



# Equation of Circulation Energy in Human Body in Time Acupuncture Medicine

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## Abstract

In this paper, the mathematical model of energy circulation in the human body is proposed and we show the health stability by using fuzzy parameters and iteration linear equation.

*Keywords :* Acupuncture medicine, Fuzzy set, Linear equation.

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## 1 Introduction

In [1], a mathematical model is proposed as follows:

$$\begin{cases} \frac{dh}{dt} = h(k - h)(1 - h) \\ h(0) = h_0 \end{cases} \quad \begin{cases} \frac{dk}{dt} = f(Q) \\ t > 0 \end{cases} \quad (1.1)$$

where  $Q = Q_p + Q_e + Q_{cd}$ . Here  $Q$  is a total amount  $Q_i$  in the person,  $Q_p$  is a production of  $Q_i$ ,  $Q_e$  is a expenditure of  $Q_i$  and  $Q_{cd}$  is a circulation and distribution of  $Q_i$ .  $h = h(t)$  denotes a function of time, which accounts for the combined health-state of a subsystem of an organism where  $0 \leq h(t) \leq 1$ .  $k = k(t)$  denotes the ability to recover from the disease where  $0 \leq k(t) \leq 1$ .

In this paper, we propose fuzzy set theory, health function and ability function to recover and its relations in section 2. Then mathematical model of energy circulation in the human body is introduced and we show the health stability using fuzzy parameters and iteration linear equation in section 3. Subsequently, conclusions are drawn in Section 4.

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## 2 Fuzzy set and Health function and ability function to recover

We can consider the health function and ability function to recover as fuzzy set. Theory of fuzzy set was proposed in 1965 by Professor Zadeh[4]. This theory has expanded its applications in different fields since then. Fuzzy sets are the generalization of the usually sets. Fuzzy sets theory is a theory for acting insinuation without certainty. This theory can give a mathematical form to most of concepts and variables and systems that are unclear in some way and are imprecise like a real world. That also provides a field for reasoning, educating, controlling and making decision in situation without certainty. Generally, the characteristic function (property function) of each usual subset  $A$  from  $X$  is a function from  $X$  to  $\{0, 1\}$  that is defined as the following:

$$\chi_A(x) = \begin{cases} 1 & x \in A \\ 0 & \text{otherwise} \end{cases} \quad (2.2)$$

Now if we expand the range of indicative function from two members set  $0, 1$  to interval  $[0, 1]$  we will have a function that will attribute a number from interval  $[0, 1]$  to each  $x$  from  $X$ . Now  $A$  is not a usual set but is something that is called a Fuzzy set, more precisely, a fuzzy subset of  $X$ . So, fuzzy set  $A$  is a set that its member's degrees can be selected from  $I = [0, 1]$ .

Regardingly, health of the person is not exact then we can propose the membership degree diagnosis health of a person on health people by a fuzzy set with membership function like  $h(t)$  which is called health function. By considering  $h(t)$  as the health function,  $h(t) = 1$  denotes the total health, and  $h(t) = 0$  denotes the total lack of health or death. Otherwise  $0 < h(t) < 1$  denotes a situation between these two extreme cases. We denoted  $k = k(t)$  as ability function to recover from disease by considering that  $k(t) = 1$  denotes that the ability is good, and  $k(t) = 0$  denotes that the ability is poor. We represent this diagrammatically as follows:

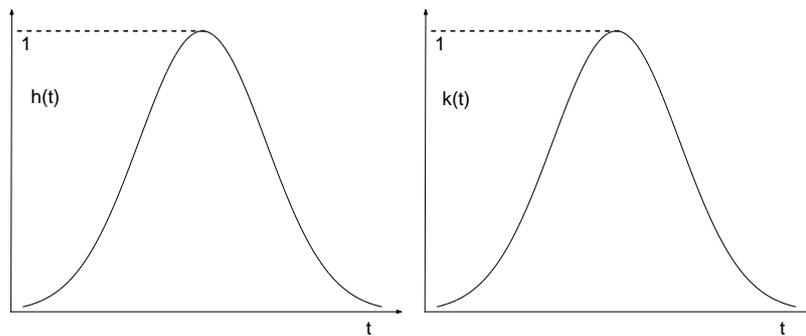


Fig. 1. represent of diagrammatically of  $h(t)$  and  $k(t)$

We can describe health balance or health unbalance of human as relation of  $h(t)$  and  $k(t)$ .

### Relation of $h(t)$ and $k(t)$ :

- (a)  $h(t) = 1 \mapsto k(t) = 1 \Rightarrow \frac{dh}{dt} = 0, \frac{dk}{dt} = 0$
- (b)  $h(t) = 0 \mapsto k(t) = 0 \Rightarrow \frac{dh}{dt} = 0, \frac{dk}{dt} = 0$
- (c)  $0 < h(t) < 1 \mapsto 0 \leq k(t) \leq 1$

$$\begin{aligned}
\text{(c-1)} \quad & \begin{cases} h(t) \leq \frac{1}{2} \\ k(t) \leq \frac{1}{2} \end{cases} \Rightarrow \text{poor health and recovery ability (high risk)} \\
\text{(c-2)} \quad & \begin{cases} h(t) \geq \frac{1}{2} \\ k(t) \geq \frac{1}{2} \end{cases} \Rightarrow \text{good health and recovery ability (low risk)} \\
\text{(c-3)} \quad & \begin{cases} h(t) \leq \frac{1}{2} \\ k(t) \geq \frac{1}{2} \end{cases} \Rightarrow \text{poor health and good recovery ability} \\
\text{(c-4)} \quad & \begin{cases} h(t) \geq \frac{1}{2} \\ k(t) \leq \frac{1}{2} \end{cases} \Rightarrow \text{good health and poor recover ability}
\end{aligned}$$

### 3 Equation of circulation energy in time acupuncture medicine

One of the basic postulations of health and recovery ability in human by acupuncture medicine is that circulation of energy appears in two forms, in form of the Yang (positive) and in the form of Yin (negative). According to this idea, an illness occurs by unbalance in distribution of energy  $Q_i$  between Yang and Yin in any of the human body subsystems. According to the views adopted in the eastern medicine, the whole human organism is separated into twelve subsystems. The acupuncture points located on the human skin and joined in a line are linked to each of these subsystems(meridian). Accordingly, each meridian has maximal activity in two hours. By Yang and Yin concept all meridians are divided into two groups: six meridians are Yang and six meridians are Yin. Circulation and connection of Yang meridian to the Yin meridian and vice versa are called "Luo-Link"s( $\leftrightarrow$ ). Circulation can have two shapes: tonic (increase or decrease the energy in a meridian) and sedative (acts with the opposite sign). We consider the following mathematical formulation for energy circulation in human body.

$$Q_i(t) = \sum_{j=1}^{12} k_{ij} Q_j(t - \tau_{ij}) + f_i(t) \quad i = 1, 2, \dots, 12 \quad t > 0 \quad (3.3)$$

Where  $Q_i(t)$  is a periodic function and shows the amount of energy in  $i$ -th meridian at the moment  $t$  with term of period  $24n$ ;  $n = 1, 2, \dots$ .  $Q_j(t - \tau_{ij})$  amount of energy in  $j$ -th meridian at the moment  $t$  with delay  $\tau_{ij}$  which is calculated as

$$\tau_{ij} = \begin{cases} t_i - t_j, & t_i - t_j \geq 0 \\ t_i - t_j + 24, & t_i - t_j \leq 0 \end{cases} \quad (3.4)$$

If  $t_i - t_j = 0$  this means that  $Q_i(t - 24) = Q_i(t)$  shows that principal direction period of  $Q_i(t)$  is a  $T = 24$ .  $f_i(t)$  is the external influence on  $i$ -th meridian.  $k_{ij}$  is a constant coefficient energy translation of  $i$ -th to  $j$ -th meridian and vice versa, if energy translates between  $i$ -th to  $j$ -th meridian is a "lou" then  $k_{ij} \geq 0$  and if energy translates between  $i$ -th to  $j$ -th meridian is a "link" then  $k_{ij} \leq 0$ .

For example,

if maximal activity of  $i$ -th and  $j$ -th meridians is  $t_i = 11$  and  $t_j = 1$  respectively, then  $\tau_{ij} = t_i - t_j = 11 - 1 = 10$ . This means that  $j$ -th meridian is the activity 10 hours before activity of  $i$ -th meridian.

if the maximal activity of  $i$ -th and  $j$ -th meridians is  $t_i = 11$  and  $t_j = 17$  respectively, then  $\tau_{ij} = t_i - t_j + 24 = 11 - 17 + 24 = 18$ . This means that  $j$ -th meridian is the activity 18 hours before the activity of  $i$ -th meridian(yesterday).

### 3.1 Delay stability of energy circulation

Now, we show the maximal activity of  $i$ -th meridian with  $Q_i(t + 24n)$  in  $n$ -th day. Regarding to (3.3):

$$Q_i(t + 24n) = \sum_{j=1}^{12} k_{ij} Q_j(t + 24n - \tau_{ij}) + f_i(t + 24n) \quad i = 1, 2, \dots, 12 \quad t > 0 \quad (3.5)$$

But, with (3.4) we define

$$Q(t + 24n - \tau_{ij}) = \begin{cases} Q(n), & t_i - t_j \geq 0 \\ Q(n - 1), & t_i - t_j < 0 \end{cases} \quad (3.6)$$

$$f(t + 24n) = f(n)$$

Hence, by replacing in (3.5) then

$$Q_i(n) = \sum_{t_i - t_j \geq 0} k_{ij} Q_j(n) + \sum_{t_i - t_j < 0} k_{ij} Q_j(n - 1) + f_i(n) \quad i = 1, 2, \dots, 12 \quad (3.7)$$

With

$$k_{ij}^* = \begin{cases} -k_{ij}, & i \neq j \\ 1 - k_{ij}, & i = j \end{cases} \quad (3.8)$$

Then formula (3.7) can be rewritten as

$$\sum_{\tau_{ij} \geq 0} k_{ij}^* Q_j(n) + \sum_{\tau_{ij} < 0} k_{ij}^* Q_j(n - 1) = f_i(n) \quad i = 1, 2, \dots, 12 \quad (3.9)$$

Let, vector  $Q(n) = (Q_1(n), \dots, Q_{12}(n))$  then matrix form of equation (3.9) is as  $AQ(n) + BQ(n - 1) = b$  where  $A, B$  is two  $12 \times 12$  matrices and  $b$  is a  $1 \times 12$  vector.

Regarding to above-mentioned interpretation energy circulation is stable if

$$\lim_{n \rightarrow 0} \|Q(n) - Q(n - 1)\| = 0 \quad (3.10)$$

where  $\|\cdot\|$  is a distance in vector space. We prove the following theorem for stability of energy circulation.

**Theorem 1.** Energy circulation is stable if  $\|A^{-1}B\| < 1$ . ( $A^{-1}$  is inverse of  $A$ )

**Proof:** It is sufficient, we show that if  $\|A^{-1}B\| < 1$  then

$$\lim_{n \rightarrow 0} \|Q(n) - Q(n - 1)\| = 0 \quad (3.11)$$

We know that  $AQ(n) + BQ(n - 1) = b$ . If  $\det(A) \neq 0$  then  $A$  is invertible. If  $AQ(n) + BQ(n - 1) = b \Rightarrow Q(n) = -A^{-1}BQ(n - 1) + A^{-1}b$

$$\begin{aligned} \|Q(n) - Q(n - 1)\| &= \|-A^{-1}BQ(n - 1) + A^{-1}b + A^{-1}BQ(n - 2) - A^{-1}b\| = \|-A^{-1}BQ(n - 1) \\ &+ A^{-1}BQ(n - 2)\| \leq \|A^{-1}B\| \|Q(n - 1) - Q(n - 2)\| \leq \|A^{-1}B\|^2 \|Q(n - 2) - Q(n - 3)\| \leq \\ &\|A^{-1}B\|^3 \|Q(n - 3) - Q(n - 4)\| \leq \dots \leq \|A^{-1}B\|^{n-1} \|Q(1) - Q(0)\| \end{aligned}$$

Therefore  $\|Q(n) - Q(n-1)\| \leq \|A^{-1}B\|^{n-1} \|Q(1) - Q(0)\|$  hence, it is evident that if  $\|A^{-1}B\| < 1$  then

$$\lim_{n \rightarrow \infty} \|Q(n) - Q(n-1)\| = 0 \quad (3.12)$$

### 3.2 Delay stability of energy circulation with fuzzy value

Regarding to Eq. (3.9) in which  $f_i(t)$  the external influence on  $i$ -th meridian, is not of exact value then we propose  $f_i(t)$  is the fuzzy and then formula (3.9) is rewritten as

$$\sum_{\tau_{ij} \geq 0} k_{ij}^* \tilde{Q}_j(n) + \sum_{\tau_{ij} < 0} k_{ij}^* \tilde{Q}_j(n-1) = \tilde{f}_i(n) \quad i = 1, 2, \dots, 12 \quad (3.13)$$

then matrix form of equation (3.13) is as  $A\tilde{Q}(n) + B\tilde{Q}(n-1) = \tilde{b}$  where  $A, B$  are two  $12 \times 12$  real matrices and  $\tilde{b}$  is a  $1 \times 12$  fuzzy vector.

Regarding to the interpretation of energy circulation, energy circulation is stable if

$$\lim_{n \rightarrow \infty} d(\tilde{Q}(n), \tilde{Q}(n-1)) = 0 \quad (3.14)$$

where  $d$  is a metric distance in fuzzy vector space.

## 4 Conclusions

In the real world there are many problems which have mathematical model. In this paper, we proposed the mathematical model in acupuncture. we consider the mathematical model of energy circulation in the human body with time acupuncture medicine and we showed the health stability by using an iteration linear equation. We proved the stability of energy circulation equation with exact value and introduced the fuzzy energy circulation equation and research the stability of iteration equation. In the end, we hope that this idea can be extended to other topics in the world.

## References

- [1] M. Friedman, S. birch, W.A. Tiller, mathematical modeling as a tool for basic research in acupuncture, the journal of alternative and complemetary medicine 3 (1) (1997) 89-99.
- [2] M .Friedman, S. Birch, modeling healing in acupuncture and traditional East Asian Medicine(TEAM), 2008, Unsubmitted.
- [3] M .Friedman, S. Birch , W.A. Tiller, Towards the development of a mathematical model for acupuncture meridians, Acup Electrother Res. Int. 14 (1989) 217-226.
- [4] H.J. Zimmermann, Fuzzy Set Theory and its Applications, second ed., Kluwer-Nijhoff, Boston, 1991.
- [5] D. Kincaid, W. Cheney, Numerical analysis matematics of scientific computing, 1929.