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Research Article

A new parametric method for ranking fuzzy numbers based on positive and negative ideal solutions

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Abstract

Ranking of fuzzy numbers play an important role in decision making, optimization, forecasting etc. Fuzzy numbers must be ranked before an action is taken by a decision maker. The main aim of this paper is to propose a new approach for the ranking of generalized fuzzy numbers. The proposed ranking approach is based on distance between positive ideal solution and negative ideal solution. The main advantage of the proposed approach is that it provides the correct ordering of generalized and normal fuzzy numbers and it is very simple and easy to apply in the real life problems. The approach is illustrated by numerical examples, showing that it overcomes several shortcomings such as the indiscriminative and counterintuitive behavior of existing fuzzy ranking approaches.

Keywords : Fuzzy numbers; Ranking fuzzy numbers; Ideal solutions; Distance.

1 Introduction

Fuzzy set theory [33] is a powerful tool to deal with real life situations. Real numbers can be linearly ordered by $\geq$ or $\leq$, however, this type of inequality does not exist in fuzzy numbers. Since fuzzy numbers are represented by possibility distribution, they can overlap with each other and it is difficult to determine clearly whether one fuzzy number is larger or smaller than the other. An efficient approach for ordering the fuzzy numbers is by the use of a ranking function $\Re : F(R) \rightarrow R$, where $F(R)$ is a set of fuzzy numbers defined on real line, which maps each fuzzy number into the real line, where a natural order exists. Thus, specific ranking of fuzzy numbers is an important procedure for decision-making in a fuzzy environment and generally has become one of the main problems in fuzzy set theory. Ranking fuzzy numbers is often a necessary step in many mathematical models, and a large number of ranking methods have been proposed to perform this task. However, few comparative studies exist and nowadays it is still unknown how similar ranking methods are in practice, i.e., how likely they are to induce the same ranking. Also, very often, fuzzy mathematical models encounter steps when it is required to rank fuzzy numbers. A basic example is a decision support system whose inputs are judgments on a finite set of alternatives $\{1, \cdots, n\}$, and whose outputs are fuzzy numbers $\{A_1, \cdots, A_n\}$, where A_i is the fuzzy number representing the possible scores of alternative i. In this example, choosing the best alternative needs the definition of an ordering relation on $\{A_1, \cdots, A_n\}$. Methods for ordering fuzzy numbers, do actually rank them, and consequently the literature regards them simply as ranking methods. It is here highly relevant to note, that dif-

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ferent ranking methods can induce different rankings. Furthermore, it is reasonable that the more different two methods are, the more likely they induce different rankings, and vice versa. Said this, discovering howsimilar different rankingmethods are is beyond mere curiosity, especially because many methods have been proposed in literature to perform this task. Nevertheless, in spite of the large number of proposals, very few comparative studies exist and with this study we investigate differences/similarities between ranking methods. The main aim of this paper is to propose a new approach for the ranking of generalized fuzzy numbers. The proposed ranking approach is based on positive ideal solution and negative ideal solution. The main advantage of the proposed approach is that it provides the correct ordering of generalized and normal fuzzy numbers and it is very simple and easy to apply in the real life problems.

This paper is organized as follows. In Sect. 2, some basic definitions, arithmetic operations and ranking function are reviewed. In this Section, a brief review of the existing approaches for the ranking of generalized fuzzy numbers are presented. Section 3 is devoted to explain themethodology and present the numerical results. We shall then discuss the results separately, in Section 4. Finally, Section 5 contains the conclusions.

2 Preliminaries

In this section some basic definitions and arithmetic operations are reviewed.

2.1 Fuzzy sets

Fuzzy numbers is an extremely suitable methodology that embraces adequately subjective knowledge and objective knowledge. Zadeh [33] stated some basic results linked to the development of fuzzy sets. Many sets encountered in reality do not have precisely defined bounds that separate the elements within the set from those outside the set. In our case, it might be said that a certain check-in counter has a long waiting time. If we denote by W the set of long waiting time at a check-in counter, the question logically arises as to the bounds of such a defined set. Does a waiting time of 5 min belong to this set? What about 10 min or 15 min? The answers to these questions are always logical and there exists some positive probability of finding a consumer that answers these questions positively.

On the other hand, it is intuitively clear that a waiting time of 10 min belongs more likely or stronger to the set W long waiting time at a check-in counter, than a time of 5 min. In other words, there is more truth in the statement that a waiting time of 10 min is a long waiting time at a check-in counter than in the statement that a waiting time of 5 min is a long waiting time at a check-in counter. Within this context, we can fully appreciate that everything is a matter of degree, so all waiting times at a check-in counter can be treated as long. If now we introduce a set called short waiting time at a check-in counter, and proceed analogously, we see that we can treat all the waiting times as short. Finally, we can ask ourselves whether a waiting time of 5 min is long, short or perhaps medium. The answer is very simple. A waiting time of 5 min is long, short and medium, all at the same time. In other words, a waiting time of 5 min belongs to the sets long waiting time at a check-in counter, short waiting time at a check-in counter and medium waiting time at a check-in counter with different intensity of membership.

Zadeh [32] and Mandami and Assilian [25] developed fuzzy logic, introducing a concept of approximate reasoning, and showed that vague logical statements enable the formation of algorithms that can use vague data to derive vague inferences. Many fields have benefited from this approach, but above all the study of complex humanistic systems, such as the study of service quality.

Let the universe of discourse X be the subset of real numbers R, $X = \{x_1, x_2, x_3, \cdots; x_n\}$. A fuzzy set $\tilde{A} = \{(x, \mu_A(x))|x \in X\}$ in X is a set of ordered pairs, where $\mu_A(x)$ is called a membership function and $\mu_A(x) : X \to [0, 1]$. The membership function for fuzzy sets can take any value from the closed interval $[0, 1]$. The greater $\mu_A(x)$ is, the greater the truth of the statement that element x belongs to set A is.

In this paper, we are going to parameterize a triangular fuzzy number $\tilde{A}$ by a triplet (a_1, a_2, a_3).

The membership function $\mu_A(x)$ is defined below.

$$\mu_A(x) = \begin{cases} \frac{x-a_1}{a_2-a_1}, & a_1 \le x \le a_2, \\ \frac{x-a_3}{a_2-a_3}, & a_2 \le x \le a_3, \\ 0, & otherwise. \end{cases} \quad (2.1)$$

Each linguistic term was characterized by a triangular fuzzy number for representing its approximate value range between 0 and 100,[1] and denoted as (a_1, a_2, a_3), where $0 \le a_1 \le a_2 \le a_3 \le 100$. a_2 is the most likely value of the linguistic term, and a_1 and a_3 are the lower and upper bounds used, respectively, to reflect the fuzziness of the term.

We could not reflect the fact that respondents may have different perceptions of these linguistic terms, but hopefully this caveat is not very important because we have used some representative default values to reflect the preferences that have been previously employed.[2]

Vagueness of linguistic terms about satisfaction degree has already been set up. So in order to provide more objective information for DMU managers, we have fuzzified satisfaction degree as triangular fuzzy numbers and aggregated group opinions of consumers according to the average fuzzy number of n triangular numbers $\tilde{A}_i = \left(a_1^{(i)}, a_2^{(i)}, a_3^{(i)}\right)$ where $i = 1, 2, \cdots, n$, as follows:

$$\tilde{A} = (a_1, a_2, a_3) = \left(\frac{1}{n}\right) \bullet \left(\tilde{A}_1 \oplus \tilde{A}_2 \oplus \cdots \oplus \tilde{A}_n\right)$$

$$= \left(\frac{\sum_{i=1}^n a_1^{(i)}, \sum_{i=1}^n a_2^{(i)}, \sum_{i=1}^n a_3^{(i)}}{n}\right), \quad (2.2)$$

where $\bullet$ is the multiplication of a scalar and a fuzzy number, and $\oplus$ is the add operation of fuzzy numbers, so $\tilde{A}$ is the overall average performance valuation of some DMU/ date (observation) under some attribute over n interviewed consumers. Eq. (2.2) shows that the average performance can be represented by a new triangular fuzzy number. To justify whether a DMU attribute is weak or strong, we need to defuzzy the information obtained above. The result of fuzzy synthetic information of each observation is a fuzzy number.

[1]We have used this range, but other ranges such as (0-7) or (0-10) would also be valid.

[2]The survey process is conducted without having in mind the treatment of fuzzy set theory methodology. However, it would be very easy that each respondent has the option of defining a new triplet more concordant with his perception.

Therefore, it is necessary to employ some non-fuzzy ranking method for fuzzy numbers during service quality comparison for each observation. In other words, Defuzzification is a technique to convert the fuzzy number into crisp real numbers. The procedure of defuzzification is to locate the Best Nonfuzzy Performance (BNP) value. This purpose can be attained by several available methods. Mean-of-Maximum, Center-of-Area, and α-cut Method [23] are some of the most common approaches. In this paper, we compare the performance of two triangular fuzzy numbers using $\upsilon_{\tilde{A}}$ defined as follows, $\upsilon_{\tilde{A}} = \dfrac{(a_1 + 2a_2 + a_3)}{4}$ for the triplet (a_1, a_2, a_3) of a triangular fuzzy number $\tilde{A}$. This method [19] has been chosen due to its simplicity and the lack of requirement of analyst's personal judgment. The method is based on Kaufmann and Guptas [15] method to compare fuzzy numbers and its logic is underpinned in the definition of the removal of a fuzzy number.

In this paper, we are going to employ a method based in the TOPSIS approach. Hwang and Yoon [14] proposed the following logic of TOPSIS, defining the ideal solution and the negative ideal solution. The positive ideal solution is the solution that maximizes the benefit criteria and minimizes the cost criteria; whereas the negative ideal solution has got the opposite logic, i.e. maximizes the cost criteria and minimizes the benefit criteria. The optimal observation is the one, which is closest to the ideal solution and farthest to the negative ideal solution. The ranking of alternatives in TOPSIS is based on the relative similarity to the ideal solution, which avoids from the situation of having same similarity to both ideal and negative ideal solutions.

To sum up, an ideal solution is composed of all best values attainable of criteria, whereas a negative ideal solution is made up of all worst values attainable of criteria. During the processes of selection of observation, the best alternative would be the one that is nearest to the positive ideal solution and farthest from the negative ideal solution. Take the objective space of the two criteria as example which is indicated in Fig. 1, A^+ and A^- are, respectively, the ideal solution and negative ideal solution, and observation A_1 is shorter in distance in regard to the ideal solution (A^+) and negative ideal solution (A^-) than A_2. As a matter of fact, the ups and downs of these two observations regarding to ideal solutions cannot be compared because there exists some tradeoff between the ups and downs. However, TOPSIS can help resolving this problem because it has defined such relative closeness so as to consider and correlate, as a whole, the distance to the ideal solution and the negative ideal solution.

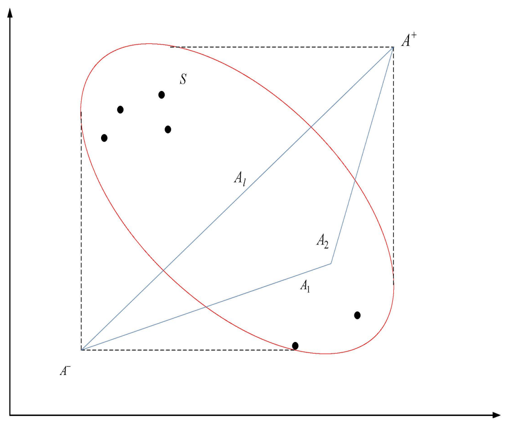

Figure 1: Distance between ideal solution and negative ideal solution.

Ideal solutions are computed based on the following equations:

$$A^{+} = \{(max\ V_{ij} \mid j \in J), \left(min\ V_{ij} \mid j \in J^{'}\right), \quad i = 1, 2, \cdots, m\}, \tag{2.3}$$

$$A^{-} = \{(min\ V_{ij} \mid j \in J), \left(max\ V_{ij} \mid j \in J^{'}\right), \quad i = 1, 2, \cdots, m\}, \tag{2.4}$$

where J and $J^{'}$ form a partition of the different criteria according to their benefit or cost characteristic.

After the determination of ideal solutions, we calculate the Euclidean distance between ideal solution and negative ideal solution for each observation as

$$S_i^{+} = dist(V_i, A^{+}) = \sqrt{\sum_{j=1}^{n} (V_{ij} - A_i^{+})^2}, \quad i = 1, 2, \cdots, m, \tag{2.5}$$

$$S_i^{-} = dist(V_i, A^{-}) = \sqrt{\sum_{j=1}^{n} (V_{ij} - A_i^{-})^2}, \quad i = 1, 2, \cdots, m, \tag{2.6}$$

Then we calculate the relative closeness to the positive ideal solution of each observation, such as

$$C_i = \frac{S_i^{-}}{S_i^{+} + S_i^{-}}, \quad i = 1, 2, \cdots, m, \tag{2.7}$$

where $0 \leq C_i \leq 1$. An observation is closer to an ideal solution as C_i approaches to 1. A set of alternatives can be sorted according to the descending order of C_i. This approach has been widely used in different decision contexts [5, 6, 7, 8, 9, 10, 11, 12]. This is mainly due to its applicability in solving different scenarios of human decision problems, and its mathematical simplicity measuring the relative performance of the alternatives. The rationale behind (2.7) is that a better performance of a pair DMU should be captured by a higher degree of similarity to the positive ideal solution and a lower degree of similarity to the negative ideal solution. The larger the performance index, the better the overall service quality performance of the alternative DMU, relative to other surveys done in the same DMU. As such, the performance index calculated is a relative concept and it indicates the relative ranking obtained dynamically for each one of the DMUs evaluated in terms of the service attributes included in the survey.

2.2 Ranking methods

In spite of their wide variety and their common scope of establishing an ordering relation on F, ranking methods can be divided into two types:

- Ranking methods of the first type map fuzzy numbers directly into the real line. That is, they are transformation functions $M : F \to \Re$ which associate each fuzzy number with a real number and then use the ordering $\geq$ on the real line. For most of the methods (with the exceptions of Y_3 and K), a higher associated value indicates a higher rank:

$$M(A_i) \geq M(A_j) \Longrightarrow A_i \succeq_M A_j$$

where $\succeq_M$ is the dominance relation induced by M. Whenwe use Y_3 and K to obtain the ranking, the rule is inverted and smaller values of M indicate higher rank. Another theoretical difference within this family of methods is that those proposed by Chen [20] and Kerre [17] employ some reference points in calculating the ranking index which is based on the set of fuzzy numbers under comparison. In other words, the real number associated to a given fuzzy number A_i depends on the whole set $\{A_1, \cdots, A_n\}$ and not only on the characteristics of A_i.

- Rankingmethods of the second type generate fuzzy binary relations. In this case themethods are functions $M : F \times F \to [0, 1]$ where the value of the relation $M(A_i, A_j) \in [0, 1]$ is the degree to which A_i is greater than A_j . Consequently, the fuzzy numbers are ranked according to the following rule.

$$M(A_i, A_j) \geq M(A_j, A_i) \Longrightarrow A_i \succeq_M A_j.$$

On the basis of the fuzzy binary relation on the set of fuzzy numbers, some procedures allowthe derivation of an ordering relation.
Hereafter, until the end of this section, we shall introduce the ranking methods included in the study, with a special consideration for the most recent ones. As it was pointed out in the previous section, there are numerous methods to rank fuzzynumbers. In this paper,we focus on themethods that gained high importance in the literature. Themethods whichwere excluded from the analysis include: probabilistic approaches [18], centroid and original point based methods [22], distance-based ranking [21], preference weighting function expectation [26].

2.2.1 Adamo

When employing the method suggested by Adamo [33], one simply evaluates the fuzzy number based on the rightmost point of the -cut for a given α:

$$AD_\alpha(A) = a_\alpha^+. \tag{2.8}$$

It is important to mention that Adamo's approach is the only one which satisfies all the reasonable properties proposed in [33] for ordering fuzzy quantities.

2.2.2 Center of maxima

The center of maxima [9] of a fuzzy number is calculated as the average value of the endpoints of the modal values interval (x is in the modal values interval if $A(x) = 1$):

$$CoM(A) = \frac{a_1^- + a_1^+}{2}. \tag{2.9}$$

In case of fuzzy numbers, this definition coincides with the mean of maxima method [9]: the average of all the values contained in the modal interval.

2.2.3 Center of gravity

The center of gravity of a fuzzy number was introduced in [12] as

$$CoG(A) = \frac{\int_{-\infty}^{+\infty} xA(x)dx}{\int_{-\infty}^{+\infty} A(x)dx}. \tag{2.10}$$

Yager [28] proposed four different approaches for the ranking of fuzzy subsets with the support in the unit interval: if we consider only this subclass of fuzzy numbers, one of the proposed indices,

$$Y_1(A) = \frac{\int_0^1 g(x)A(x)dx}{\int_0^1 A(x)dx}, \tag{2.11}$$

where $g(x)$ measures the importance of x, can be seen as a generalization of the ranking based on the center of gravity.

2.2.4 Median

The concept of themedian value of a fuzzynumber generalizes the definition of median to fuzzy numbers by minimizing the following expression

$$\mid \int_{-\infty}^{Med(A)} A(x)dx - \int_{Med(A)}^{-\infty} A(x)dx \mid \tag{2.12}$$

The median can be interpreted as the center of area (CoA) of a fuzzy number A as it divides the area under the membership function into two equal parts.

2.2.5 Credibilistic mean

Liu and Liu [27] proposed the concept of credibility measure based on four axiomatic properties and proved that the original definition is equivalent to the following formulation:

$$Cr(B) = \frac{Pos(B) + Nec(B)}{2}, \tag{2.13}$$

where $B \subset R$, i.e., the credibilitymeasure is the arithmeticmean of the possibility and necessity measures. Using this novel concept, they defined the credibilistic expectation of a fuzzy variable A [27]:

$$CrMean(A) = \int_{-\infty}^{0} Cr(A \geq x)dx - \int_{0}^{+\infty} Cr(A \leq x)dx. \tag{2.14}$$

2.2.6 Changs method

Chang [2] proposed a ranking method based on the index

$$C(A) = \int_{x \in suppA} xA(x)dx. \quad (2.15)$$

It can be observed that $CoG(A) = \frac{C(A)}{\int_{-\infty}^{+\infty} A(x)dx}$.

2.2.7 Possibilistic mean

The possibilistic mean value [10] of a fuzzy number $A \in F$ is the weighted average of the middle points of the α-cuts of a fuzzy number A:

$$Ep(A) = \int_0^1 \alpha(a_\alpha^- + a_\alpha^+)d\alpha \quad (2.16)$$

The definition of possibilistic mean is based on the ordering proposed by Saneifard et al [10]. Saneifard et al, [11] extended the original definition by replacing the weight with a general weighting function $f(\alpha)$.

2.2.8 Yager's approaches

In [28], additionally to Eq. (2.11), Yager proposed three other different ranking methods for fuzzy quantities in the unit interval. The first index is

$$Y_2(A) = \int_0^{hgt(A)} M(A_\alpha)d\alpha, \quad (2.17)$$

where $hgt(A) = sup_{xsuppA}A(x)$ is the height of A and M is the mean value operator, can be used to rank fuzzy numbers with arbitrary support. In this case, $hgt(A) = 1$ and $M(A_\alpha) = \frac{a_\alpha^- + a_\alpha^+}{2}$.

Note 1. The index Y_2 coincides with other methods when we consider the class F: the signed distance [], the nearest point of a fuzzy number [28], the total integral valuewith index of optimism $\alpha = 0.5$ [28], the average index of a fuzzy number with optimism-pessimism degree $\lambda = 0.5$ [28], and the approach proposed by Fortemps and Roubens [4]. Additionally, it is proportional to the signed distance proposed in [6] with the constant 0.5.
The two other methods proposed by Yager are the following:

$$Y_3(A) = \int_0^1 \mid x - A(x) \mid dx, \quad (2.18)$$

$$Y_4(A) = sup_{x[0,1]} min(x, A(x)). \quad (2.19)$$

2.2.9 Chen's method

Chen [20] defined a ranking method using the concepts of fuzzy maximizing and minimizing sets:

$$A_{max}(x) = \left(\frac{x - x_{min}}{x_{max} - x_{min}}\right)^k,$$

$$A_{min}(x) = \left(\frac{x_{max} - x}{x_{max} - x_{min}}\right)^k$$

where $x_{max} = sup \bigcup_{i=1}^n suppA_i$ and $x_{min} = inf \bigcup_{i=1}^n suppA_i$ and $k > 0$ is a real number. Using these two sets, Chen defined the left and right utility of a fuzzy number A_i as

$$L(Ai) = sup_{xR} min(A_{min}(x), A_i(x)),$$

$$R(Ai) = sup_{xR} min(A_{max}(x), A_i(x)),$$

and finally the ranking index is obtained as

$$CH^k(A_i) = \frac{1}{2}(R(A_i) + 1 - L(A_i)). \quad (2.20)$$

2.2.10 Kerre's method

Kerre [17] defined a ranking index based on the Hamming-distance of fuzzy numbers: the index is calculated by determining the distance between A_i and $m\tilde{a}x(A_1, \cdots, A_n)$:

$$K(A_i) = \int_{x \in S} \mid A_i(x) - m\tilde{a}x(A_1, \cdots, A_n)(x) \mid dx, \quad (2.21)$$

where $S = \bigcup_{i=1}^n suppA_i$.

2.2.11 Baas and Kwakernaak's method

The method of Baas and Kwakernaak belongs to the second family of methods. The value of the relation $P_{BK}(A_i, A_j)$ quantifies the degree to which A_i is greater than A_j as follows:

$$P_{BK}(A_i, A_j) = sup_{x_i \geq x_j} min(A_i(x_i), A_j(x_j)).$$

The ranking index for a fuzzy number is then obtained as

$$BK(A_i) = min_{j \neq i} P_{BK}(A_i, A_j).$$

It is worth noting that P_{BK} coincides with the fuzzy relation PD introduced by Dubois and

Prade in [3]. It is important to mention that the rankings produced by the two methods can be different: Baas and Kwakernaak's approach is based on the minimum value of P_{BK} and Dubois and Prade's PD relation can be used according to the ordering procedure described in [3].

2.2.12 Nakamura's method

Nakamura [24] defined the following parametric method based on the fuzzy relation

$$P_{N^{\lambda}}(A_i, A_j) =$$

$$\frac{\lambda d_H(\underline{A}_i, \tilde{min}(\underline{A}_i, \underline{A}_j))}{\lambda d_H(\underline{A}_i, \underline{A}_j) + (1-\lambda)(d_H(\overline{A}_i, \overline{A}_j))}$$

$$+\frac{(1-\lambda)(d_H(\overline{A}_i, \tilde{min}(\overline{A}_i, \overline{A}_j)))}{\lambda d_H(\underline{A}_i, \underline{A}_j) + (1-\lambda)(d_H(\overline{A}_i, \overline{A}_j))}, \ \lambda \in [0,1],$$

where $d_H(A_i, A_j) = \int_R \mid A_i(x) - A_j(x) \mid dx$ is the Hamming distance between two fuzzy numbers, $\underline{A}_i(x) = sup_{y \le x} A_i(y)$ and $\overline{A}_i(x) = sup_{y \ge x} A_i(y)$. When $\lambda d_H(\underline{A}_i, \underline{A}_j) + (1-\lambda)(d_H(\overline{A}_i, \overline{A}_j)) = 0$, the value of the relation is defined as $P_{N_\lambda}(A_i, A_j) = 0.5$.

Remark 2.1 *Wang and Kerre proved in [30, 31] that $P_{N^0.5}$ induces the same ranking of fuzzy numbers as Yuan's method [29], Saade and Schwarzlander's approach [13] and the two methods proposed by Kolodziejczyk in [16].*

3 Ranking fuzzy numbers based on the positive and negative ideal solutions

In this section, a novel method based on on the positive and negative ideal solutions of fuzzy numbers is presented for ranking fuzzy numbers. By this method, we will resolve the shortcomings discussed in other mthods.

Definition 3.1 *Let A_i and A_j be two fuzzy numbers characterized by (2.1) and C_i and C_j are the relative closeness to the positive ideal solution of each observation.*
Since this article wants to approximate a fuzzy number by a scalar value, thus the researchers have to use an operator $C : F \to R$ which transforms fuzzy numbers into a family of real line. Operator C is a crisp approximation operator. Since every above defuzzification can be used as a crisp approximation of a fuzzy number, therefore the resultant value is used to rank the fuzzy numbers. Thus, C is used to rank fuzzy numbers. The larger C, the larger fuzzy number

Let A_i and A_j be two arbitrary fuzzy numbers. Define the ranking of A_i and A_j by C on F as follows:

(i) $A_i \succ A_j$ if and only if $C(A_i) > C(A_j)$,

(ii) $A_i \prec A_j$ if and only if $C(A_i) < C(A_j)$,

(iii) $A_i \sim A_j$ if and only if $C(A_i) = C(A_j)$.

Here, the following reasonable axioms that Wang and Kerre [30, 31] have proposed for fuzzy quantities ranking are considered.

$\mathcal{A}_1$: For an arbitrary finite subset Γ of E and $A \in \Gamma$, $A \succeq A$.

$\mathcal{A}_2$: For an arbitrary finite subset Γ of E and $(A, B) \in \Gamma^2$, $A \succeq B$ and $B \succeq A$, we should have $A \sim B$.

$\mathcal{A}_3$: For an arbitrary finite subset Γ of E and $(A, B, C) \in \Gamma^3$, $A \succeq B$ and $B \succeq C$, we should have $A \succeq C$.

$\mathcal{A}_4$: For an arbitrary finite subset Γ of E and $(A, B) \in \Gamma^2$, $\inf supp(A) > \sup supp(B)$ we should have $A \succeq B$.

$\mathcal{A}'_4$: For an arbitrary finite subset Γ of E and $(A, B) \in \Gamma^2$, $\inf supp(A) > \sup supp(B)$ we should have $A \succeq B$.

$\mathcal{A}_5$: Let Γ and Γ' be two arbitrary finite subset of E in which A and B are in $\Gamma \bigcap \Gamma'$. We obtain the ranking order $A \succ B$ by defuzzification $RM_a(A)$ on Γ' if only and if $A \succ B$ by $RM_a(A)$ on Γ.

$\mathcal{A}_6$: Let A, B, $A + B$ and $B + C$ be elements of E. If $A \succeq B$ then $A + C \succeq B + C$.

$\mathcal{A}'_6$: Let A, B, A+B and B+C be elements of E. If $A \succeq B$ then $A + C \succ B + C$.

In addition for the axioms $A_1, A_2, \cdots, A_6$ we can consider two following properties:

$\mathcal{A}_7$: For an arbitrary finite subset Γ of E and $A \in \Gamma$, the defuzzification must belong to its support function.

$\mathcal{A}_8$: For an arbitrary finite subset Γ of E and $A \in \Gamma$, distance between A and its defuzzification will be minimized.

Since C is a real number, then the proof of above axioms is evident.

Remark 3.1 *If $A_i \preceq A_j$, then $-A_i \succeq -A_j$.*

Hence, this article can infer ranking order of the images of the fuzzy numbers.

4 Numerical example

In this section, some examples are used to illustrate the proposed approach to ranking fuzzy numbers.

Example 4.1 *Consider the following fuzzy numbers $A_1 = (0.2, 0.5, 0.8)$ and $A_2 = (0.4, 0.5, 0.6)$ taken from [10]. Now using our method, we have $A_1 = 0.149$, $A_2 = 0.184$. Therefore, the ranking order is $A_1 \prec A_2$. The ranking indices are $d_1 = 0.0909$, $d_2 = 0.1566$ and the order by the deviation degree method is: $A_1 \prec A_2$. Therefore, the results of the two methods are the same. The left and the right areas of A_1 and A_2, as well as their centroid points are the same. This example has also been solved in [11], where the deviation degree method was used. The following example shows that the proposed method can handle the cases where some of A_is may have non-linear membership functions.*

Example 4.2 *Consider the triangular fuzzy number $A = (1, 2, 5)$, and the fuzzy number $B = (1, 2, 2, 4, 1)$ shown in Figure 2 taken from [10]. The non-linear membership function of A_2 is defined by*

$$\mu_B(x) = \begin{cases} \sqrt{1-(x-2)^2} & \text{when } x \in [1,2], \\ \sqrt{1-\frac{1}{4}(x-2)^2} & \text{when } x \in [2,4], \\ 0 & \text{otherwise.} \end{cases}$$

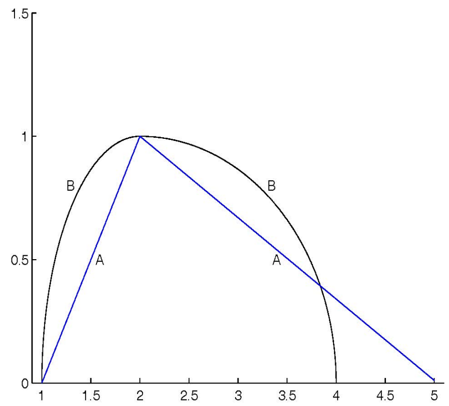

Figure 2: Fuzzy Numbers A and B.

5 Conclusion

This paper presents a new approach for ranking $L-R$ fuzzy numbers. The examples given in this paper illustrate that the proposed approach gives the correct ordering of fuzzy numbers. Comparing with the existing approaches, it is efficient and simple. For the validation of the proposed ranking function, it is shown that this ranking function satisfies all the reasonable properties of fuzzy quantities (I) and (II) proposed by X. Wang, E. E. Kerre [30, 31].

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