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Extension of TOPSIS for Group Decision-Making Based on the Type-2 Fuzzy Positive and Negative Ideal Solutions

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Abstract

In this paper based on the interval type-2 fuzzy sets, we introduce an extension of fuzzy TOPSIS for handling fuzzy multiple attributes group decision making problems. In the proposed method the fuzzy positive ideal solution and fuzzy negative ideal solution are obtained in the form of interval type-2 fuzzy sets without ranking the elements of decision matrix, using the proposed method the solution of decision problem is obtained with less computational attempt than existing methods.

Keywords : Fuzzy TOPSIS; Fuzzy positive ideal solution; Fuzzy negative ideal solution; Fuzzy group decision making; Fuzzy multiple attributes group decision making; Interval type-2 fuzzy set.

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1 Introduction

Multiple attributes decision making (MADM) is an approach employed to solve problems involving selection from among a finite number of alternatives [8]. Among the many existing methods for solving MADM problems TOPSIS that was introduced by Hwang and Yoon in 1981 [8] is one of the well-known methods. The basic principle of TOPSIS is that the chosen alternative should have the shortest distance from the positive ideal solution and the farthest from the negative ideal solution. In TOPSIS the performance ratings and the weights of the attributes are given as crisp values [1]. Chen [2] introduced an extension of TOPSIS under fuzzy environment. Triantaphyllou and Lin [9] developed a fuzzy version of TOPSIS method based on fuzzy arithmetic operations. Wang and Lee [10] generalized TOPSIS to fuzzy multiple criteria group decision making (FMCGDM) in a fuzzy environment. Ashtiani et.al [1] introduced an extension of fuzzy TOPSIS based

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on interval-valued fuzzy sets. Chen and Lee [3] developed TOPSIS for fuzzy multiple attributes group decision making based on the interval type-2 fuzzy sets. In this paper we develop the type-2 fuzzy TOPSIS based on the type-2 fuzzy positive and negative ideal solutions to handle MADM problems. The rest of this paper is organized as following: In the next section required definitions are introduced. Section 3 is allocated to classic TOPSIS method, proposed method is presented in section 4, finally in section 5 two numerical examples are introduced, as will be seen the proposed method leads to correct solutions with less computational attempt than the methods in references [3, 4].

2 Type-2 Fuzzy Sets

Definition 2.1. *A type-2 fuzzy set $\tilde{\tilde{A}}$ in universe of discourse X can be represented by type-2 fuzzy membership function $\mu_{\tilde{\tilde{A}}}$ as follows:*

$$\tilde{\tilde{A}} = \{((x,u), \mu_{\tilde{\tilde{A}}}(x,u)) | \forall x \in X, \forall u \in J_x \subseteq [01], 0 \leq \mu_{\tilde{\tilde{A}}} \leq 1\}$$

Type-2 fuzzy set $\tilde{\tilde{A}}$ also can be represented as follows:

$$\tilde{\tilde{A}} = \int_{x\in X} \int_{u\in J_x} \mu_{\tilde{\tilde{A}}}(x,u)/(x,u) \qquad , J_x \subseteq [0,1]$$

where $\int\int$ denotes the union over all admissible x and u.

Definition 2.2. *Let $\tilde{\tilde{A}}$ be a type-2 fuzzy set in universe of discourse X which is represented by type-2 membership function $\mu_{\tilde{\tilde{A}}}$. $\tilde{\tilde{A}}$ is an interval type-2 fuzzy set if all $\mu_{\tilde{\tilde{A}}}(x,u)$ =1. This special case of a type-2 fuzzy set can be represented as follows:*

$$\tilde{\tilde{A}} = \int_{x\in X} \int_{u\in J_x} 1/(x,u) \qquad , J_x \subseteq [0,1]$$

Definition 2.3. *The interval type-2 fuzzy set $\tilde{\tilde{A}}$, can be represented as $\tilde{\tilde{A}}$=$(\tilde{A}^U, \tilde{A}^L)$ where $\tilde{A}^U$, $\tilde{A}^L$ are upper membership function and lower membership function, respectively. Note that $\tilde{A}^U$ and $\tilde{A}^L$ are type-1 fuzzy sets.*

Definition 2.4. *The trapezoidal interval type-2 fuzzy set $\tilde{\tilde{A}}$ can be represented as follows:*

$$\tilde{\tilde{A}} = (\tilde{A}^U, \tilde{A}^L) = ((a_1^U, a_2^U, a_3^U, a_4^U; H_1(\tilde{A}^U), H_2(\tilde{A}^U)), (a_1^L, a_2^L, a_3^L, a_4^L; H_1(\tilde{A}^L), H_2(\tilde{A}^L)))$$

where a_i^U and a_i^L, $(1 \leq i \leq 4)$ are the parameters of a_i^U and a_i^L respectively, and $H_i(\tilde{A}^U)$ and $H_i(\tilde{A}^L)$,$(1 \leq i \leq 2)$ denote the membership values of elements a_i^U and a_i^L, $(2 \leq i \leq 3)$ respectively, $H_i(\tilde{A}^U) \in [0,1]$ and $H_i(\tilde{A}^L) \in [0,1]$,$(1 \leq i \leq 2)$.

Definition 2.5. *The addition operation between two trapezoidal interval type-2 fuzzy sets*

$$\tilde{\tilde{A}}_1 = (\tilde{A}_1^U, \tilde{A}_1^L) = ((a_{11}^U, a_{12}^U, a_{13}^U, a_{14}^U; H_1(\tilde{A}_1^U), H_2(\tilde{A}_1^U)), (a_{11}^L, a_{12}^L, a_{13}^L, a_{14}^L; H_1(\tilde{A}_1^L), H_2(\tilde{A}_1^L)))$$

and

$\tilde{\tilde{A}}_2 = (\tilde{A}_2^U, \tilde{A}_2^L) = ((a_{21}^U, a_{22}^U, a_{23}^U, a_{24}^U; H_1(\tilde{A}_2^U), H_2(\tilde{A}_2^U)), (a_{21}^L, a_{22}^L, a_{23}^L, a_{24}^L; H_1(\tilde{A}_2^L), H_2(\tilde{A}_2^L)))$

is defined as follows:

$$\begin{aligned}\tilde{\tilde{A}}_1 \oplus \tilde{\tilde{A}}_2 &= (\tilde{A}_1^U, \tilde{A}_1^L) \oplus (\tilde{A}_2^U, \tilde{A}_2^L) \\ &= ((a_{11}^U \oplus a_{21}^U, a_{12}^U \oplus a_{22}^U, a_{13}^U \oplus a_{23}^U, a_{14}^U \oplus a_{24}^U; min(H_1(\tilde{A}_1^U), H_1(\tilde{A}_2^U)), min(H_2(\tilde{A}_1^U) \\ &\quad , H_2(\tilde{A}_2^U)); (a_{11}^L \oplus a_{21}^L, a_{12}^L \oplus a_{22}^L, a_{13}^L \oplus a_{23}^L, a_{14}^L \oplus a_{24}^L; min(H_1(\tilde{A}_1^L), H_1(\tilde{A}_2^L)) \\ &\quad , min(H_2(\tilde{A}_1^L), H_2(\tilde{A}_2^L))))\end{aligned}$$

Definition 2.6. *The multiplication operation between type-2 fuzzy sets $\tilde{\tilde{A}}_1$ and $\tilde{\tilde{A}}_2$ is defined as follows:*

$$\begin{aligned}\tilde{\tilde{A}}_1 \otimes \tilde{\tilde{A}}_2 &= (\tilde{A}_1^U, \tilde{A}_1^L) \otimes (\tilde{A}_2^U, \tilde{A}_2^L) \\ &= ((a_{11}^U \otimes a_{21}^U, a_{12}^U \otimes a_{22}^U, a_{13}^U \otimes a_{23}^U, a_{14}^U \otimes a_{24}^U; min(H_1(\tilde{A}_1^U), H_1(\tilde{A}_2^U), min \\ &\quad (H_2(\tilde{A}_1^U), H_2(\tilde{A}_2^U))), (a_{11}^L \otimes a_{21}^L, a_{12}^L \otimes a_{22}^L, a_{13}^L \otimes a_{23}^L, a_{14}^L \oplus a_{24}^L; min(H_1 \\ &\quad (\tilde{A}_1^L), H_1(\tilde{A}_2^L), H_2(\tilde{A}_2^L)))\end{aligned}$$

3 Classic TOPSIS Method

The TOPSIS method was developed by Hwang and Yoon (1981). This method is based on the concept that chosen alternative should have the shortest Euclidean distance from the ideal solution, and the farthest from the negative ideal solution. The ideal solution and negative ideal solutions are hypothetical solutions have in the best and worst cases respectively. TOPSIS thus gives a solution that is not only closest to the hypothetically best, that is also the farthest from the hypothetically worst. The mail procedure of the TOPSIS method for the selection of the best alternative from among those available is described bellow [8]:

Step 1: The first step is to determine the objective, and to identify the pertinent evaluation attributes.

Step 2: This step represents a matrix based on all the information available on attributes. Each row of this matrix is allocated to one alternative, and each column to one attribute. Let there exist m attributes and n alternatives so the decision matrix is as following:

$$D = (d_{ij})_{m\times n}$$

Step 3: Normalize the decision matrix as following:

$$R = (r_{ij})_{m\times n} \quad ; \quad r_{ij} = d_{ij}/(\sum_{i=1}^{m} d_{ij}^2)^{1/2}$$

Step 4: Decide on the relative importance (i.e. weights) of different attributes with respect to the objective. A set of weight w_i $(for\ \ j = 1, 2, \ldots, m)$ such that $\sum w_i = 1$ may be decided upon.

Step 5: Obtain the weighted normalized matrix V. This is done by the multiplication of each element of the column of the matrix R with associated weight w_i:

$$V = (v_{ij})_{m\times n} \quad ; \quad v_{ij} = w_i \times r_{ij}$$

Step 6: Obtain the ideal (best) and negative ideal (worst) solution in this step. The ideal and negative ideal solutions can be expressed as:

$$x^+ = (v_1^+, v_2^+, \ldots, v_m^+) \quad , \quad x^- = (v_1^-, v_2^-, \ldots, v_m^-)$$

where

$$v_i^+ = \begin{cases} \max v_{ij} & , f_i \in F_1 \\ \min v_{ij} & , f_i \in F_2 \end{cases} \quad , \quad v_i^- = \begin{cases} \min v_{ij} & , f_i \in F_1 \\ \max v_{ij} & , f_i \in F_2 \end{cases}$$

Step 7: Obtain the separation measures. The separation of each alternative from the ideal one is given by the Euclidean distance in the following equations:

$$S_j^+ = (\sum_{i=1}^{m}(v_{ij} - v_j^+)^2)^{1/2} \qquad ; \quad j = 1, 2, \ldots, n$$

$$S_j^- = (\sum_{i=1}^{m}(v_{ij} - v_j^-)^2)^{1/2} \qquad ; \quad j = 1, 2, \ldots, n$$

Step 8: The relative closeness of a particular alternative to the ideal solution, P_i, can be expressed in this step as follows:

$$P_j = S_j^+/(S_j^+ + S_j^-) \qquad ; \quad j = 1, 2, \ldots, n$$

Step 9: A set of alternatives is generated in the descending order in this step, according to the value of P_j indicating the most preferred and least preferred feasible solution. P_i may also be called the overall or composite performance score of alternative A_j.

4 Proposed Method

In this section our proposed TOPSIS for group decision making is introduced. The proposed method is as following:

Step 1: Construct decision matrix Y_p for pth decision maker and then construct the average decision matrix $\overline{Y}$, as following:

$$Y_p = (\tilde{\tilde{f}}{}^p_{ij})_{m\times n} = \begin{matrix} \\ f_1 \\ f_2 \\ \vdots \\ f_m \end{matrix} \begin{matrix} x_1 & x_2 & \cdots & x_n \\ \left(\begin{matrix} \tilde{\tilde{f}}{}^p_{11} & \tilde{\tilde{f}}{}^p_{12} & \cdots & \tilde{\tilde{f}}{}^p_{1n} \\ \tilde{\tilde{f}}{}^p_{21} & \tilde{\tilde{f}}{}^p_{22} & \cdots & \tilde{\tilde{f}}{}^p_{2n} \\ \vdots & \vdots & & \vdots \\ \tilde{\tilde{f}}{}^p_{m1} & \tilde{\tilde{f}}{}^p_{m2} & \cdots & \tilde{\tilde{f}}{}^p_{mn} \end{matrix}\right) \end{matrix},$$

$$\overline{Y} = (\tilde{\tilde{f}}_{ij})_{m\times n} \quad ; \tilde{\tilde{f}}_{ij} = \frac{\tilde{\tilde{f}}{}^1_{ij} \oplus \tilde{\tilde{f}}{}^2_{ij} \oplus \cdots \oplus \tilde{\tilde{f}}{}^k_{ij}}{k}$$

where $\tilde{\tilde{f}}_{ij}$ is an interval type-2 fuzzy set, $1 \le i \le m$, $1 \le j \le n$, $1 \le p \le k$ and k denotes the number of decision-makers.

Step 2: Construct the weighting matrix W_p of the attributes for pth decision-maker and then obtain the average weighting matrix $\overline{W}$ as following:

$$W_p = (\tilde{\tilde{w}}_i^p)_{1\times m} = \begin{array}{c} \begin{matrix} f_1 & f_2 & \dots & f_n \end{matrix} \\ \begin{pmatrix} \tilde{\tilde{w}}_1^p & \tilde{\tilde{w}}_2^p & \dots & \tilde{\tilde{w}}_m^p \end{pmatrix} \end{array},$$

$$\overline{W}_{1\times m} = (\tilde{\tilde{w}}_i) \quad ; \tilde{\tilde{w}}_i = \frac{\tilde{\tilde{w}}_i^1 \oplus \tilde{\tilde{w}}_i^2 \oplus \dots \oplus \tilde{\tilde{w}}_i^k}{k}$$

where $\tilde{\tilde{w}}_i$ is an interval type-2 fuzzy set, $1 \le i \le m$, $1 \le p \le k$ and k denotes the number of decision-makers.

Step 3: Construct the weighted decision matrix $\overline{Y}_w$,

$$\overline{Y}_w = (\tilde{\tilde{v}}_{ij})_{m\times n} = \begin{array}{c} \\ f_1 \\ f_2 \\ \vdots \\ f_m \end{array} \begin{array}{c} \begin{matrix} x_1 & x_2 & \dots & x_n \end{matrix} \\ \begin{pmatrix} \tilde{\tilde{v}}_{11} & \tilde{\tilde{v}}_{12} & \dots & \tilde{\tilde{v}}_{1n} \\ \tilde{\tilde{v}}_{21} & \tilde{\tilde{v}}_{22} & \dots & \tilde{\tilde{v}}_{2n} \\ \vdots & \vdots & & \vdots \\ \tilde{\tilde{v}}_{m1} & \tilde{\tilde{v}}_{m2} & \dots & \tilde{\tilde{v}}_{mn} \end{pmatrix} \end{array},$$

where $\tilde{\tilde{v}}_{ij} = \tilde{\tilde{w}}_i \otimes \tilde{\tilde{f}}_{ij}, 1 \le i \le m, 1 \le j \le n.$

Step 4: Obtain fuzzy positive ideal solution (FPIS) and fuzzy negative ideal solution (FNIS) as following (FPIS and FNIS are represented as S^+ and S^- respectively):

$$S^+ = \begin{cases} (1,1,1,1;1,1) & , for\ benefit\ attributes \\ (0,0,0,0;1,1) & , for\ cost\ attributes \end{cases}$$

$$S^- = \begin{cases} (0,0,0,0;1,1) & , for\ benefit\ attributes \\ (1,1,1,1;1,1) & , for\ cost\ attributes \end{cases}$$

Step 5: According to [1, 3, 6] and using the fact that interval-valued fuzzy sets and interval type-2 fuzzy sets are equivalent [4] the normalized Euclidean distance between each alternative x_j and FPIS can be calculated as following:

$$d^{U+}(x_j) = \sum_{i=1}^{m} d^{U+}(S^+, \tilde{v}_{ij}^U) \quad , \quad d^{L+}(x_j) = \sum_{i=1}^{m} d^{L+}(S^+, \tilde{v}_{ij}^L)$$

where

$$\tilde{\tilde{v}}_{ij} = (\tilde{v}_{ij}^U, \tilde{v}_{ij}^L) = ((v_{ij}^{1U}, v_{ij}^{2U}, v_{ij}^{3U}, v_{ij}^4 U; H_1(\tilde{v}_{ij}^U), H_2(\tilde{v}_{ij}^U)), (v_{ij}^{1L}, v_{ij}^{2L}, v_{ij}^{3L}, v_{ij}^{4L}; H_1(\tilde{v}_{ij}^L), H_2(\tilde{v}_{ij}^L)))$$

So for benefit attributes we have:

$$d^{U+}(S^+, \tilde{v}_{ij}^U) = \sqrt{\frac{1}{4}\sum_{k=1}^{4}(v_{ij}^{kU} - 1)^2} \quad , \quad d^{L+}(S^+, \tilde{v}_{ij}^L) = \sqrt{\frac{1}{4}\sum_{k=1}^{4}(v_{ij}^{kL} - 1)^2}$$

and for cost attributes we have:

$$d^{U+}(S^+,\tilde{v}_{ij}^U)=\sqrt{\frac{1}{4}\sum_{k=1}^{4}(v_{ij}^{kU}-0)^2}\quad,\quad d^{L+}(S^+,\tilde{v}_{ij}^L)=\sqrt{\frac{1}{4}\sum_{k=1}^{4}(v_{ij}^{kL}-0)^2}$$

Similarly the distance between each alternative x_j and FNIS is obtained as following:

$$d^{U-}(x_j)=\sum_{i=1}^{m}d^{U-}(S^-,\tilde{v}_{ij}^U)\quad,\quad d^{L-}(x_j)=\sum_{i=1}^{m}d^{L-}(S^-,\tilde{v}_{ij}^L)$$

where for benefit attributes we have:

$$d^{U-}(S^-,\tilde{v}_{ij}^U)=\sqrt{\frac{1}{4}\sum_{k=1}^{4}(v_{ij}^{kU}-0)^2}\quad,\quad d^{L-}(S^+,\tilde{v}_{ij}^L)=\sqrt{\frac{1}{4}\sum_{k=1}^{4}(v_{ij}^{kL}-0)^2}$$

and for cost attributes we have:

$$d^{U-}(S^-,\tilde{v}_{ij}^U)=\sqrt{\frac{1}{4}\sum_{k=1}^{4}(v_{ij}^{kU}-1)^2}\quad,\quad d^{L-}(S^-,\tilde{v}_{ij}^L)=\sqrt{\frac{1}{4}\sum_{k=1}^{4}(v_{ij}^{kL}-1)^2}$$

Step 6: Calculate the relative degree of closeness $C^*(x_j)$ of alternative x_j with respect to the step 5 as follows:

$$C^*(x_j)=\frac{C_1(x_j)+C_2(x_j)}{2}$$

where $C_1(x_j)=\frac{d^{U-}(x_j)}{d^{U+}(x_j)+d^{U-}(x_j)}$, $C_2(x_j)=\frac{d^{L-}(x_j)}{d^{L+}(x_j)+d^{L-}(x_j)}$ and $1\le j\le n$.

Step 7: Sort the values of $C^*(x_j)$, $(1\le j\le n)$ in an ascensional sequence. The larger value of $C^*(x_j)$ the higher preference of the corresponding alternative x_j,$(1\le j\le n)$.

5 Numerical examples

In this section we introduce 2 numerical examples and solve them using our proposed TOPSIS method, the results are attractive.

Example 5.1. *, [7]. Table 1 shows the linguistic terms "Very Low" (VL), "low" (L), " Medium Low" (ML), "Medium" (M), "Medium High" (MH),"High" (H), "Very High" (VH), and their corresponding type-1 fuzzy sets, respectively. Table 2 shows the linguistic terms, "Very Poor" (VP), "Poor" (P), "Medium Poor" (MP), "Fair" (F), "Medium Good" (MG), "Good" (G), "Very Good" (VG) and their corresponding type-1 fuzzy sets, respectively. Assume that there are three decision-makers* D_1, D_2 *and* D_3 *of a software company to hire a system analysis engineer and assume that there are three alternatives* x_1, x_2 *and* x_3 *and five attributes "Emotional Steadiness", "Oral Communication Skill", "Personality", "Past Experience", "Self-Confidence". Let* X *be the set of alternatives, where* $X=\{x_1,x_2,x_3\}$ *and let* F *be the set of attributes, where F=Emotional Steadiness,* *Oral Communication Skill, Personality, Past Experience, Self-Confidence. Assume that the three decision-makers D_1, D_2 and D_3 use the linguistic terms shown in Table 1 to represent the weights of the five attributes, respectively, as shown in Table 3. In Table 3, five benefit attributes are considered, including "Emotional Steadiness", "Oral Communication Skill", "Personality", "Past Experience" and "Self Confidence". Assume that three decision-makers D_1, D_2 and D_3 use the linguistic terms shown in Table 2 to represent the evaluating values of the alternatives with respect to different attributes, respectively, as shown in Table 4. Based on the interval type-2 fuzzy set representation method, the linguistic terms shown in Table 1 and Table 2 can be represented by interval type-2 fuzzy sets, as shown in Table 5 and Table 6 respectively.*

Table 1

Linguistic terms of weights of the attributes and their corresponding type-1 fuzzy sets.

Linguistic terms	Type-1 fuzzy sets
Very Low(VL)	(0,0,0,0.1;1,1)
Low(L)	(0,0.1,0.1,0.3;1,1)
Medium Low(ML)	(0.1,0.3,0.3,0.5;1,1)
Medium(M)	(0.3,0.5,0.5,0.7;1,1)
Medium High(MH)	(0.5,0.7,0.7,0.9;1,1)
High(H)	(0.7,0.9,0.9,0.1;1,1)
Very High(VH)	(0.9,1,1,1;1,1)

Table 2

Linguistic terms for the ratings and their corresponding type-1 fuzzy sets.

Linguistic terms	Type-1 fuzzy sets
Very Poor(VP)	(0,0,0,1;1,1)
Poor(P)	(0,1,1,3;1,1)
Medium Poor(MP)	(1,3,3,5;1,1)
Fair(F)	(3,5,5,7;1,1)
Medium Good(MG)	(5,7,7,9;1,1)
Good(G)	(7,9,9,10;1,1)
Very Good(VG)	(9,10,10,10;1,1)

Table 3

Weights of the attributes evaluated by decision-makers.

Attributes	Decision-makers		
	D_1	D_2	D_3
Emotional Steadiness	H	VH	MH
Oral Communication Skill	VH	VH	VH
Personality	VH	H	H
Past Experience	VH	VH	VH
Self-Confidence	M	MH	MH

Table 4
Evaluating values of the alternatives given by the decision-makers with respect to different attributes.

Attributes	Alternatives	Decision-makers		
Emotional Steadiness	x_1	MG	G	MG
	x_2	G	G	MG
	x_3	VG	G	F
Oral Communication Skill	x_1	G	MG	F
	x_2	VG	VG	VG
	x_3	MG	G	VG
Personality	x_1	G	G	G
	x_2	VG	VG	G
	x_3	G	MG	VG
Past Experience	x_1	VG	G	VG
	x_2	VG	VG	VG
	x_3	G	VG	MG
Self-Confidence	x_1	F	F	F
	x_2	VG	MG	G
	x_3	G	G	MG

Table 5
Linguistic terms of weights of the attributes and their corresponding type-2 fuzzy sets.

Linguistic terms	Type-2 fuzzy sets
Very Low(VL)	((0,0,0,0.1;1,1),(0,0,0,0.1;1,1))
Low(L)	((0,0.1,0.1,0.3;1,1),(0,0.1,0.1,0.3;1,1))
Medium Low(ML)	((0.1,0.3,0.3,0.5;1,1),(0.1,0.3,0.3,0.5;1,1))
Medium(M)	((0.3,0.5,0.5,0.7;1,1),(0.3,0.5,0.5,0.7;1,1))
Medium High(MH)	((0.5,0.7,0.7,0.9;1,1),(0.5,0.7,0.7,0.9;1,1))
High(H)	((0.7,0.9,0.9,0.1;1,1),(0.7,0.9,0.9,0.1;1,1))
Very High(VH)	((0.9,1,1,1;1,1),(0.9,1,1,1;1,1))

Table 6
Linguistic terms for the ratings and their corresponding type-2 fuzzy sets.

Linguistic terms	Type-2 fuzzy sets
Very Poor(VP)	((0,0,0,1;1,1),(0,0,0,1;1,1))
Poor(P)	((0,0,0,1;1,1),(0,0,0,1;1,1))
Medium Poor(MP)	((1,3,3,5;1,1),(1,3,3,5;1,1))
Fair(F)	((3,5,5,7;1,1),(3,5,5,7;1,1))
Medium Good(MG)	((5,7,7,9;1,1),(5,7,7,9;1,1))
Good(G)	((7,9,9,10;1,1),(7,9,9,10;1,1))
Very Good(VG)	((9,10,10,10;1,1),(9,10,10,10;1,1))

Step 1: *Based on Tables 4, 5 and 6 construct the decision matrices Y_1, Y_2 and Y_3 and obtain the average decision matrix as follows:(Since in Table 2 the universe of discourse of the linguistic terms is [0,10] the decision matrices can be reconstructed by division all*

elements of them by 10 so the average decision matrix obtain as following:)

$$\overline{Y} = \begin{array}{l} \\ \textit{Emotional Steadiness} \\ \textit{Oral Communication Skill} \\ \textit{Personality} \\ \textit{Past Experience} \\ \textit{Self-Confidence} \end{array} \begin{array}{c} \begin{array}{ccc} x_1 & x_2 & x_3 \end{array} \\ \begin{pmatrix} \tilde{\tilde{f}}_{11} & \tilde{\tilde{f}}_{12} & \tilde{\tilde{f}}_{13} \\ \tilde{\tilde{f}}_{21} & \tilde{\tilde{f}}_{22} & \tilde{\tilde{f}}_{23} \\ \tilde{\tilde{f}}_{31} & \tilde{\tilde{f}}_{32} & \tilde{\tilde{f}}_{33} \\ \tilde{\tilde{f}}_{41} & \tilde{\tilde{f}}_{42} & \tilde{\tilde{f}}_{43} \\ \tilde{\tilde{f}}_{51} & \tilde{\tilde{f}}_{52} & \tilde{\tilde{f}}_{53} \end{pmatrix} \end{array}$$

where

$\tilde{\tilde{f}}_{11} = ((0.57, 0.77, 0.77, 0.93; 1, 1), (0.57, 0.77, 0.77, 0.93; 1, 1))$
$\tilde{\tilde{f}}_{12} = ((0.63, 0.83, 0.83, 0.97; 1, 1), (0.63, 0.83, 0.83, 0.97; 1, 1))$
$\tilde{\tilde{f}}_{13} = ((0.63, 0.8, 0.8, 0.9; 1, 1), (0.63, 0.8, 0.8, 0.9; 1, 1))$
$\tilde{\tilde{f}}_{21} = ((0.5, 0.7, 0.7, 0.87; 1, 1), (0.5, 0.7, 0.7, 0.87; 1, 1))$
$\tilde{\tilde{f}}_{22} = ((0.9, 1, 1, 1; 1, 1), (0.9, 1, 1, 1; 1, 1))$
$\tilde{\tilde{f}}_{23} = ((0.7, 0.87, 0.87, 0.97; 1, 1), (0.7, 0.87, 0.87, 0.97; 1, 1))$
$\tilde{\tilde{f}}_{31} = ((0.57, 0.77, 0.77, 0.9; 1, 1), (0.57, 0.77, 0.77, 0.9; 1, 1))$
$\tilde{\tilde{f}}_{32} = ((0.83, 0.97, 0.97, 1; 1, 1), (0.83, 0.97, 0.97, 1; 1, 1))$
$\tilde{\tilde{f}}_{33} = ((0.7, 0.87, 0.87, 0.97; 1, 1), (0.7, 0.87, 0.87, 0.97; 1, 1))$
$\tilde{\tilde{f}}_{41} = ((0.83, 0.97, 0.97, 1; 1, 1), (0.83, 0.97, 0.97, 1; 1, 1))$
$\tilde{\tilde{f}}_{42} = ((0.9, 1, 1, 1; 1, 1), (0.9, 1, 1, 1; 1, 1))$
$\tilde{\tilde{f}}_{43} = ((0.7, 0.87, 0.87, 0.97; 1, 1), (0.7, 0.87, 0.87, 0.97; 1, 1))$
$\tilde{\tilde{f}}_{51} = ((0.3, 0.5, 0.5, 0.7; 1, 1), (0.3, 0.5, 0.5, 0.7; 1, 1))$
$\tilde{\tilde{f}}_{52} = ((0.7, 0.87, 0.87, 0.97; 1, 1), (0.7, 0.87, 0.87, 0.97; 1, 1))$
$\tilde{\tilde{f}}_{53} = ((0.63, 0.83, 0.83, 0.97; 1, 1), (0.63, 0.83, 0.83, 0.97; 1, 1))$

Step 2: *Based on Tables 3, 5 and 6 construct weighting matrices W_1, W_2 and W_3 and then obtain the average weighting matrix $\overline{W}$ as following:*

$$\overline{W} = [\tilde{\tilde{w}}_1 \quad \tilde{\tilde{w}}_2 \quad \tilde{\tilde{w}}_3 \quad \tilde{\tilde{w}}_4 \quad \tilde{\tilde{w}}_5]$$

where

$\tilde{\tilde{w}}_1 = ((0.7, 0.87, 0.87, 0.97; 1, 1), (0.7, 0.87, 0.87, 0.97; 1, 1))$
$\tilde{\tilde{w}}_2 = ((0.9, 1, 1, 1, ; 1, 1), (0.9, 1, 1, 1, ; 1, 1))$
$\tilde{\tilde{w}}_3 = ((0.77, 0.93, 0.93, 1; 1, 1), (0.77, 0.93, 0.93, 1; 1, 1))$
$\tilde{\tilde{w}}_4 = ((0.9, 1, 1, 1; 1, 1), (0.9, 1, 1, 1; 1, 1))$
$\tilde{\tilde{w}}_5 = ((0.43, 0.63, 0.63, 0.83; 1, 1), (0.43, 0.63, 0.63, 0.83; 1, 1))$

Step 3: *The weighted decision matrix $\overline{Y}_W$ is obtained bellow:*

$$\overline{Y}_W = \begin{array}{l} \\ \textit{Emotional Steadiness} \\ \textit{Oral Communication Skill} \\ \textit{Personality} \\ \textit{Past Experience} \\ \textit{Self-Confidence} \end{array} \begin{array}{c} \begin{array}{ccc} x_1 & x_2 & x_3 \end{array} \\ \begin{pmatrix} \tilde{\tilde{v}}_{11} & \tilde{\tilde{v}}_{12} & \tilde{\tilde{v}}_{13} \\ \tilde{\tilde{v}}_{21} & \tilde{\tilde{v}}_{22} & \tilde{\tilde{v}}_{23} \\ \tilde{\tilde{v}}_{31} & \tilde{\tilde{v}}_{32} & \tilde{\tilde{v}}_{33} \\ \tilde{\tilde{v}}_{41} & \tilde{\tilde{v}}_{42} & \tilde{\tilde{v}}_{43} \\ \tilde{\tilde{v}}_{51} & \tilde{\tilde{v}}_{52} & \tilde{\tilde{v}}_{53} \end{pmatrix} \end{array}$$

where

$\tilde{\tilde{v}}_{11} = ((0.4, 0.66, 0.66, 0.9; 1, 1), (0.4, 0.66, 0.66, 0.9; 1, 1))$
$\tilde{\tilde{v}}_{12} = ((0.44, 0.72, 0.72, 0.93; 1, 1), (0.44, 0.72, 0.72, 0.93; 1, 1))$
$\tilde{\tilde{v}}_{13} = ((0.44, 0.69, 0.69, 0.87; 1, 1), (0.44, 0.69, 0.69, 0.87; 1, 1))$
$\tilde{\tilde{v}}_{21} = ((0.45, 0.7, 0.7, 0.87; 1, 1), (0.45, 0.7, 0.7, 0.87; 1, 1))$
$\tilde{\tilde{v}}_{22} = ((0.81, 1, 1, 1; 1, 1), (0.81, 1, 1, 1; 1, 1))$
$\tilde{\tilde{v}}_{23} = ((0.63, 0.87, 0.87, 0.97; 1, 1), (0.63, 0.87, 0.87, 0.97; 1, 1))$
$\tilde{\tilde{v}}_{31} = ((0.43, 0.72, 0.72, 0.9; 1, 1), (0.43, 0.72, 0.72, 0.9; 1, 1))$
$\tilde{\tilde{v}}_{32} = ((0.64, 0.9, 0.9, 1; 1, 1), (0.64, 0.9, 0.9, 1; 1, 1))$
$\tilde{\tilde{v}}_{33} = ((0.54, 0.81, 0.81, 0.97; 1, 1), (0.54, 0.81, 0.81, 0.97; 1, 1))$
$\tilde{\tilde{v}}_{41} = ((0.75, 0.97, 0.97, 1; 1, 1), (0.75, 0.97, 0.97, 1; 1, 1))$
$\tilde{\tilde{v}}_{42} = ((0.81, 1, 1, 1; 1, 1), (0.81, 1, 1, 1; 1, 1))$
$\tilde{\tilde{v}}_{43} = ((0.63, 0.87, 0.87, 0.97; 1, 1), (0.63, 0.87, 0.87, 0.97; 1, 1))$
$\tilde{\tilde{v}}_{51} = ((0.13, 0.32, 0.32, 0.58; 1, 1), (0.13, 0.32, 0.32, 0.58; 1, 1))$
$\tilde{\tilde{v}}_{52} = ((0.3, 0.55, 0.55, 0.81; 1, 1), (0.3, 0.55, 0.55, 0.81; 1, 1))$
$\tilde{\tilde{v}}_{53} = ((0.27, 0.53, 0.53, 0.81; 1, 1), (0.27, 0.53, 0.53, 0.81; 1, 1))$

Step 4: *The FPIS and FNIS are as follows:*

$$S^+ = (1, 1, 1, 1; 1, 1) \quad , \quad S^- = (0, 0, 0, 0; 1, 1)$$

Step 5: *The values of* $d^{U+}(x_j)$, $d^{L+}(x_j)$, $d^{U-}(x_j)$ *and* $d^{L-}(x_j)$, $(1 \le j \le 3)$ *are listed in the following table:*

Table 7
The values of $d^{U+}(x_j)$, $d^{L+}(x_j)$, $d^{U-}(x_j)$, $d^{L-}(x_j)$

j	$d^{U+}(x_j)$	$d^{L+}(x_j)$	$d^{U-}(x_j)$	$d^{L-}(x_j)$
1	1.9	1.9	3.39	3.39
2	1.19	1.19	4.09	4.09
3	1.55	1.55	3.74	3.74

Step 6: *The values of* $C_1(x_j)$, $C_2(x_j)$ *and* $C^*(x_j)$, $(1 \le j \le 3)$ *are listed bellow in Table 8:*

Table 8
The values of $C_1(x_j)$, $C_2(x_j)$ *and* $C^*(x_j)$

j	$C_1(x_j)$	$C_2(x_j)$	$C^*(x_j)$
1	0.64	0.64	0.64
2	0.77	0.77	0.77
3	0.71	0.71	0.71

Step 7: *We have* $C^*(x_1) < C^*(x_3) < C^*(x_2)$ *so the preference of alternatives is:* $x_1 < x_3 < x_2$ *as it can be seen the results are similar to the results of Chen's method [7]. It should be noted that the number of operations of proposed algorithm in steps 4, 5 and 6 is about 120 while in the Chen's method [7] it is about 1670.*

Example 5.2. *, [7,8]. Table 9 shows the linguistic terms "Very Low" (VL), "low" (L), "Medium" (M), "Medium High" (MH),"High" (H), "Very High" (VH), and their corresponding type-2 fuzzy sets, respectively. Assume that there are three decision-makers* D_1,D_2 *and* D_3 *to evaluate cars and assume that there are three alternatives* x_1, x_2 *and* x_3 *and four attributes "Safety", "Price", "Appearance", "performance". Let* X *be the set of alternatives, where* $X = \{x_1, x_2, x_3\}$*, and let* F *be the set of attributes, where* $F= \{$*Safety, Price, Appearance, performance*$\}$*. Assume that the three decision-makers* D_1, D_2 *and* D_3 *use the linguistic terms shown in Table 9 to represent the weights of the four attributes, respectively, as shown in Table 10. In Table 10, three benefit attributes are considered, including "Safety", "Appearance", and "Performance" and one cost attribute is considered, i.e. "Price". Assume that the three decision-makers* D_1, D_2 *and* D_3 *use the linguistic terms shown in Table 9 to represent the evaluating values of the alternatives with respect to different attributes, respectively, as shown in Table 11.*

Table 9
Linguistic terms and their corresponding interval type-2 fuzzy sets.

Linguistic terms	Type-2 fuzzy sets
Very Low(VL)	((0, 0, 0, 0.1; 1, 1), (0, 0, 0, 0.05; 0.9, 0.9))
Low(L)	((0, 0.1, 0.1, 0.3; 1, 1), (0.05, 0.1, 0.1, 0.2; 0.9, 0.9))
Medium Low(ML)	((0.1, 0.3, 0.3, 0.5; 1, 1), (0.2, 0.3, 0.3, 0.4; 0.9, 0.9))
Medium(M)	((0.3, 0.5, 0.5, 0.7; 1, 1), (0.4, 0.5, 0.5, 0.6; 0.9, 0.9))
Medium High(MH)	((0.5, 0.7, 0.7, 0.9; 1, 1), (0.6, 0.7, 0.7, 0.8; 0.9, 0.9))
High(H)	((0.7, 0.9, 0.9, 1; 1, 1), (0.8, 0.9, 0.9, 0.95; 0.9, 0.9))
Very High(VH)	((0.9, 1, 1, 1; 1, 1), (0.95, 1, 1, 1; 0.9, 0.9))

Table 10
Weights of the attributes evaluated by decision-makers.

Attributes	Decision-makers		
	D_1	D_2	D_3
Safety	VH	H	VH
Price	VH	VH	VH
Appearance	M	MH	MH
Performance	VH	H	H

Table 11
Evaluating values of the alternatives given by decision-makers with respect to different attributes.

Attributes	*Alternatives*	*Decision-makers*		
Safety	x_1	*MH*	*H*	*MH*
	x_2	*H*	*MH*	*H*
	x_3	*VH*	*H*	*MH*
Price	x_1	*H*	*VH*	*H*
	x_2	*MH*	*H*	*VH*
	x_3	*VH*	*VH*	*H*
Appearance	x_1	*VH*	*H*	*H*
	x_2	*H*	*VH*	*VH*
	x_3	*M*	*MH*	*MH*
Performance	x_1	*VH*	*H*	*H*
	x_2	*H*	*VH*	*H*
	x_3	*H*	*VH*	*VH*

Step 1: *Based on Table 11 construct the decision matrices Y_1, Y_2 and Y_3 and obtain the average decision matrix as follows:*

$$\overline{Y} = \begin{matrix} & \\ Safety \\ Price \\ Appearance \\ Performence \end{matrix} \begin{matrix} x_1 & x_2 & x_3 \\ \end{matrix} \begin{pmatrix} \tilde{\tilde{f}}_{11} & \tilde{\tilde{f}}_{12} & \tilde{\tilde{f}}_{13} \\ \tilde{\tilde{f}}_{21} & \tilde{\tilde{f}}_{22} & \tilde{\tilde{f}}_{23} \\ \tilde{\tilde{f}}_{31} & \tilde{\tilde{f}}_{32} & \tilde{\tilde{f}}_{33} \\ \tilde{\tilde{f}}_{41} & \tilde{\tilde{f}}_{42} & \tilde{\tilde{f}}_{43} \end{pmatrix}$$

where

$$\tilde{\tilde{f}}_{11} = ((0.57, 0.77, 0.77, 0.93; 1, 1), (0.67, 0.77, 0.77, 0.85; 0.9, 0.9))$$
$$\tilde{\tilde{f}}_{12} = ((0.63, 0.83, 0.83, 0.97; 1, 1), (0.73, 0.83, 0.83, 0.9; 0.9, 0.9))$$
$$\tilde{\tilde{f}}_{13} = ((0.7, 0.87, 0.87, 0.97; 1, 1), (0.78, 0.87, 0.87, 0.92; 0.9, 0.9))$$
$$\tilde{\tilde{f}}_{21} = ((0.77, 0.93, 0.93, 1.00; 1, 1), (0.85, 0.93, 0.93, 0.97; 0.9, 0.9))$$
$$\tilde{\tilde{f}}_{22} = ((0.7, 0.87, 0.87, 0.97; 1, 1), (0.78, 0.87, 0.87, 0.92; 0.9, 0.9))$$
$$\tilde{\tilde{f}}_{23} = ((0.83, 0.97, 0.97, 1; 1, 1), (0.9, 0.97, 0.97, 0.98; 0.9, 0.9))$$
$$\tilde{\tilde{f}}_{31} = ((0.77, 0.93, 0.93, 1; 1, 1), (0.85, 0.93, 0.93, 0.97; 0.9, 0.9))$$
$$\tilde{\tilde{f}}_{32} = ((0.83, 0.97, 0.97, 1; 1, 1), (0.90, 0.97, 0.97, 0.98; 0.9, 0.9))$$
$$\tilde{\tilde{f}}_{33} = ((0.43, 0.63, 0.63, 0.83; 1, 1), (0.53, 0.63, 0.63, 0.73; 0.9, 0.9))$$
$$\tilde{\tilde{f}}_{41} = ((0.77, 0.93, 0.93, 1; 1, 1), (0.85, 0.93, 0.93, 0.97; 0.9, 0.9))$$
$$\tilde{\tilde{f}}_{42} = ((0.83, 0.97, 0.97, 1; 1, 1), (0.90, 0.97, 0.97, 0.98; 0.9, 0.9))$$
$$\tilde{\tilde{f}}_{43} = ((0.77, 0.93, 0.93, 1.0; 1, 1), (0.85, 0.93, 0.93, 0.97; 0.9, 0.9))$$

Step 2: *Based on Table 10 construct weighting matrices W_1, W_2 and W_3 and then obtain the average weighting matrix $\overline{W}$ as following:*

$$\overline{W} = [\tilde{\tilde{w}}_1 \quad \tilde{\tilde{w}}_2 \quad \tilde{\tilde{w}}_3 \quad \tilde{\tilde{w}}_4]$$

where

$$\tilde{\tilde{w}}_1 = ((0.83, 0.97, 0.97, 1; 1, 1), (0.9, 0.97, 0.97, 0.98, 0.9, 0.9))$$
$$\tilde{\tilde{w}}_2 = ((0.83, 0.97, 0.97, 1; 1, 1), (0.9, 0.97, 0.97, 0.98; 0.9, 0.9))$$
$$\tilde{\tilde{w}}_3 = ((0.43, 0.63, 0.63, 0.83; 1, 1), (0.53, 0.63, 0.63, 0.73; 0.9, 0.9))$$
$$\tilde{\tilde{w}}_4 = ((0.77, 0.93, 0.93, 1; 1, 1), (0.85, 0.93, 0.93, 0.97; 0.9, 0.9))$$

Step 3: *The weighted decision matrix* $\overline{Y}_W$ *is obtained bellow:*

$$\overline{Y}_W = \begin{array}{c} \\ Safety \\ Price \\ Appearance \\ Performance \end{array} \begin{array}{c} \quad x_1 \quad\; x_2 \quad\; x_3 \\ \begin{pmatrix} \tilde{\tilde{v}}_{11} & \tilde{\tilde{v}}_{12} & \tilde{\tilde{v}}_{13} \\ \tilde{\tilde{v}}_{21} & \tilde{\tilde{v}}_{22} & \tilde{\tilde{v}}_{23} \\ \tilde{\tilde{v}}_{31} & \tilde{\tilde{v}}_{32} & \tilde{\tilde{v}}_{33} \\ \tilde{\tilde{v}}_{41} & \tilde{\tilde{v}}_{42} & \tilde{\tilde{v}}_{43} \end{pmatrix} \end{array}$$

where

$$\tilde{\tilde{v}}_{11} = ((0.47, 0.74, 0.74, 0.93; 1, 1), (0.6, 0.74, 0.74, 0.84; 0.9, 0.9))$$
$$\tilde{\tilde{v}}_{12} = ((0.53, 0.81, 0.81, 0.97; 1, 1), (0.66, 0.81, 0.81, 0.89; 0.9, 0.9))$$
$$\tilde{\tilde{v}}_{13} = ((0.58, 0.84, 0.84, 0.97; 1, 1), (0.71, 0.84, 0.84, 0.9; 0.9, 0.9))$$
$$\tilde{\tilde{v}}_{21} = ((0.64, 0.9, 0.9, 1; 1, 1), (0.77, 0.9, 0.9, 0.95; 0.9, 0.9))$$
$$\tilde{\tilde{v}}_{22} = ((0.58, 0.84, 0.84, 0.97; 1, 1), (0.71, 0.84, 0.84, 0.9; 0.9, 0.9))$$
$$\tilde{\tilde{v}}_{23} = ((0.69, 0.93, 0.93, 1; 1, 1), (0.81, 0.93, 0.93, 0.97; 0.9, 0.9))$$
$$\tilde{\tilde{v}}_{31} = ((0.33, 0.59, 0.59, 0.83; 1, 1), (0.45, 0.59, 0.59, 0.71; 0.9, 0.9))$$
$$\tilde{\tilde{v}}_{32} = ((0.36, 0.61, 0.61, 0.83; 1, 1), (0.48, 0.61, 0.61, 0.72; 0.9, 0.9))$$
$$\tilde{\tilde{v}}_{33} = ((0.19, 0.4, 0.4, 0.69; 1, 1), (0.28, 0.4, 0.4, 0.54; 0.9, 0.9))$$
$$\tilde{\tilde{v}}_{41} = ((0.59, 0.87, 0.87, 1; 1, 1), (0.72, 0.87, 0.87, 0.93; 0.9, 0.9))$$
$$\tilde{\tilde{v}}_{42} = ((0.59, 0.87, 0.87, 1; 1, 1), (0.72, 0.87, 0.87, 0.93; 0.9, 0.9))$$
$$\tilde{\tilde{v}}_{43} = ((0.64, 0.9, 0.9, 1; 1, 1), (0.77, 0.9, 0.9, 0.95; 0.9, 0.9))$$

Step 4: *The FPIS and FNIS are as follows:*

$$S^+ = \begin{cases} (1,1,1,1;1,1) & ,for\ benefit\ attributes \\ (0,0,0,0;1,1) & ,for\ cost\ attributes \end{cases}$$

$$S^- = \begin{cases} (0,0,0,0;1,1) & ,for\ benefit\ attributes \\ (1,1,1,1;1,1) & ,for\ cost\ attributes \end{cases}$$

Step 5: *The values of* $d^{U+}(x_j)$, $d^{L+}(x_j)$, $d^{U-}(x_j)$ *and* $d^{L-}(x_j)$, $(1 \leq j \leq 3)$ *are listed in following table:*

Table 12
The values of $d^{U+}(x_j)$, $d^{L+}(x_j)$, $d^{U-}(x_j)$, $d^{L-}(x_j)$

j	$d^{U+}(x_j)$	$d^{L+}(x_j)$	$d^{U-}(x_j)$	$d^{L-}(x_j)$
1	1.87	1.76	2.39	2.32
2	1.75	1.62	2.51	2.45
3	1.93	1.84	2.31	2.23

Step 6: *The values of* $C_1(x_j)$, $C_2(x_j)$ *and* $C^*(x_j)$, $(1 \leq j \leq 3)$ *are listed bellow in Table 13:*

Table 13
The values of $C_1(x_j)$, $C_2(x_j)$ and $C^*(x_j)$,

j	$C_1(x_j)$	$C_2(x_j)$	$C^*(x_j)$
1	0.56	0.57	0.565
2	0.59	0.6	0.595
3	0.54	0.55	0.545

***Step 7**: We have $C^*(x_3) < C^*(x_1) < C^*(x_2)$ so the preference of alternatives is: $x_3 < x_1 < x_2$, as can be seen the results are completely similar to the results in [7, 8]. It should be noted that the number of operations of the proposed method in steps 4, 5 and 6 is about 108 while in Chen's method [7] the number is about 1342 and in the method of [8] is very bigger.*

6 Conclusion

In this paper we have introduced an extension of fuzzy TOPSIS based on the interval type-2 fuzzy sets, for handling fuzzy multiple attributes group decision-making problems. The proposed method is based on obtaining FPIS and FNIS in the form of interval type-2 fuzzy sets without ranking the elements of decision matrix. As numerical examples have shown the proposed method can cause to correct solution with less computational attempt than existing methods.

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