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Using DEA to Compute Most Favourable and Least Favourable Sets of Weights in ABC Inventory Classification

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Abstract

Inventory classification using ABC analysis is one of the most widely employed techniques in organizations. The need to consider multiple criteria for inventory classification is stressed in the literature. A DEA approach is proposed in this paper for computing most favourable and least favourable sets of weights in multiple-criteria inventory classification. To illustrate the model capability the proposed methodology is applied to a real data set consisting of the 47 items.

Keywords : ABC inventory classification; Data envelopment analysis; Multiple criteria analysis

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1 Introduction

In an organization even with moderate size, there may be thousands of inventory stock keeping units. To have an efficient control of these huge amount of inventory items, traditional approach is to classify the inventory into different groups. Different inventory control policies can then applied to different groups. ABC analysis is a well known and practical classification based on the Pareto principle. ABC classification allows organizations to separate stock keeping units into three classes: A- very important; B- moderately important; and C- least important. The amount of time, effort, money and other resources spent on inventory planning and control should be in the relative importance of each item. Thus, the purpose of classifying items into groups is to establish appropriate levels of control over each item.

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Traditionally, the classification of inventory into the A, B, or C categories has generally been based on dollar value per unit multiplied by annual usage rate, commonly known as dollar usage [2]. In this classification, items are ordered in descending order of their annual dollar usage values. The relatively small number of items at the top of the list (approx. 10%) controlling the majority of the total annual dollar usage constitute class A, and the majority of the items at the bottom of the list (approx. 60%) controlling a relatively small portion of the total annual dollar usage constitute class C. Items between the two classes constitute class B (approx. 30%). Class A items require tight inventory control because they represent such a large percentage of the total dollar value of inventory. This requires accurate demand forecasts and detailed record keeping. In addition, close attention should be given to purchasing policies and procedures if the inventory items are acquired from sources outside the firm. Class C items should receive a flexible control, such as a simple two bin system. Finally, class B items should have a control effort that lies between these two extremes. The reader interested in the details of inventory control policies for the above classes is referred to Silver et al. [15]

There are many instances when other criteria, other than the annual use value, become important [6] in deciding the importance of an inventory item. This problem of multi-criteria inventory classification (MCIC) has been addressed by some studies in the literature. Some of the criteria considered in the literature include inventory cost, part criticality, lead time, commonality, obsolescence, substitutability, number of requests for the item in a year, scarcity, durability, substitutability, repairability, order size requirement, stockability, demand distribution, and stock-out penalty cost [5, 6, 8, 10]. Complex computational tools are needed for multi-criteria ABC classification. Flores et al. [5] provide a matrix-based methodology. A joint criteria matrix is developed in the case of two criteria. However, the methodology is relatively difficult to use when more criteria have to be considered. Several multiple criteria decision-making (MCDM) tools have also been employed for the purpose. Cohen and Ernst [2] and Ernst and Cohen [4] have used cluster analysis to group similar items. The analytic hierarchy process (AHP) [14] has been employed in many MCIC studies [5, 11, 12]. When AHP is used, the general idea is to derive a single scalar measure of importance of inventory items by subjectively rating the criteria and/or the inventory items [5, 6]. The single most important issue associated with AHP-based studies is the subjectivity involved in the analysis. Heuristic approaches based on artificial intelligence, such as genetic algorithms [6] and artificial neural networks [10], have also been applied to address the MCIC problem. Clearly, these approaches are heuristics and need not provide optimal solutions at all environments. To overcome the mentioned shortcomings, Ramanathan [13] proposed a weighted linear optimization model for multi-criteria ABC inventory classification. Despite its many advantages, his model could lead to a situation where an item with a high value in an unimportant criterion is inappropriately classified as a class A item. Zhou and Fan [16] present an extended version of the Ramanathan's model by incorporating some balancing features for MCIC. Zhou and Fan model, hereafter ZF-model, uses two sets of weights that are most favourable and least favourable for each item. Ng [9] proposes a simple model for MCIC. The model converts all criteria measures of an inventory item into a scalar score. With proper transformation, Ng obtains the scores of inventory items without a linear optimizer. The Ng-model is flexible as it could easily integrate additional information from decision makers for inventory classification. But, Ng-model leads to a situation where the score of each item is independent of the weights obtained from the model. That is, the weights do not have any role for determining total score of each item and this may lead to a situation where an item is inappropriately classified. More recently, we [7] proposed a nonlinear programming model to rectify this flaw. Our model not only incorporates multiple criteria for ABC classification, but also maintains the effects of weights in the final solution, an improvement over the model proposed by Ng.

In this paper, we would like to propose a DEA approach for the MCIC problems. The mathematical formulation is presented in Section 2. An illustration is provided in Section 3 with comparisons to the result from those in the literature. Short conclusions are given in Section 4.

2 The proposed model

We consider a situation in which a set of M items is available. The manager would like to classify these items based on N criteria. The measure of item m under criteria n is denoted as x_{mn} $(m = 1, 2, ..., M, n = 1, 2, ..., N)$. We evaluate an item m $(m = 1, 2, ..., M)$ by converting multiple measures under all criteria into a single score. A common scale for all measures is also an important issue. A particular criterion measure, in a large scale, may always dominate the score. For this, we propose normalizing all measures x_{mn} into a 0-1 scale. We denote all transformed measures as y_{mn}. In order to transform the performance ratings, the performance ratings are normalized into the range of [0, 1] by the following equations [1].

(i) The larger the better type:

$$y_{mn} = \frac{x_{mn} - \min\{x_{mn}\}}{\max\{x_{mn}\} - \min\{x_{mn}\}} \tag{2.1}$$

(ii) The smaller the better type:

$$y_{mn} = \frac{\max\{x_{mn}\} - x_{mn}}{\max\{x_{mn}\} - \min\{x_{mn}\}} \tag{2.2}$$

The score of an item is expressed as the weighted sum of transformed measures. Now let w_n be the relative importance weight attached to the nth criteria $(n = 1, 2, ..., N)$ and y_{mn} be the the performance of mth inventory item in terms of nth criteria. We enable the inventory manager to incorporate the ranking of the importance of the criteria in the decision making process. We require the user to rank the criteria importance in a sequence, rather than specifying exact weight values or exact degrees of relative preferences. Following [9] we assume the criteria are arranged in the descending order of importance (i.e. $w_1 \geq w_2 \geq ... \geq w_n$). The score of each item in terms of most favourable weights is defined as

$$gI_m = \sum_{n=1}^{N} y_{mn} w_n^g, \quad m = 1, 2, ..., M, \tag{2.3}$$

similarly, the score of each item in terms of least favourable weights is defined as

$$bI_m = \sum_{n=1}^{N} y_{mn} w_n^b, \quad m = 1, 2, ..., M. \tag{2.4}$$

Both of (2.3) and (2.4) are linear functions of the relative importance weights. Once the weights are given or determined, items can be classified using their total scores. To determine the relative importance weights, most favourable and least favourable, we suggest the following DEA models, respectively.

$$\begin{aligned} \max \quad & \alpha \\ s.t. \quad & \alpha \leq gI_m = \sum_{n=1}^{N} y_{mn} w_n^g \leq 1, && m = 1,2,...,M \\ & w_n^g \geq w_{n+1}^g, && n = 1,2,...,N-1 \\ & w_n^g \geq 0, && n = 1,2,...,N. \end{aligned} \tag{2.5}$$

$$\begin{aligned} \min \quad & \beta \\ s.t. \quad & \beta \geq bI_m = \sum_{n=1}^{N} y_{mn} w_n^b \geq 1, && m = 1,2,...,M \\ & w_n^b \geq w_{n+1}^b, && n = 1,2,...,N-1 \\ & w_n^b \geq 0, && n = 1,2,...,N. \end{aligned} \tag{2.6}$$

Models (2.5) and (2.6) are two linear programming problems. Model (2.5) maximizes the minimum of the scores of the M items and determines a common set of most favourable weights for all the items. The model (2.5) requires the score of each item to be equal to or less than one. On the other hand, model (2.6) minimizes the maximum of the scores of the M items and determines a common set of least favourable weights for all the items. The model (2.5) requires the score of each item to be equal to or greater than one. Once the weights are determined, the total score of each item can be computed as follows

$$sI_m(\lambda) = \lambda.\frac{gI_m - \alpha^*}{gI^* - \alpha^*} + (1-\lambda).\frac{bI_m - bI^-}{\beta^* - bI^-} \qquad m = 1,2,...,M, \tag{2.7}$$

where α^* and β^* are the optimal value of (2.5) and (2.6), respectively, and $gI^* = \max\{gI_m : m = 1,2,...,M\}$, $bI^- = \min\{bI_m : m = 1,2,...,M\}$ and $0 \leq \lambda \leq 1$ is a control parameter which may reflect the preference of decision maker. If $\lambda = 1, sI_m(\lambda)$ will become a normalized version of the gI_m. If $\lambda = 0, sI_m(\lambda)$ will become a normalized version of the bI_m. If inventory managers have no strong preference, $\lambda = 0.5$ would be a fairly neutral and reasonable choice.

3 Illustrative example

For illustration purpose, we apply our method, with $\lambda = 0.5$, to an inventory classification problem in literature [5, 9, 13, 16]. Following [9, 16] let us consider three criteria: Annual Dollar Usage (ADU), Average Unit Cost (AUC) and Lead Time (LT) for inventory classification. All the criteria are positive related to the score of the inventory items. An inventory with 47 items and measurement of performance under each of the criteria considered are shown in Table 1. This table also shows the maximal and minimal measures under each criteria as well as transformed measures in a scale of 0-1 as suggested in Section 2.

For comparison purpose, we maintain the same distribution of class A, B and C items as in literature studies [13, 16], i.e. 10 class A, 14 class B and 23 class C. Table 2 shows the classification based on our proposed model.The classification with the three criteria by

ZF-model, Ng-model and traditional ABC analysis using annual dollar usage are listed in this table as well. . As shown in Table 2, the ABC classification using our approach provides different results compared with the other methods. The difference could be because of the underlying assumption behind the methods.

Comparing to traditional ABC analysis based on only annual dollar usage, only 28 out of 47 items are kept in the same classes when ABC classification using proposed model with multi-criteria. In other words, more than half of the inventory items are re-classified by the proposed model. Eight out of the ten class A items in traditional ABC classification is still classified as class A items when multiple criteria is considered in proposed model. The other two (S2 and S10) are re-classified as class B and C using our model. For the 14 class B items, only 5 is remained in class B when criteria other than annual dollar usage are considered. Seven of the class B items are re-classified as C in our proposed model while the remaining 2 are moved up to class A. For the 23 class C items, 15 are kept as class C and eight of the class C items are moved up to class B. For more explanation, consider Item S8. This item is considered as a class A item based on annual dollar usage as it has one of the highest annual dollar usage. It has been classified as a class A item by our approach as well, for the same reason. However, Item S8 is classified as class B item by the two other methods because of the weighting scheme adopted in these methods.

Compared with the Ng-model, it can be seen from Table 2 that 28 out of the 47 items do not have the same classification. Of class A items identified in our proposed model, six of 10 items are classified as class A in both models. Similarly, 2 out of 14 class B items are classified as class B items in both models, and 11 out of 23 class C items are classified as class C items in both models. The difference in classification of the two approaches is because of the method of score computation for each item, the method of normalization of all measures and the schemes of weights generation in scoring.

When compared with ZF-model, 15 out of 47 items are coincided. For class A items identified by our proposed model, three items are classified as class A items in both models. And 3 out of 14 class B items are matched in both models. While for class C items, 9 out of 23 items are cross-matching. The difference in classification of the two approaches is because of the newly introduction of ranking in criteria and the normalization of all measures.

Table 1
Source and transformed measures of items under criteria

Item	ADU	AUC	LT	ADU (transformed)	AUC (transformed)	LT (transformed)
S1	5840.64	49.92	2	1.0000	0.7813	0.8333
S2	5670	210	5	0.9707	0.0000	0.3333
S3	5037.12	23.76	4	0.8619	0.9090	0.5000
S4	4769.56	27.73	1	0.8159	0.8896	1.0000
S5	3478.8	57.98	3	0.5939	0.7419	0.6666
S6	2936.67	31.24	3	0.5007	0.8725	0.6666
S7	2820	28.2	3	0.4806	0.8873	0.6666
S8	2640	55	4	0.4497	0.7565	0.5000
S9	2423.52	73.44	6	0.4124	0.6665	0.1666
S10	2407.5	160.5	4	0.4097	0.2416	0.5000
S11	1075.2	5.12	2	0.1806	1.0000	0.8333
S12	1043.5	20.87	5	0.1751	0.9231	0.3333
S13	1038	86.5	7	0.1742	0.6027	0.0000
S14	883.2	11 0.4	5	0.1476	0.4861	0.3333
S15	854.4	71.2	3	0.1426	0.6774	0.6666
S16	810	45	3	0.1350	0.8053	0.6666
S17	703.68	14.66	4	0.1167	0.9534	0.5000
S18	594	49.5	6	0.0978	0.7833	0.1666
S19	570	47.5	5	0.0937	0.7931	0.3333
S20	467.6	58.45	4	0.0761	0.7397	0.5000
S21	463.6	24.4	4	0.0754	0.9058	0.5000
S22	455	65	4	0.0739	0.7077	0.5000
S23	432.5	86.5	4	0.0701	0.6027	0.5000
S24	398.4	33.2	3	0.0642	0.8629	0.6666
S25	370.5	37.05	1	0.0594	0.8441	1.0000
S26	338.4	33.84	3	0.0539	0.8598	0.6666
S27	336.12	84.03	1	0.0535	0.6148	1.0000
S28	313.6	78.4	6	0.0496	0.6423	0.1666
S29	268.68	134.34	7	0.0419	0.3692	0.0000
S30	224	56	1	0.0342	0.7515	1.0000
S31	216	72	5	0.0328	0.6735	0.3333
S32	212.08	53.02	2	0.0322	0.7662	0.8333
S33	197.92	49.48	5	0.0297	0.7834	0.3333
S34	190.89	7.07	7	0.0285	0.9904	0.0000
S35	181.8	60.6	3	0.0269	0.7292	0.6666
S36	163.28	40.	3	0.0238	0.8257	0.6666
S37	150	30	5	0.0215	0.8785	0.3333
S38	134.8	67.4	3	0.0189	0.6960	0.6666
S39	119.2	59.6	5	0.0162	0.7340	0.3333
S40	103.36	51.68	6	0.0135	0.7727	0.1666
S41	79.2	19.8	2	0.0093	0.9283	0.8333
S42	75.4	37.7	2	0.0087	0.8409	0.8333
S43	59.78	29.89	5	0.0060	0.8790	0.3333
S44	48.3	48.3	3	0.0040	0.7892	0.6666
S45	34.4	34.4	7	0.0016	0.8570	0.0000
S46	28.8	28.8	3	0.0006	0.8844	0.6666
S47	25.38	8.46	5	0.0000	0.9836	0.3333

Table 2
ABC classifications by different models

Item	ADU	AUC	LT	sI	ABC classification			
					Our model	ZF	Ng	Traditional
S1	5840.64	49.92	2	1.0000	A	A	A	A
S3	5037.12	23.76	4	0.9924	A	A	A	A
S4	4769.56	27.73	1	0.9446	A	C	A	A
S6	2936.67	31.24	3	0.7021	A	C	A	A
S7	2820	28.2	3	0.6984	A	C	B	A
S5	3478.8	57.98	3	0.6748	A	B	A	A
S8	2640	55	4	0.5802	A	B	B	A
S11	1075.2	5.12 1	2	0.5615	A	C	C	B
S12	1043.5	20.87	5	0.5014	A	B	B	B
S9	2423.52	73.44	6	0.4873	A	A	A	A
S17	703.68	14.66	4	0.4809	B	C	C	B
S34	190.89	7.07	7	0.4435	B	B	B	C
S47	25.38	8.46	5	0.4178	B	C	C	C
S21	463.6	24.4	4	0.4160	B	C	C	B
S2	5670 2	10	5	0.4084	B	A	A	A
S16	810	45	3	0.3862	B	C	C	B
S41	79.2	19.8	2	0.3809	B	C	C	C
S24	398.4	33.2	3	0.3765	B	C	C	B
S26	338.4	33.84	3	0.3668	B	C	C	C
S25	370.5	37.05	1	0.3593	B	C	C	C
S37	150	30	5	0.3567	B	B	C	C
S19	570	47.5	5	0.3471	B	B	B	B
S43	59.78	29.89	5	0.3460	B	C	C	C
S46	28.8	28.8	3	0.3458	B	C	C	C
S18	594	49.5	6	0.3430	C	A	B	B
S45	34.4	34.4	7	0.3397	C	B	B	C
S42	75.4	37.7	2	0.3200	C	C	C	C
S36	163.28	40.82	3	0.3199	C	C	C	C
S15	854.4	71.2	3	0.2984	C	C	C	B
S20	467.6	58.45	4	0.2953	C	B	C	B
S33	197.92	49.48	5	0.2933	C	B	B	C
S32	212.08	53.02	2	0.2826	C	C	C	C
S44	48.3	48.3	3	0.2788	C	C	C	C
S40	103.36	51.68	6	0.2737	C	B	B	C
S30	224	56	1	0.2733	C	C	C	C
S22	455	65	4	0.2703	C	B	C	B
S13	1038	86.5	7	0.2669	C	A	A	B
S35	181.8	60.6	3	0.2517	C	C	C	C
S39	119.2	59.6	5	0.2474	C	B	B	C
S38	134.8	67.4	3	0.2217	C	C	C	C
S31	216	72	5	0.2154	C	B	B	C
S28	313.6	78.4	6	0.2049	C	A	B	C
S23	432.5	86.5	4	0.1909	C	B	B	B
S27	336.12	84.03	1	0.1876	C	C	C	C
S10	2407.5	160.5	4	0.1753	C	A	A	A
S14	883.2	110.4	5	0.1624	C	A	B	B
S29	268.68	134.34	7	0.1523	C	A	A	C

4 Conclusion

In this paper, a simple approach for inventory classification was proposed when multiple criteria were considered. To do so, we introduced a two stage DEA approach. To illustrate the model capability the proposed methodology was applied to a real data set consisting of the 47 items and the obtained results were compared with those in the literature.

References

[1] C.H. Cheng, Evaluating weapon systems using ranking fuzzy numbers, Fuzzy Sets and Systems 107 (1999) 25-35.

[2] M.A. Cohen and R. Ernst, Multi-item classification and generic inventory stock control policies, Production and Inventory Management Journal 29 (1988) 6-8.

[3] W.W. Cooper, L.M. Seiford and K. Tone, Data envelopment analysis: A comprehensive text with models, applications, references and DEA-solver software, Kluwer Academic Publisher. 1999

[4] R. Ernst and M.A. Cohen, Operations related groups (ORGs): a clustering procedure for production/inventory systems, Journal of Operations Management 9 (1990) 574-598.

[5] B.E. Flores, D.L. Olson and V.K. Dorai, Management of multicriteria inventory classification, Mathematical and Computer Modelling 16 (1992) 71-82.

[6] H.A. Guvenir and E. Erel, Multicriteria inventory classification using a genetic algorithm, European Journal of Operational Research 105 (1998) 29-37.

[7] A. Hadi-Vencheh, An improvement to multiple criteria ABC inventory classification , European Journal of Operational Research 201 (2010) 962-965.

[8] P. Hautaniemi and T. Pirttilä, The choice of replenishment policies in an MRP environment, International Journal of Production Economics 59 (1999) 85-92.

[9] W.L. Ng, A simple classifier for multiple criteria ABC analysis, European Journal of Operational Research 177 (2007) 344-353.

[10] F.Y. Partovi and M. Anandarajan, Classifying inventory using an artificial neural network approach, Computers and Industrial Engineering 41 (2002) 389-404.

[11] F.Y. Partovi and J. Burton, Using the analytic hierarchy process for ABC analysis, International Journal of Production and Operations Management 13 (1993) 29-44.

[12] F.Y. Partovi and W.E. Hopton, The analytic hierarchy process as applied to two types of inventory problems, Production and Inventory Management Journal 35 (1993)13-19.

[13] R. Ramanathan, ABC inventory classification with multiple-criteria using weighted linear optimization, Computers & Operations Research 33 (2006) 695-700.

[14] T.L. Saaty, The analytic hierarchy process. McGraw-Hill: New York; 1980.

[15] E.A. Silver, D.F. Pyke and R. Peterson, Inventory management and production planning and scheduling, Wiley, New York. 1998

[16] P. Zhou and L. Fan, A note on multi-criteria ABC inventory classification using weighted linear optimization, European Journal of Operational Research 182 (2007) 1488-1491.