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Research Article

# Ranking of DMUs by using TOPSIS and different ranking models in DEA

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**Abstract**

As regards of the necessity of ranking efficient units different Data Envelopment Analysis (DEA) models are introduced. Each of the existing models has advantages and deals with ranking efficient units from special aspects. But, there exist no model has all these benefits in a unified manner. The aim of this paper is to present a new ranking method which can incorporate, to a great extent, advantages of various ranking models. In doing so a Multiple Criteria Decision Making (MCDM) technique utilized. For demonstration of the presented method an application of this new method in banking system is provided.

*Keywords* : Data envelopment analysis; Ranking; Multiple Criteria Decision Making; TOPSIS.

## 1 Introduction

Data Envelopment Analysis is now famous as a non-parametric technique based on mathematical programming for the efficiency assessment of a set of Decision Making Units (DMUs). Different DEA models are provided in literature for evaluating a set of homogenous DMUS in various occasions. In DEA technique an envelope constructed through observed units, which surrounded the coordinates of units in corresponding space and this yield a frontier. According to a comparison process to this frontier DEA estimate relative efficiency. Those units which are located onto the frontier are called efficient and others called inefficient. An important issue in efficiency evaluation is comparing units under evolution with each other for better analysand the system. A suffering in DEA analysis is the existence of multiple efficient units. Ranking efficient DMUs has become the interest of many DEA researches. Thus variety of DEA models have been formulated for ranking efficient DMUs. A new DEA area which is called supper efficiency first introduced by DEA researchers. Anderson and peterson (A.P) [1] proposed a method for ranking efficient DMUs. Their method was based on the position of each eliminated efficient DMU in relation to its corresponding new Production Possibility Set (PPS). Sexton et al. [14] proposed another approach known as the cross-evaluation method, which can be utilized for ranking DMUs. Thrall [17] in a paper showed that nonstability may be occure in A.P model. For overcoming this difficulty, Mehrabian et al. [12] introduced a method (MAJ) for ranking efficient DMUs. This method is stable but in some cases it would be infeasible. Moreover, to overcome the problem

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**Table 1:** A Multiple Attribute Decision problem.

| | **Criter 1** | **Criter 2** | ... | **Criter n** |
|---|---|---|---|---|
| **Altern 1** | $x_{11}$ | $x_{12}$ | ... | $x_{1n}$ |
| **Altern 2** | $x_{21}$ | $x_{22}$ | ... | $x_{2n}$ |
| $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ |
| **Altern m** | $x_{m1}$ | $x_{m2}$ | ... | $x_{mn}$ |

of instability in A.P model and to rank efficient units Sueyoshi [16] use the modified slacks-based model For avoiding mentioned difficulties in ranking efficient units Tone [18] presented the slack based model (SBM). This model rank efficicnt units while dealing with slack variables and according to the idea of one-leave-out. In their paper Tohidi et al. [7] provided a method, in accordance to gradient line, for ranking efficient DMUs. As Tohidi et al. (2004) discussed the advantage of this method is its stability and robustness. Considering DEA technique in a paper Jahanshahloo et al. [5] presented a method for ranking extreme efficient units. This issue is of great importance since is not easy solve the suffering of ranking extreme efficient units. The idea is based upon the leave-one-out efficient unit and l1-norm. Noted that this model is always feasible and stable. Moreover, Jahanshahloo et al. [8] presented a model based on common weights analysis (CWA) to determine the ranking order of units. Also, for overcoming mentioned difficulties of A.P. model, Jahanshahloo et al. [6] proposed Modified MAJ model in which the mentioned problems are fixed. In ding so, also, Li et al [9] proposed a super-efficiency model that does not have the suffering in previous methods. Based upon the omission of the efficient DMUs from the reference set of the inefficient DMUs, Jahanshahloo et al. [4] proposed a new ranking system for extreme efficient DMUs. Liu and Peng [10] introduced common weights analysis (CWA) to determine the common set of weights for DMUs and ranked DMUs based this idea. For ranking efficiency units on basis of considered voting analytic hierarchy process (VAHP) Soltanifar and Hosseinzadeh Lotfi [15] presented a paper. In their paper Rezai Balf et al. [13] provided a method, which has more ability over other existing methods, based on Tchebycheff Norm for ranking efficient units.

Decision making can be interpreted as the procedure of finding the best alternative among a set of feasible ones. An obvious issue is that these alternative have confliction with each other, thus finding a set of alternatives as an optimal solution is not an easy task. In literature those problems accounted for several criteria are called multi criteria decision making (MCDM) problems. In literature there exist several methods for solving MCDM problems and one famous method is TOPSIS which is technique for order preference by similarity to an ideal solution which is presented by Hwang and Yoon [11] In This method ranking order of units is based upon the distance from ideal and negative-ideal points.

In this paper the aim is to provide a new ranking method based on the existing ranking models in literature. As each of the presented models have both advantages and disadvantages, thus here it is tried to get use of all these methods to present a new ranking method.

This paper unfolds as follows: at first some preliminaries about DEA technique will be reviewed. Then, in Section 2 the new method for ranking efficient units considering different ranking models will be presented. Sections 4 and 5 give a numerical illustration of an application of the presented model in banking system and conclude the paper.

## 2 DEA Priliminaries

Data envelopment (DEA) a is a mathematical based programming for assessing a set of homogenous Decision Making Units (DMUs). Charnes et al. [2] presented CCR model which considers constant returns to scale technology and deriving relative efficiency of DMUs. Banker et al. [3] presented BBC model which considerers units in variable returns to scale technology. After that many DEA models have been presented each of which deals with specific issues and helps managers for better decision making. According to DEA efficiency score DMUs classified into two subsets, efficient and inefficient DMUs. Those DMUs performs efficiently construct DEA frontier and others, inefficient ones, are far away from this frontier. Noted that, efficient DMUs can also be considered as benchmark units for those units which are inefficient. This issue has an important role since in regard of these benchmarks managers can better guide system in future.

Considering basic assumptions of DEA and assume there exist a set of $n$ DMUs with $m$ inputs and $s$ outputs to be evaluated. It should be noted that in DEA assumptions it is assumed that the input and output vectors are all semipositive. Consider CCR model in input orientation as follows:

$$\begin{aligned} \min \quad & \theta - \varepsilon(1s^- + 1s^+) \\ s.t. \quad & \sum_{j=1}^{n} \lambda_j x_{ij} + s^- = \theta x_{io}, \quad i = 1, ..., m, \\ & \sum_{j=1}^{n} \lambda_j y_{rj} - s^+ = y_{ro}, \quad r = 1, ..., s, \\ & \lambda_j \geq 0, \quad j = 1, ..., n. \end{aligned} \tag{2.1}$$

The dual of the above model which is called multiplier form is as follows:

$$\begin{aligned} \max \quad & \sum_{r=1}^{s} u_r y_{ro} \\ s.t. \quad & \sum_{i=1}^{m} v_i x_{io} = 1, \\ & \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0, \quad j = 1, ..., n, \\ & u_r \geq 0, \quad v_i \geq 0, \ r = 1, ..., s, \ i = 1, ..., m \end{aligned} \tag{2.2}$$

In the above- mentioned model $v$ and $u$ are the input and output weight vectors.

Both of the aforesaid models are in input orientation where the input reduction is due to maximized. The above models can be written in output orientation where the output shortfall is due to minimized.

## 3 TOPSIS method

Decision making is the process of finding the best alternative from among all existing feasible ones. An important issue in this subject is that these feasible alternatives are in confliction with each other. In situations where decision maker considering several criteria the problem is called multi-criteria decision making (MCDM) problems. Considering such problems decision maker (DM) wants to find the best alternative among the existing ones. As regards of an MCDM problem with finite possibilities it can be indicated as follows, see Table 1 where $A_1; A_2; ...; A_m$ are possible alternatives to be selected by DMU and $C_1; C_2; ...; C_n$ are criteria. Let $x_{ij}$ be the rating of alternative $i$ with respect to criterion $j$ and $w_j$ is the weight of criterion $i$. There are several methods for solving MCDM problems. One of the famous methods for solving an MCDM problem is TOPSIS which is a technique for order preference by similarity to an ideal solution that presented by Hwang and Yoon [11]. By introducing an ideal and anti ideal point and measuring the distance of the unit under evaluation from these two points ranking order of the units will be obtained. Let $n$ units with $m$ inputs and $s$ outputs are to be evaluated. All the inputs gains cost and all outputs yield revenue. Also let $x_o \in R^m_+$ and $y_o \in R^s_+$ to be, respectively, the inputs and outputs vectors of $DMU_o$ (unit under evaluation). As mentioned before TOPSIS technique is based upon the distance of under evaluation unit from the ideal and anti ideal units. Now Consider the following steps in TOPSIS method.

Step 1. Normalization: As regards of the following formula all of the input-output data need to be normalized.

$$v_{ij} = \frac{x_{ij}}{\sum_{j=1}^{n} x_{ij}^2}, \quad u_{rj} = \frac{y_{rj}}{\sum_{j=1}^{n} y_{rj}^2}, \quad \forall i, \forall r, \forall j \tag{3.3}$$

Step 2. Weighting: Let $W_i^{'}$ be the weight of the $ith$ input $(i = 1, ..., m)$ and $W_r^{''}$ be the weight of the $rth$ input $(r = 1, ..., s)$. Now from the following relations all the data should be Weighted.

**Table 2:** An MCDM problem.

| | **R.M. 1** | **R.M. 2** | ... | **R.M. n** |
|---|---|---|---|---|
| **DMU 1** | $r_{11}$ | $r_{12}$ | ... | $r_{1n}$ |
| **DMU 2** | $r_{21}$ | $r_{22}$ | ... | $r_{2n}$ |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |
| **DMU m** | $r_{m1}$ | $r_{m2}$ | ... | $r_{mn}$ |

**Table 3:** Matrix of properties.

| | **Property 1** | **Property 2** | ... | **Property t** |
|---|---|---|---|---|
| **R.M. 1** | $w_{11}$ | $w_{12}$ | ... | $w_{1t}$ |
| **R.M. 2** | $w_{21}$ | $w_{22}$ | ... | $w_{2t}$ |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |
| **R.M. n** | $w_{n1}$ | $w_{n2}$ | ... | $w_{nt}$ |

**Table 4:** Inputs, Outputs and efficiency scores.

| $DMU_p$ | $I_1$ | $I_2$ | $I_3$ | $O_1$ | $O_2$ | $O_3$ | *CCR Efficiency* |
|---|---|---|---|---|---|---|---|
| $DMU_1$ | 0.950 | 0.700 | 0.155 | 0.190 | 0.521 | 0.293 | 1.0000 |
| $DMU_2$ | 0.796 | 0.600 | 1.000 | 0.227 | 0.627 | 0.462 | 0.8333 |
| $DMU_3$ | .798 | 0.750 | 0.513 | 0.228 | 0.970 | 0.261 | 0.9911 |
| $DMU_4$ | 0.865 | 0.550 | 0.210 | 0.193 | 0.632 | 1.000 | 1.0000 |
| $DMU_5$ | 0.815 | 0.850 | 0.268 | 0.233 | 0.722 | 0.246 | 0.8974 |
| $DMU_6$ | 0.842 | 0.650 | 0.500 | 0.207 | 0.603 | 0.569 | 0.7483 |
| $DMU_7$ | 0.719 | 0.600 | 0.350 | 0.182 | 0.900 | 0.716 | 1.0000 |
| $DMU_8$ | 0.785 | 0.750 | 0.120 | 0.125 | 0.234 | 0.298 | 0.7978 |
| $DMU_9$ | 0.476 | 0.600 | 0.135 | 0.080 | 0.364 | 0.244 | 0.7877 |
| $DMU_{10}$ | 0.678 | 0.550 | 0.510 | 0.082 | 0.184 | 0.049 | 0.290 |
| $DMU_{11}$ | 0.711 | 1.000 | 0.305 | 0.212 | 0.318 | 0.403 | 0.6045 |
| $DMU_{12}$ | 0.811 | 0.650 | 0.255 | 0.123 | 0.923 | 0.628 | 1.0000 |
| $DMU_{13}$ | 0.659 | 0.850 | 0.340 | 0.176 | 0.645 | 0.261 | 0.8166 |
| $DMU_{14}$ | 0.976 | 0.800 | 0.540 | 0.144 | 0.514 | 0.243 | 0.4693 |
| $DMU_{15}$ | 0.685 | 0.950 | 0.450 | 1.000 | 0.262 | 0.098 | 1.0000 |
| $DMU_{16}$ | 0.613 | 0.900 | 0.525 | 0.115 | 0.402 | 0.464 | 0.6390 |
| $DMU_{17}$ | 1.000 | 0.600 | 0.205 | 0.090 | 1.000 | 0.161 | 1.0000 |
| $DMU_{18}$ | 0.634 | 0.650 | 0.235 | 0.059 | 0.349 | 0.068 | 0.4727 |
| $DMU_{19}$ | 0.372 | 0.700 | 0.238 | 0.039 | 0.190 | 0.111 | 0.4088 |
| $DMU_{20}$ | 0.583 | 0.550 | 0.500 | 0.110 | 0.615 | 0.764 | 1.0000 |

$$\alpha_{ij} = v_{ij}.W_i^{'}, \quad \forall i, \forall j \tag{3.4}$$

$$\beta_{ij} = u_{rj}.W_i^{''}r, \quad \forall r, \forall j \tag{3.5}$$

Step 3. Finding Ideal and anti-ideal points: According to this assumption that reduction in inputs will increase the performance and any increment in outputs will also increase the performance consider the following relations.

$$A^+ = (A_1^+, ..., A_m^+, A_{m+1}^+, ..., A_{m+s}^+) \tag{3.6}$$

$$A^- = (A_1^-, ..., A_m^-, A_{m+1}^-, ..., A_{m+s}^-) \tag{3.7}$$

where

$$A_i^+ = \begin{cases} Min\{\alpha_{ij} : j = 1, ..., n\}, & 1 \leq i \leq m, \\ Max\{\beta_{ij} : j = 1, ..., n\}, & m+1 \leq i \leq m+s, \end{cases} \tag{3.8}$$

$$A_i^- = \begin{cases} Max\{\alpha_{ij} : j = 1, ..., n\}, & 1 \leq i \leq m, \\ Min\{\beta_{ij} : j = 1, ..., n\}, & m+1 \leq i \leq m+s, \end{cases} \tag{3.9}$$

Step 4. Measuring the distance from the ideal and anti ideal points: In this term, Euclidean distance of the DMUs, considering their new coordinations as derived in step 2, from the ideal and anti ideal points, as introduced in step3, will be calculated.

$$d_j^+ = (\sum_{i=1}^{m}(\alpha_{ij} - A_i^+)^2 + \sum_{i=m+1}^{m+s}(\beta_{ij} - A_i^+)^2)^{\frac{1}{2}} \tag{3.10}$$

$$d_j^- = (\sum_{i=1}^{m}(\alpha_{ij} - A_i^-)^2 + \sum_{i=m+1}^{m+s}(\beta_{ij} - A_i^-)^2)^{\frac{1}{2}} \tag{3.11}$$

Step 5. Ranking criterion: In this method, TOPSIS, decision making units according to their distances from the ideal and anti ideal points will be ranked. Meaning that a DMU has a better rank which is close to the ideal point and far from the anti ideal point. Thus ranking criterion is defined as follows:

$$R_j = \frac{d_j^-}{d_j^- + d_j^+} \tag{3.12}$$

It is evident that: $R_j \in [0, 1]$.

For all j $R_j = 0$ if and only if $d_j^- = 0$ which means $DMU_j$ is located onto the anti ideal point and has the worst coordination. For all j $R_j = 1$ if and only if $d_j^+ = 0$ which means $DMU_j$ is located onto the ideal point and has the best coordination. Therefore, the more $R_j$ is the better corresponding rank order will be.

## 4 Proposed method

As stated in literature there exist different ranking method each of which is based upon various principles. Each of these ranking methods has advantages and disadvantages and until now there is no method which has all the ability of these method and a power for ranking all DMUs while considering this issue from various aspects. Here according to the content of TOPSIS method and the fact that in this method both pessimistic and optimistic viewpoints are incorporated, which is of major importance, a new method will be introduced for completely ranking units.
Considering CCR model and identify efficient units. As stated in literature the quantity of $\theta^*$ for inefficient units is a criterion for ranking them but efficient units, as they all have $\theta^* = 1$, can not be ranked in this manner. In this new method, for ranking efficient units, consider efficient units as alternatives and various ranking method as different criteria. Consider an MCDM problem with possibilities as the following matrix. See Table 2 where $DMU_1; DMU_2; ...; DMU_m$ are possible alternatives (efficient units) to be selected through solving CCR model and $R.M._1; R.M._2; ...; R.M._n$ are criteria (different ranking methods). Let $r_{ij}$ be the ranking order of $DMU_i$ with respect to ranking method $j$.

Consider $t$ criterion to show the property of each ranking method. According to these prop-

**Table 5:** Ranking scores.

| E.D | R.M1 | R.M2 | R.M3 | R.M4 | R.M5 | R.M6 |
|---|---|---|---|---|---|---|
| 1 | 0.604 | 0.683 | 0.649 | 0.825 | 0.743 | 0.660 |
| 4 | 0.303 | 0.246 | 0.270 | 0.288 | 0.253 | 0.304 |
| 7 | 0.459 | 0.320 | 0.409 | 0.426 | 0.340 | 0.406 |
| 12 | 0.522 | 0.602 | 0.578 | 0.690 | 0.646 | 0.488 |
| 15 | 0.384 | 0.359 | 0.370 | 0.161 | 0.324 | 0.349 |
| 17 | 0.317 | 0.526 | 0.310 | 0.690 | 0.447 | 0.321 |
| 20 | 0.371 | 0.425 | 0.498 | 0.554 | 0.547 | 0.581 |

Continue Table (5)

| R.M7 | R.M8 | R.M9 | R.M10 | R.M11 | R.M12 | R.M13 |
|---|---|---|---|---|---|---|
| 0.311 | 0.321 | 0.335 | 0.188 | 0.224 | 0.355 | 0.356 |
| 0.314 | 0.389 | 0.190 | 0.359 | 0.140 | 0.233 | 0.231 |
| 0.491 | 0.461 | 0.577 | 0.484 | 0.128 | 0.174 | 0.175 |
| 0.350 | 0.407 | 0.498 | 0.668 | 0.742 | 0.620 | 0.620 |
| 0.418 | 0.371 | 0.600 | 0.564 | 0.168 | 0.181 | 0.181 |
| 0.588 | 0.294 | 0.704 | 0.618 | 0.154 | 0.221 | 0.221 |

**Table 6:** Matrix of Property's weighs.

| | $t_1$ | $t_2$ | $t_3$ | $t_4$ | $t_5$ | $t_6$ | $t_7$ |
|---|---|---|---|---|---|---|---|
| **R.M 1** | 0 | 0 | 0.9 | 0 | 1 | 1 | 0 |
| **R.M 2** | 0.2 | 0 | 0.9 | 0.8 | 1 | 1 | 0.2 |
| **R.M 3** | 1 | 0 | 0.9 | 1 | 1 | 0 | 0.2 |
| **R.M 4** | 1 | 0 | 0 | 1 | 0.7 | 0 | 0.2 |
| **R.M 5** | 1 | 0 | 1 | 1 | 1 | 0 | 0.2 |
| **R.M 6** | 1 | 0 | 0 | 1 | 1 | 0 | 0.2 |
| **R.M 7** | 1 | 0 | 0 | 1 | 1 | 0 | 0.2 |
| **R.M 8** | 1 | 0 | 0 | 1 | 1 | 0.2 | 0.2 |
| **R.M 9** | 0 | 0.7 | 0.9 | 1 | 0 | 0 | 0 |
| **R.M 10** | 0 | 0.7 | 0.5 | 1 | 0 | 0 | 0 |
| **R.M 11** | 1 | 0 | 1 | 1 | 1 | 0 | 0.2 |
| **R.M 12** | 1 | 0 | 1 | 1 | 1 | 0 | 0.2 |
| **R.M 13** | 1 | 0 | 1 | 1 | 0 | 0 | 0 |

erties criterion, ranking method will be weighted. In this manner considering Table 3 $w_i t$ is weight of ranking method $i$ in accordance to property $t$. As an instance these properties can be feasibility, ranking extreme efficient units, stability and etc. Thus according to the above table data can be weighted to be used in TOPSIS. In this way ideal and anti ideal ranking orders will be identified and ranking order of each efficient unit will be compared to these two points. Finally considering (3.12) new ranking order will be obtained in regards of ranking order of efficient units via different ranking methods.

## 5 Application

In this section, an empirical example about application of the proposed approach into Commercial banks is given. We consider twenty Commercial banks of Iran which input-output data are tabulated in Table 4. These data are gathered through all branches of each bank. In summary, the input and output sets are as follows. Also the result of CCR model as reviewed in previous sections, are listed in this table. As it can be seen seven units are efficient, DMUS 1,4,7,12,15, 17 and 20.

***Inputs***:

- Staff.
- Computer terminal.
- Space.

***Outputs***:

- Deposits.
- Loans Granted.
- Charge.

Consider some of the important ranking methods in literature as follows. R.M1: A.P. model (Andersen and Petersen, 1993) is based upon the the idea of one-leave- out and measuring the distance of the under evaluation from the new production possibility set.

R.M2: MAJ model (Mehrabian et al., 1999) presented for ranking efficient units which is also based upon the the idea of one-leave- out. This model is always stable but might be infeasible in some cases.

R.M3: Modified MAJ model (Jahanshahloo et al., 2006) overcomes the problem might be occurred in MAJ model.

R.M4: A new model based on the idea of changing the reference set of the inefficienct units (Jahanshahloo et al., 2007)

R.M5: A model presented by Li et al. [9] is based upon the super-efficiency method that does not have the suffering in previous methods

R.M6: Slack based model (Tone, 2002) is based upon the input and output variables.

R.M7: SA DEA model (Sueyoshi, 1999) overcomes the problem of infeasibility existed in A.P. model.

R.M8: Cross efficiency (Sexton et al., 1986) is provided based on using weights of other units in optimality for other units.

R.M9: A model based on finding common set of weights (Liu and Peng, 2008) which determine the common set of weights for DMUs and ranked DMUs based this idea.

R.M10: A model based on finding common set of weights (Jahanshahloo et al., 2005) for ranking efficient units.

R.M11: L1-norm model (Jahanshahloo et al., 2004) the idea is based upon the leave-one-out efficient unit and l1-norm. Noted that this model is always feasible and stable.

R.M12: $L_\infty$-norm model (Jahanshahloo et al., 2012) provided a method, which has more ability over other existing methods, based on Tchebycheff Norm for ranking efficient units.

R.M13: Gradient line model (Jahanshahloo et al., 2004) provided a method, in accordance to gradient line, for ranking efficient DMUs.

In the following table ranking order of efficient units in regards of the above mentioned ranking methods are listed. In this table E.D. shows efficient DMUs, and $R.Mj, (j = 1, ..., 14)$ are those explained in the above table.

As regards of properties of different ranking models for ranking efficient units and the factors defined by the decision maker consider the following Table 6 and the weights given to each factor. These factors are as follows:

- Feasibility.
- ranking extreme efficient units.
- Complexity in computation.
- Stability.
- Absence of multiple optimal solution.
- Independency to $\theta$ and slacks.
- Independency to the number of efficient and inefficient units.

In accordance to these factors and given weights as mentioned in TOPSIS data needs to

**Table 7:** New Ranking order

| *E.D.* | *Scale of ranking* | *Rank order* |
|---|---|---|
| 1 | 1.54609E-09 | 7 |
| 4 | 0.011305044 | 2 |
| 7 | 0.000519718 | 3 |
| 12 | 0.000191301 | 6 |
| 15 | 0.951636271 | 1 |
| 17 | 0.000431195 | 4 |
| 20 | 0.000205404 | 5 |

be weighted and then according to these new coordinations distances from ideal and anti-ideal point calculated. Finally according to (3.12) ranking order of efficient units acquired as follows in Table 7:

In this regards the more the scale of the efficient unit is the better corresponding ranking order will be. As it can be seen this new ranking order is not much differ from the mentioned ranking obtained from the existing ranking methods but some how more accurate since in considers major aspects of ranking models and obtain the ranking order from the order of efficient units from other methods. $DMU_{15}$ which is in the first place in most of the ranking methods is in the fist place and $DMU_1$ which has the worst ranking place in all the mentioned units got this place as well. According to the Table 5 the results obtained from this method is some how an aggregation of the different ranking DEA models. As seen in Table 5, $DMU_1$ in most of the ranking models has the worst place and in this new ranking order is has the worst place as well.

## 6 Conclusion

In this paper the aim is to rank efficient units while considering advantages of some of the important ranking methods, existing in literature. As each of existing ranking methods have some major benefits that other do not have and the fact that it is not possible to gather all these advantages in a unified model, thus is seems significant to provide a new ranking method which considered all the good aspects of these models. In doing so, MCDM method is considered. As regards of the obtained ranking orders form different ranking models and a matrix of weights, corresponds to the different property of these methods, TOPSIS is accounted for in order to consider different aspect of these methods and a new method introduced.
For further research on this subject other aspect of MCDM technique can also be accounted for in order to obtain a new ranking order on basis of the existing ranking methods.

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