Teaching Thermodynamics to Engineering Students: New Approach for Interpolation of Steam tables

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Abstract
This article is for teaching steam table interpolation to the first year engineering students. We present a systematic approach for interpolating steam tables. In the thermodynamics book, a double interpolation approach is mentioned to find state properties at a temperature and pressure which are not mentioned in the table. See the following thermodynamics books [Engineering Thermodynamics by J.B. Jones and R.E. Dugan; Thermodynamics by W.Z. Black and J.G. Hartley, and Applied Thermodynamics for Engineering Technologists by T.D. Eastop and A. McConley]. In this work, we present bilinear interpolation approach for teaching such concepts. We found our approach easy to teach to engineering students.

Keywords: Poisson Boltzmann equation; Nonlinear solver; Newton solver.

1 Introduction

For properties which are not mentioned exactly in the steam tables, it is necessary to interpolate between the values tabulated. For example, to find the state properties (specific enthalpy, specific internal energy, specific volume and entropy) of saturated vapour at 9.8 bar, it is required to interpolate between the two surrounding values given in the table. In the table, the properties of dry saturated steam at 9.0 bar and 10.0 bar are mentioned.

Let us denote the unknown state property at the pressure $p$ by the symbol $f$. From steam tables, we can read the state properties at the two nearest pressures: at the pressure $p_1$ the state property is $f_1$, and at the pressure $p_2$ the state property is $f_2$. Here, $p_1 < p < p_2$. These data are presented in the Fig. 1 and the Table 1.

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Table 1
State properties of saturated vapour at different pressures.

<table>
<thead>
<tr>
<th>Pressure [bar]</th>
<th>State property</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_0$</td>
<td>$f_0$</td>
</tr>
<tr>
<td>$p$</td>
<td>$f$</td>
</tr>
<tr>
<td>$p_1$</td>
<td>$f_1$</td>
</tr>
</tbody>
</table>

2 Main Results

Let us define a variable, $c$, from the given pressures

$$c = \frac{p - p_0}{p_1 - p_0}.$$  

(2.1)

The other state variables $f$ ($f$ may be temperature, entropy, specific enthalpy, specific volume and specific internal energy) at the pressure $p$ can be computed as follows

$$f = (1 - c) f_0 + c f_1.$$  

(2.2)

In the case of liquid, wet steam and superheated vapour table, we need two state properties. In the thermodynamics book a double interpolation approach is mentioned [2, 3, 4] to find state properties at a temperature and pressure which are not mentioned in the table. For example, to find the state properties (specific enthalpy, specific entropy, specific internal energy) of superheated steam at 18.5 bar and 432 °C. Here, we are presenting a more systematic approach for solving double interpolation of steam table. During our teaching experience, we found this approach much easier to convey to students.

Our task is to find the state properties when neither the temperature nor the pressure is given in the steam table. Let the state property be (enthalpy, entropy or specific internal energy) at the temperature $T_i$ and pressure $p_j$ be $f_{ij}$. Here, indexes $i$ and $j$ can take values 0 and 1. See the Table 3 for state properties at different temperature and pressure conditions. Let us assume that these state property can be read from the steam table. Our task is to find out state property at the temperature $T$ and pressure $p$. Here, temperature $T$ lies between the temperatures $T_1$ and $T_2$, and the pressure $p$ lies between the pressures $p_1$ and $p_2$. That is $T_1 < T < T_2$, and $p_1 < p < p_2$. These data are presented in the Table 2 and Fig. 2.

Table 1
Enthalpies of superheated steam at different temperatures and pressures.

<table>
<thead>
<tr>
<th>Pressure [bar]</th>
<th>Temperature [°C]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$T_0$</td>
</tr>
<tr>
<td>$p_0$</td>
<td>$f_{00}$</td>
</tr>
<tr>
<td>$p$</td>
<td>$f$</td>
</tr>
<tr>
<td>$p_1$</td>
<td>$f_{01}$</td>
</tr>
</tbody>
</table>
Let us define two variables $a$ and $b$ from temperature and pressure as follows

$$a = \frac{T - T_0}{T_1 - T_0} \quad \text{and} \quad b = \frac{p - p_0}{p_1 - p_0} \quad (2.3)$$

Now, the state property at the temperature $T$ and pressure $p$ can be computed by the following expression

$$f = (1 - a) (1 - b) f_{00} + a (1 - b) f_{10} + (1 - a) b f_{01} + a b f_{11}. \quad (2.4)$$

Let us figure out enthalpy of superheated steam at 18.5 bar and 432 °C. Steam table does not mention the enthalpy neither at 18.5 bar (for any temperature) nor at 432 °C (for any pressure). We need to interpolate between 15 bar and 20 bar, and between 400 °C and 450 °C. Steps to find state properties when neither the temperature nor the pressure is given in the table, are as follows:

1. Find two nearest pressures from steam table. These are 15 bar ($p_0$) and 20 bar ($p_1$).
2. Find two nearest temperatures from steam table. These are 400 °C ($T_0$) and 450 °C ($T_1$).
3. From steam table, read enthalpies at temperature 400 °C and 15 bar, and at 450 °C and 20 bar.
4. Present the date in the tabular form. See the Table 3.
5. Find $a$ and $b$
6. Use the relation (2.4) for finding state properties at 18.5 bar and 432 °C.

### Table 3

Entalpies [kJ/kg] of superheated steam at different temperatures and pressures.

<table>
<thead>
<tr>
<th>Pressure [bar]</th>
<th>Temperature °C</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>400</td>
</tr>
<tr>
<td>15.0</td>
<td>3256</td>
</tr>
<tr>
<td>18.5</td>
<td></td>
</tr>
<tr>
<td>20.0</td>
<td>3248</td>
</tr>
</tbody>
</table>

Let us first figure out $a$ and $b$. By the equation (2.3)

$$a = \frac{432 - 400}{450 - 400} = \frac{32}{50} = 0.064,$$

$$b = \frac{18.5 - 15.0}{20.0 - 15.0} = \frac{1.5}{5} = 0.300.$$
Thus, enthalpy of superheated steam at 432 °C and 18.5 bar is by the equation (2.4)

\[ h = (1 - 0.064)(1 - 0.3)(3256) + (0.064)(1 - 0.3)(3364) + (1 - 0.064)(0.3)(3248) + (0.064)(0.3)(3357), \]

\[ h = 3320 \text{ kJ/kg}. \]

Let us first figure out \( a \) and \( b \), by the equation (2.3):

\[ a = \frac{432 - 400}{450 - 400} = \frac{32}{50} = 0.064, \]

\[ b = \frac{18.5 - 15.0}{20.0 - 15.0} = \frac{1.5}{5} = 0.300. \]

3 Conclusions

This article has outlined an approach for teaching steam table interpolation to the first year engineering students. We have shown that the present method is very easy and effective.

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References


