,t) = \frac{(it)^2}{2!},$$

$$w_3(x,t) = \frac{(it)^3}{3!}, \ldots$$

So, $w(x,t) = w_0(x,t) + w_1(x,t) + w_2(x,t) + w_3(x,t) + \ldots = 1 + it + \frac{(it)^2}{2!} + \frac{(it)^3}{3!} + \ldots = e^{it}$ which is the exact solution. Table 1 shows the errors of the method at the given point $(2,1)$ and for different values of n.

We observe that the HAM is convergent when n increases. Figure 1, illustrates the h-curve of $w(x,t)$ at (2,1) with $n = 11$. In this case, the convergence of the method is guranteed when $-1.5 \lesssim h \lesssim -0.5$.

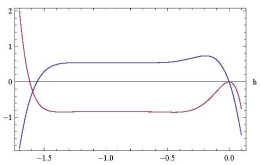

Figure 1: The h-curve of 11-approximation of the example when x=2 and t=1, for real and imaginary parts.

6 Conclusion

In this paper, the homotopy analysis method (HAM) was applied to solve the Schrodinger equation with a power law nonlinearity and the theorem of convergence of HAM was proved.

Also, an example was solved to determine the efficiency and convergence of the proposed method. Therefore, HAM can be a reliable and powerful method to obtain the analytical or approximate solution of a nonlinear partial differential equation with complicated nonlinearity.

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Mohammad Ali Fariborzi Araghi was born in Tehran, in 1967. He received B.SC. degree in mathematics from Tehran Teacher Training university, M.SC. degree in applied mathematics from Islamic Azad university, Karaj branch and Ph.D. degree in applied mathematics, numerical analysis field, from Islamic Azad university, Science and Research branch. He is the associate professor of Islamic Azad university, Central Tehran branch. His research interests are numerical solution of integral equations and partial differential equations and fuzzy mathematics.

Soheila Naghshband was born in Tehran, in 1973. She received B.SC. degree in applied mathematics from Shahid Beheshti university and M.SC. degree in pure mathematics from Tehran university. She is the Ph.D. student of Islamic Azad university, Central Tehran branch. She is a faculty member of West Tehran branch, Islamic Azad university. Her research interests are numerical solution of nonlinear partial differential equations and related topics.