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Two-stage Malmquist Productivity Index with Intermediate Products

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Abstract

Different processes in Decision Making Unit (DMU) are the same as subprocesses in that DMU when it is not considered as blackbox. Most of the time these subprocesses are mooted in a series structure and frequently used in real world applications. When it is aimed to evaluate the performance of a unit with its subprocesses and what it did in the past, those techniques which show progress and regress can be used. One of these famous techniques is Malmquisr Productivity Index (MPI). Here MPI is developed and used for series structural DMUs, with two components, in which intermediate inputs and outputs exist.

Keywords : Malmquist Productivity Index; Data Envelopment Analysis; Multi-component model; Progress; Regress.

1 Introduction

Having a thorough knowledge relating to the performance of under evaluation DMUs, which are under the manager's control, is considered as the main concern of the manager to guide those units. Complexity in information, effects of external factors, and impression of rivals on performance are of fundamental importance that managers without scientific attitude can not solve. In regard to the widespread usage of the Malmquist Productivity Index for measuring the total factor productivity units, a growing body of literature has been developed. In accordance with this fact that one of the major sources of economic development is productivity growth, Nowadays an in-depth interpretation of the factors which affects productivity is very prominent.

Malmquist S. [8] , in 1953, published a quantity index in which input distance functions are used to make comparison among two or more consumption bundles. Later in 1982,

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in production analysis Caves D.W., Christensen L.R. and Diewert W.E. [1] (CCD), introduced Malmquist Productivity Index on basis of what malmquist had proposed. Nowadays applications of the Malmquist Productivity Index have enjoyed a great deal of attention. Grifell-tatj E. and Lovell C.A.K. [5] in their paper tried to adopt a different approach to the use of DEA with panel data and consequently, created a malmquist index of productivity change and provided a new decomposition of it. They indicated that by omitting the bias effect, if it is important, an inappropriate allocation of its effect to the two included components can be caused. Thus, their decomposition allocates the measured productivity change to three mutually exclusive and exhaustive components: efficiency change, the magnitude of technical change, and the bias of technical change. In their paper, G.R. Jahanshahloo, R. Shahverdi and M. Rostamy-Malkhalifeh [7] used Malmquist productivity index to evaluate the decision making Units with interval data. Also, a method for assessing Malmquist productivity index using cost efficiency also was developed.

Chen Y. [2], extended the Malmquist Productivity Index into a non- radial index on bases of the fact that DEA-based Malmquist Productivity Index measures the technical and productivity changes over time. The advantage of this index is that, while non-zero slacks are being considered, it eliminates possible ineffficiency. In their paper, G. R. Jahanshahloo, F. Hosseinzadeh Lotfi, A. A. Noora and B. Rahmani Parchikolaei [6] used malmquist productivity index inorder to determine progress and regression of human development index in some Asian countries.

In a paper provided by M. Navanbakhsh, G. R. Jahanshahloo, F. Hossienzadeh Lotfi and Taeb, Z. [9] Revenue Malmquist productivity index was developed. It should be noted that this index could be calculated when the price of each outputs is available and progress and regress of output revenue is the basis.
Here the aim is to develop malmquist productivity index in order to show the progress or regress of subprocesses that may exist within a DMU. This is mooted and also, the related models and equations are formulated while two-stage series structural DMUs are considered. Finally, the progress and regress of each subprocess and the entire DMU is obtained.

The current article proceeds as follows: first some preliminaries about DEA and MPI are reviewed. In section 3 the main idea is mooted and section 4 concludes the paper.

2 Priliminaries

2.1 DEA

The relative efficiency can be acquired from various viewpoints. In this section we briefly review the DEA models which yield the relative efficiency scores.
Solving the following model which is called "CCR" after the names of the authors, Charnes et al. [2] , the relative efficiency under the constant returns to scale technology can be obtained. In the following models there exist n $DMUs$ with m inputs and s outputs, x_j and y_j are the given input and output vectors of DMU_j, j=1,...,n, whose elements are all

positive.

$$\begin{aligned} \min \quad & \theta \\ s.t. \quad & X\lambda \leq \theta X, \\ & Y\lambda \geq Y, \\ & \lambda \geq 0. \end{aligned} \tag{2.1}$$

The dual of the above model which is called multiplier form is as follows:

$$\begin{aligned} \max \quad & U^tY_o \\ s.t. \quad & V^tX_o = 1, \\ & U^tY_j - V^tX_j \leq 0, \quad j = 1, \ldots, n, \\ & U \geq 0, V \geq 0. \end{aligned} \tag{2.2}$$

where v and u are the input and output weight vectors.
Both of the aforesaid models are in input orientation. For an input- oriented projection, one seeks a projection such that the proportional reduction in inputs is maximized.

2.2 Malmquist Productivity Index

It is possible to estimate the Malmquist Productivity Index, which can be used in order to measure the productivity changes over time by using DEA methodology. DEA models are linear programming (LP) models that derive the frontier production function of the DMU included in the sample. While working with DEA the idea behind efficiency analysis is to use data collected for DMUs to develop the best practice frontier. The DMUs that operate best practice are technically efficient, whereas the degree of technical inefficiency of the rest is calculated on the basis of the Euclidian distance of their input-output ratio from the frontier of production function. It is of great importance to include this aspect of the production process that constituents of the best practice frontier can obviously change over time. The Malmquist DEA approach calculates an efficiency measure for one year relative to the prior year, while allowing the best frontier to shift. Between these time points (time t and t+1) the frontier function has shifted from frontier t to frontier t+1. The Malmquist Productivity Index can be further decomposed into technical efficiency change (EC) and technological change (TE) relative to the frontier. Hence the malmquist growth is the product of technical efficiency change and technological change.
Malmquist Productivity Index is defined with the assimilation of efficiency changes of each unit and technology changes. MPI can be calculated via several functions, such as distance function. In this paper, the DEA is used to compute the distance functions of Malmquist Productivity Index. First, the notion underlying the Malmquist Productivity Index and data envelopment analysis models are discussed.
Let DMU_l denote a unit from a total n units in which relative efficiency is being evaluated. Define $x_l \in R_+^m$ and $y_l \in R_+^s$ as semipositive input and output vectors of DMU_l. The most general way of characterization of production technology is production possibility set T, which is defined with a set of semipositive (x, y) as:

$$T = \{(x, y)| \quad x \geq X\lambda, \quad y \leq Y\lambda, \quad \lambda \geq 0\}$$

As discussed earlier Malmquist Productivity Index can be calculated via several functions, such as distance function:

$$D(X_l, Y_l) = Min\{\theta : (\theta X_l, Y_l) \in T\}$$

The resultant distance function can be computed by solving linear programming problems. Consider an input-oriented CCR model as follows:

$$\begin{aligned} D^f(x_l^k, y_l^k) = \min \quad & \theta \\ s.t. \quad & X^f\lambda \leq \theta X^k, \\ & Y^f\lambda \geq Y^k, \\ & \lambda \geq 0. \end{aligned} \tag{2.3}$$

in which l is the unit under assessment and each of k and f vary between time t and t+1. As an instance for assessing DMU_l consider k=t and f=t+1, $D^{t+1}(x_l^t, y_l^t)$, this means that DMU_l is considered in time t while technology is considered in time t+1. Considering this notification, four LP problems can be defined.
With regard to this subject, Caves D.W., Christensen L.R. and Diewert W.E. [1] have introduced the Malmquis Productivity Index as follows in which the results obtained from the mentioned models are being used.

$$M(x_l^{t+1}, y_l^{t+1}, x_l^t, y_l^t) = \frac{D^t(x_l^{t+1}, y_l^{t+1})D^{t+1}(x_l^{t+1}, y_l^{t+1})}{D^t(x_l^t, y_l^t)D^{t+1}(x_l^t, y_l^t)} \tag{2.4}$$

In which x_l^t and y_l^t are the input and output vectors for unit l, used in period t. Also, x_l^{t+1} and y_l^{t+1} are the input and output vectors for unit l, used in period t+1. This index measures the productivity of unit l at the production (x_l^{t+l}, y_l^{t+l}) relative to (x_l^t, y_l^t).
The above equation can be further decomposed into two components one for measuring the change in technical efficiency and the other for measuring the technical change which means the technology frontier shift between the two time periods, t and t+l:

$$M(x_l^{t+1}, y_l^{t+1}, x_l^t, y_l^t) = \frac{D^{t+1}(x_l^{t+1}, y_l^{t+1})}{D^t(x_l^t, y_l^t)}[\frac{D^t(x_l^{t+1}, y_l^{t+1})D^{t+1}(x_l^{t+1}, y_l^{t+1})}{D^t(x_l^t, y_l^t)D^{t+1}(x_l^t, y_l^t)}]^{\frac{1}{2}} \tag{2.5}$$

The efficiency change share in the above-mentioned equation is equal to the ratio of the Farrell technical efficiency measure at time t+1, which is divided by the Farrell technical efficiency measure at time t. The technical change part is captured by the geometric average of the two ratios reflecting the shifts in the frontier at time t, and t+1. The interpretation of this equation is that $M(x_l^{t+1}, y_l^{t+1}, x_l^t, y_l^t) > 1$ indicates an improvement in total productivity, $M(x_l^{t+1}, y_l^{t+1}, x_l^t, y_l^t) < 1$ indicates a decline, and $M(x_l^{t+1}, y_l^{t+1}, x_l^t, y_l^t) = 1$ shows an unchanged productivity growth, Caves D.W., Christensen L.R. and Diewert W.E. [1].

3 Main subject

Consider, there exist n DMUs to be evaluated each of which has two subprocesses with intermediate input and output. The above mentioned can be schematicaly portrayed as follows:

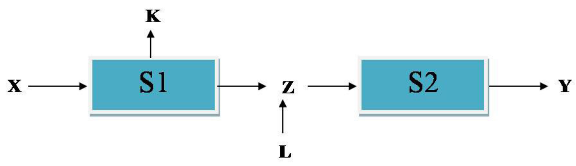

Fig. 1. *A DMU with two stages.*

Let X be the input vector of the first subprocess and K be the output vector of this process, which is a portion of the final output. Also, let L be the input vector of the second process ,Z be the output of the first process and input of the second process (intermediate product) and Y be the final output of the second process. Thus each of the DMUs has the input-output vector as (X, K, Z, L, Y) where $x_j \in R^m, Y_j \in R^s, Z \in R^k, L \in R^I, K_j \in R^b$. Therefore the efficiencies of the first and the second processes and aggregate efficiency for DMU_p is as follows:

$$e_p^1 = \frac{W\,Z_p + \overline{U}K_p}{V\,X_p}, \quad e_p^2 = \frac{U\,Y_p}{W\,Z_p + \overline{V}\,L_p}, \quad e_p^a = \frac{U\,Y_p + \overline{U}K_p}{V\,X_p + \overline{V}L_p} \tag{3.6}$$

$$e_p^1 = \frac{W\,Z_p + \overline{U}K_p}{V\,X_p}, \quad e_p^2 = \frac{U\,Y_p}{W\,Z_p + \overline{V}\,L_p}, \quad e_p^a = \frac{U\,Y_p + \overline{U}K_p}{V\,X_p + \overline{V}L_p} \tag{3.7}$$

in which V, $\bar{U}$, W, $\bar{V}$ and U are, respectively, the weight corresponds to the input-output vector (X, K, Z, L, Y). It should be noted that $\theta > 0$.
Thus the relative efficiency score of the DMU_p is calculated by solving model (3.8):

$$\begin{aligned}
Max \quad & \frac{U\,Y_p + \overline{U}K_p}{V\,X_p + \overline{V}L_p} \\
S.t. \quad & \frac{U\,Y_j + \overline{U}K_j}{V\,X_j + \overline{V}L_j} \le 1, \quad j = 1, \ldots, n, \\
& \frac{W\,Z_j + \overline{U}K_j}{V\,X_j} \le 1, \quad j = 1, \ldots, n, \\
& \frac{U\,Y_j}{W\,Z_j \,+ \overline{V}\,L_j} \le 1, \quad j = 1, \ldots, n, \\
& U \ge 1\varepsilon, \quad V \ge 1\varepsilon, \quad W \ge 1\varepsilon, \\
& \overline{U} \ge 1\varepsilon, \quad \overline{V} \ge 1\varepsilon.
\end{aligned} \tag{3.8}$$

For solving this fractional model, first, it is converted to its linear counterpart.

$$\begin{aligned}
Max \quad & U\,Y_p + \overline{U}K_p \\
S.t. \quad & UY_j + \overline{U}K_j - VX_j - \overline{V}L_j \le 0, \quad j = 1, \ldots, n, \\
& WZ_j + \overline{U}K_j - VX_j \le 0, \quad j = 1, \ldots, n, \\
& UY_j - WZ_j - \overline{V}L_j \le 0, \quad j = 1, \ldots, n, \\
& VX_p + \overline{V}L_p = 1, \\
& U \ge 1\varepsilon, \quad V \ge 1\varepsilon, \quad W \ge 1\varepsilon, \\
& \overline{U} \ge 1\varepsilon\overline{V} \ge 1\varepsilon.
\end{aligned} \tag{3.9}$$

After solving model (3.9), in regard to the above-mentioned equations, the efficiency scores of the first and second subprocesses and the aggregate efficiency score of DMU_p can be calculated.
In the aforesaid model the first bundles of constraints can be derived from summing the second and the third bundles of constraint, thus it is redundant. As regards this model can be written as:

$$\begin{array}{ll} Max & U\,Y_p + \overline{U}K_p \\ S.t & WZ_j + \overline{U}K_j - VX_j \leq 0, \quad j = 1, \ldots, n. \\ & UY_j - WZ_j - \overline{V}L_j \leq 0, \quad j = 1, \ldots, n, \\ & VX_p + \overline{V}L_p = 1, \\ & U \geq 1\varepsilon, \quad V \geq 1\varepsilon, \quad W \geq 1\varepsilon, \\ & \overline{U} \geq 1\varepsilon, \quad \overline{V} \geq 1\varepsilon. \end{array} \tag{3.10}$$

Moreover the dual to this problem is:

$$\begin{array}{ll} Max & \theta - \varepsilon(1SX + 1SZ + 1SK + 1SL + 1SY) \\ S.t. & \sum_{j=1}^{n} \lambda_j^1 X_j + SX = \theta X_p, \\ & \sum_{j=1}^{n} \lambda^1 z_j - \sum_{j=1}^{n} \lambda^2 z_j - SZ = 0, \\ & \sum_{j=1}^{n} \lambda^1 K_j - SK = K_p, \\ & \sum_{j=1}^{n} \lambda^2 L_j + SL = \theta L_p, \\ & \sum_{j=1}^{n} \lambda_j^2 Y_j - SY = Y_p, \\ & \lambda_j^1 \geq 0, \quad j = 1, \ldots, n, \quad \lambda_j^2 \geq 0, \quad j = 1, \ldots, n, \\ & SX \geq 0, \quad SZ \geq 0, \\ & SK \geq 0, \quad SL \geq 0, \quad SY \geq 0. \end{array} \tag{3.11}$$

Now, let a set of DMUs with the current situation in two different times, t and t+1, be at hand. In other words let $(X_j^t, K_j^t, Z_j^t, L_j^t, Y_j^t)$ and $(X_j^{t+1}, K_j^{t+1}, Z_j^{t+1}, L_j^{t+1}, Y_j^{t+1})$ be the coordinate vector of DMU_j in time t and $t+1$. Thus for calculating the malmquist productivity index of the first and the second subprocesses and the aggregate malmquist productivity index, four problems need to be solved.
The efficiency score of DMU_p in time t while technology is considered in time t is derived

from solving model (3.12):

$$\begin{aligned} \theta^t(t) = Max \quad & UY_p^t + \overline{U}K_p^t \\ S.t. \quad & WZ_j^t + \overline{U}K_j^t - VX_j^t \leq 0, \quad j = 1, \ldots, n, \\ & UY_j^t - WZ_j^t - \overline{V}L_j^t \leq 0, \quad j = 1, \ldots, n, \\ & VX_p^t + \overline{V}L_p^t = 1, \\ & U \geq 1\varepsilon, \quad V \geq 1\varepsilon, \quad W \geq 1\varepsilon, \\ & \overline{U} \geq 1\varepsilon, \quad \overline{V} \geq 1\varepsilon. \end{aligned} \tag{3.12}$$

Let $(V^*, \bar{U}^*, W^*, \bar{V}^*, U^*)$ be the optimal solution of the above model, thus the efficiency scores of the first and the second subprocesses and the aggregate efficiency score in time t, while technology is considered in time t, can be obtained from expression (3.13).

$$\theta_p^{1,t}(t) = \frac{W^* Z_p^t + \overline{U}^* K_p^t}{V^* X_p^t}, \quad \theta_p^{2,t}(t) = \frac{U^* Y_p^t}{W^* Z_p^t + \overline{V}^* L_p^t}, \quad \theta_p^{a,t}(t) = \frac{U^* Y_p^t + \overline{U}^* K_p^t}{V^* X_p^t + \overline{V}^* L_p^t} \tag{3.13}$$

The efficiency score of DMU_p in time t while technology is considered in time $t+1$ is derived from solving model (3.14):

$$\begin{aligned} \theta^{t+1}(t) = Max \quad & UY_p^t + \overline{U}K_p^t \\ S.t. \quad & WZ_j^{t+1} + \overline{U}K_j^{t+1} - VX_j^{t+1} \leq 0, \quad j = 1, \ldots, n, \\ & UY_j^{t+1} - WZ_j^{t+1} - \overline{V}L_j^{t+1} \leq 0, \quad j = 1, \ldots, n, \\ & VX_p^t + \overline{V}L_p^t = 1, \\ & U \geq 1\varepsilon, \quad V \geq 1\varepsilon, \quad W \geq 1\varepsilon, \\ & \overline{U} \geq 1\varepsilon, \quad \overline{V} \geq 1\varepsilon. \end{aligned} \tag{3.14}$$

Let $(V^*, \bar{U}^*, W^*, \bar{V}^*, U^*)$ be the optimal solution of the above model, thus the efficiency scores of the first and the second subprocesses and the aggregate efficiency score in time t, while technology is considered in time $t+1$, can be obtained from expression (3.15).

$$\begin{aligned} \theta_p^{1,t+1}(t) &= \frac{W^* Z_p^t + \overline{U}^* K_p^t}{V^* X_p^t}, \\ \theta_p^{2,t+1}(t) &= \frac{U^* Y_p^t}{W^* Z_p^t + \overline{V}^*}, \\ \theta_p^{a,t+1}(t) &= \frac{U^* Y_p^t + \overline{U}^* K_p^t}{V^* X_p^t + \overline{V}^* L_p^t} \end{aligned} \tag{3.15}$$

The efficiency score of DMU_p in time $t+1$ while technology is considered in time $t+1$ is derived from solving model (3.16):

$$\begin{aligned} \theta^{t+1}(t+1) = Max \quad & UY_p^{t+1} + \overline{U}K_p^{t+1} \\ S.t. \quad & WZ_j^{t+1} + \overline{U}K_j^{t+1} - VX_j^{t+1} \leq 0, \quad j=1,\ldots,n, \\ & UY_j^{t+1} - WZ_j^t - \overline{V}L_j^{t+1} \leq 0, \quad j=1,\ldots,n, \\ & VX_p^{t+1} + \overline{V}L_p^{t+1} = 1 =, \\ & U \geq 1\varepsilon, \quad V \geq 1\varepsilon, \quad W \geq 1\varepsilon, \\ & \overline{U} \geq 1\varepsilon, \quad \overline{V} \geq 1\varepsilon. \end{aligned} \tag{3.16}$$

Let $(V^*, \bar{U}^*, W^*, \bar{V}^*, U^*)$ be the optimal solution of the above model, thus the efficiency scores of the first and the second subprocesses and the aggregate efficiency score in time $t+1$, while technology is considered in time $t+1$, can be obtained from expression (3.17).

$$\begin{aligned} \theta_p^{1,t+1}(t+1) &= \frac{W^* Z_p^{t+1} + \overline{U}^* K_p^{t+1}}{V^* X_p^{t+1}}, \\ \theta_p^{2,t+1}(t+1) &= \frac{U^* Y_p^{t+1}}{W^* Z_p^{t+1} + \overline{V}^* L_p^{t+1}}, \\ \theta_p^{a,t+1}(t+1) &= \frac{U^* Y_p^{t+1} + \overline{U}^* K_p^{t+1}}{V^* X_p^{t+1} + \overline{V}^* L_p^{t+1}} \end{aligned} \tag{3.17}$$

The efficiency score of DMU_p in time $t+1$ while technology is considered in time t, is derived from solving model (3.18):

$$\begin{aligned} \theta^{t}(t+1) = Max \quad & UY_p^{t+1} + \overline{U}K_p^{t+1} \\ S.t. \quad & WZ_j^{t} + \overline{U}K_j^{t} - VX_j^{t} \leq 0, \quad j=1,\ldots,n, \\ & UY_j^{t} - WZ_j^t - \overline{V}L_j^{t} \leq 0, \quad j=1,\ldots,n, \\ & VX_p^{t+1} + \overline{V}L_p^{t+1} = 1, \\ & U \geq 1\varepsilon, \quad V \geq 1\varepsilon, \quad W \geq 1\varepsilon, \\ & \overline{U} \geq 1\varepsilon, \quad \overline{V} \geq 1\varepsilon. \end{aligned} \tag{3.18}$$

Let $(V^*, \bar{U}^*, W^*, \bar{V}^*, U^*)$ be the optimal solution of the above model, thus the efficiency scores of the first and the second subprocesses and the aggregate efficiency score in time $t+1$ while technology is considered in time t, can be obtained from expressions (3.19).

$$\begin{aligned} \theta_p^{1,t}(t+1) &= \frac{W^* Z_p^{t+1} + \overline{U}^* K_p^{t+1}}{V^* X_p^{t+1}}, \\ \theta_p^{2,t}(t+1) &= \frac{U^* Y_p^{t+1}}{W^* Z_p^{t+1} + \overline{V}^* L_p^{t+1}}, \end{aligned} \tag{3.19}$$

$$\theta_p^{a,t}(t+1) = \frac{U^* Y_p^{t+1} + \overline{U}^* K_p^{t+1}}{V^* X_p^{t+1} + \overline{V}^* L_p^{t+1}}$$

According to the acquired efficiency scores, malmquist productivity index for the first subprocess can be calculated form expression (3.20).

$$M_P^1 = \left(\frac{\theta_P^{1,t}(t+1).\theta_p^{1,t+1}(t+1)}{\theta_p^{1,t}(t).\theta_p^{1,t+1}(t)} \right)^{1/2} = \frac{\theta_P^{1,t+1}(t+1)}{\theta_p^{1,t}(t)} . \left(\frac{\theta_P^{1,t}(t+1).\theta_p^{1,t}(t)}{\theta_p^{1,t+1}(t).\theta_p^{1,t+1}(t+1)} \right)^{1/2} \tag{3.20}$$

According to the above mentioned relation, there exist three cases:

1. if $M_p^1 > 1$ then the first subprocess of DMU_p in time $t+1$ in relation to time t has made progress.
2. if $M_p^1 < 1$ then the first subprocess of DMU_p in time $t+1$ in relation to time t has made regress.
3. if $M_p^1 = 1$ then the first subprocess of DMU_p in time $t+1$ in relation to time t has neither made progress nor regress.

Equivalently, in accordance with the obtained efficiency scores the malmquist productivity index for the second subprocess can be obtained via expression (3.21).

$$M_P^2 = \left(\frac{\theta_P^{2,t}(t+1).\theta_p^{2,t+1}(t+1)}{\theta_p^{2,t}(t).\theta_p^{2,t+1}(t)} \right)^{1/2} = \frac{\theta_P^{2,t+1}(t+1)}{\theta_p^{2,t}(t)} . \left(\frac{\theta_P^{2,t}(t+1).\theta_p^{2,t}(t)}{\theta_p^{2,t+1}(t).\theta_p^{2,t+1}(t+1)} \right)^{1/2} \tag{3.21}$$

According to the above mentioned relation, there exist three cases:

1. if $M_p^a > 1$ then DMU_p in time $t+1$ in relation to time t has made progress.
2. if $M_p^a < 1$ then DMU_p in time $t+1$ in relation to time t has made regress.
3. if $M_p^a = 1$ then DMU_p in time $t+1$ in relation to time t has neither made progress nor regress.

Finally, considering the acquired aggregate efficiency scores aggregate malmquist productivity index can be obtained via expression (3.22).

$$M_P^a = \left(\frac{\theta_P^{a,t}(t+1).\theta_p^{at+1}(t+1)}{\theta_p^{a,t}(t).\theta_p^{a,t+1}(t)} \right)^{1/2} = \frac{\theta_P^{a,t+1}(t+1)}{\theta_p^{a,t}(t)} . \left(\frac{\theta_P^{a,t}(t+1).\theta_p^{a,t}(t)}{\theta_p^{a,t+1}(t).\theta_p^{a,t+1}(t+1)} \right)^{1/2} \tag{3.22}$$

According to the above mentioned relation, there exist three cases:

1. if $M_p^2 > 1$ then the second subprocess of DMU_p in time $t+1$ in relation to time t has made progress.
2. if $M_p^2 < 1$ then the second subprocess of DMU_p in time $t+1$ in relation to time t has made regress.
3. if $M_p^2 = 1$ then the second subprocess of DMU_p in time $t+1$ in relation to time t has neither made progress nor regress.

4 Conclusion

Models which are formulated for real world problems contain special details formulating which needs investigating different relations. In this paper, MPI in series structural DMUs, with two subprocesses and intermediate inpouts and outputs, has been accounted for. With the contribution of the proposed model, progress and regress of DMUs can be calculated in two various periods. In accordance with what has been provided here, it is suggested to develop this method for series structural DMUs with multiple subprocesses and intermediate products.

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