



# An Approach for Solving Fuzzy Fredholm Integro-Difference Equations with Mixed Argument

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## Abstract

In this work, the Taylor polynomial approximation for the solution of fuzzy Fredholm integro-difference equations with mixed argument and variable coefficients under the conditions is proposed. To do this, a Taylor matrix method is introduced. In this method, the truncated Taylor expansions of the functions are taken in the fuzzy Fredholm integro-difference equation and then their matrix forms is substituted into the mentioned equation. Hence by solving the matrix equation, unknown fuzzy Taylor coefficient can be found. Finally, the proposed method is illustrated by solving an example.

*Keywords* : Taylor polynomial; Fuzzy Fredholm equation; Fuzzy difference equations; Fuzzy polynomial approximation

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## 1 Introduction

The fuzzy integral equation method is used for solving many problems in mathematical physics and engineering. This problems are often reduced to fuzzy integro-difference equations. Taylor approach to solve linear differential, integral and integro-differential equations have been presented in many papers [6, 8]. But this article presents the Taylor approach for approximation of the solution of the fuzzy Fredholm integro-difference equation with mixed argument. In this paper, operational matrix of Taylor polynomial is used and the fuzzy Fredholm integro-difference equation is reduced to the fuzzy linear system of algebraic equations by it that can be solved directly.

The concept of fuzzy numbers and arithmetic operations with these numbers were first introduced and investigated by Zadeh [11] and, etc. Consequently, the fuzzy integral which

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is the same as that of Dubois and Prade in [3]. Park et al. in [7] have considered the existence of solution of fuzzy integral equation in Banach space and Subrahmaniam and Sudarsanam in [10] have proved the existence of solution of fuzzy functional equations. This paper is organized as follows:

In Section 2, the basic concept of fuzzy number operation is brought. In Section 3, the main section of the paper, Fuzzy Difference and Fredholm Integro-Difference Equation is introduced and in Section 4, Taylor matrix method is discussed in details and the proposed idea is illustrated by some examples. Finally conclusion is drawn in Section 5.

## 2 Preliminaries

There are various definitions for the concept of fuzzy numbers ([3, 5]).

**Definition 2.1.** *An arbitrary fuzzy number  $u$  in the parametric form is represented by an ordered pair of functions  $(u_r^-, u_r^+)$  which satisfy the following requirements:*

1.  $u_r^-$  is a bounded left-continuous non-decreasing function over  $[0, 1]$ .
2.  $u_r^+$  is a bounded left-continuous non-increasing function over  $[0, 1]$ .
3.  $u_r^- \leq u_r^+$ ,  $0 \leq r \leq 1$ .

**Definition 2.2.** *Let  $E$  be a set of all fuzzy numbers, we say that  $f(x)$  is a fuzzy valued function if  $f: \mathfrak{R} \rightarrow E$ .*

We use the Hausdorff distance between fuzzy numbers. This fuzzy number space as shown in [2] can be embedded into Banach space  $B = \bar{c}[0, 1] \times \bar{c}[0, 1]$  where the metric is usually defined as follows: Let  $E$  be the set of all upper semicontinuous normal convex fuzzy numbers with bounded  $r$ -level sets. Since the  $r$ -cuts of fuzzy numbers are always closed and bounded, the intervals are written as  $u[r] = [\underline{u}(r), \bar{u}(r)]$ , for all  $r$ . We denote by  $\omega$  the set of all nonempty compact subsets of  $\mathbb{R}$  and by  $\omega_c$  the subsets of  $\omega$  consisting of nonempty convex compact sets. Recall that

$$\rho(x, A) = \min_{a \in A} \|x - a\|$$

is the distance of a point  $x \in \mathbb{R}$  from  $A \in \omega$  and the Hausdorff separation  $\rho(A, B)$  of  $A, B \in \omega$  is defined as

$$\rho(A, B) = \max_{a \in A} \rho(a, B).$$

Note that the notation is consistent, since  $\rho(a, B) = \rho(\{a\}, B)$ . Now,  $\rho$  is not a metric. In fact,  $\rho(A, B) = 0$  if and only if  $A \subseteq B$ . The Hausdorff metric  $d_H$  on  $\omega$  is defined by

$$d_H(A, B) = \max\{\rho(A, B), \rho(B, A)\}.$$

The metric  $d_\infty$  is defined on  $E$  as

$$d_\infty(u, v) = \sup\{d_H(u[r], v[r]) : 0 \leq r \leq 1\}, \quad u, v \in E.$$

for arbitrary  $(u, v) \in \bar{c}[0, 1] \times \bar{c}[0, 1]$ . The following properties are well-known. (see e.g. [5, 9])

- (i)  $d_\infty(u + w, v + w) = d_\infty(u, v), \quad \forall u, v, w \in E,$
- (ii)  $d_\infty(k.u, k.v) = |k|d_\infty(u, v), \quad \forall k \in R, u, v \in E,$
- (iii)  $d_\infty(u + v, w + e) \leq d_\infty(u, w) + d_\infty(v, e), \quad \forall u, v, w, e \in E,$
- (iv)  $d_\infty(u, v) = d_\infty(v, u), \quad \forall u, v \in E,$

**Theorem 2.1.** , [9].

- (i) If we define  $\tilde{0} = \chi_0$ , then  $\tilde{0} \in E$  is a neutral element with respect to addition, i.e.  $u + \tilde{0} = \tilde{0} + u = u$ , for all  $u \in E$ .
- (ii) With respect to  $\tilde{0}$ , none of  $u \in E \setminus R$ , has opposite in  $E$ .
- (iii) For any  $a, b \in R$  with  $a, b \geq 0$  or  $a, b \leq 0$  and any  $u \in E$ , we have  $(a+b).u = a.u + b.u$ ; however, this relation dose not necessarily hold for any  $a, b \in R$ , in general.
- (iv) For any  $\lambda \in R$  and any  $u, v \in E$ , we have  $\lambda.(u + v) = \lambda.u + \lambda.v$ ;
- (v) For any  $\lambda, \mu \in R$  and any  $u \in E$ , we have  $\lambda.(\mu.u) = (\lambda.\mu).u$ .

**Definition 2.3.** Consider  $x, y \in E$ . If there exists  $z \in E$  such that  $x = y + z$ , then  $z$  is called the H-difference of  $x$  and  $y$  and it is denoted by  $x \ominus y$ .

In this paper, the sign "  $\ominus$  " always stands for H-difference and note that  $x \ominus y \neq x + (-y)$ . Let us recall the definition of strongly generalized differentiability introduced in [2].

**Lemma 2.1.** ,[2]. Let  $u, v \in E$  be such that  $u(1) - \underline{u}(0) > 0, \bar{u}(0) - u(1) > 0$  and  $len(v) = (\bar{v}(0) - \underline{v}(0)) \leq \min\{u(1) - \underline{u}(0), \bar{u}(0) - u(1)\}$ . Then the H-difference  $u \ominus v$  exists.

**Definition 2.4.** ,[1]. Let  $f : (a, b) \times E \rightarrow E$  and  $x_0 \in (a, b)$ . We Define the  $n$ th-order differential of  $f$  as follow: We say that  $f$  is strongly generalized differentiable of the  $n$ th-order at  $x_0$ . If there exists an element  $f^{(s)}(x_0) \in E, \quad \forall s = 1, \dots, n$ , such that

- (i) for all  $h > 0$  sufficiently small,

$$\exists f^{(s-1)}(x_0 + h) \ominus f^{(s-1)}(x_0), \quad \exists f^{(s-1)}(x_0) \ominus f^{(s-1)}(x_0 - h)$$

and the limits(in the metric  $d_\infty$ )

$$\lim_{h \searrow 0} \frac{f^{(s-1)}(x_0 + h) \ominus f^{(s-1)}(x_0)}{h} = \lim_{h \searrow 0} \frac{f^{(s-1)}(x_0) \ominus f^{(s-1)}(x_0 - h)}{h} = f^{(s)}(x_0)$$

or

- (ii) for all  $h > 0$  sufficiently small,

$$\exists f^{(s-1)}(x_0) \ominus f^{(s-1)}(x_0 + h), \quad \exists f^{(s-1)}(x_0 - h) \ominus f^{(s-1)}(x_0)$$

and the limits(in the metric  $d_\infty$ )

$$\lim_{h \searrow 0} \frac{f^{(s-1)}(x_0) \ominus f^{(s-1)}(x_0 + h)}{-h} = \lim_{h \searrow 0} \frac{f^{(s-1)}(x_0 - h) \ominus f(x_0)}{-h} = f^{(s)}(x_0)$$

or

(iii) for all  $h > 0$  sufficiently small,

$$\exists f^{(s-1)}(x_0 + h) \ominus f^{(s-1)}(x_0), \quad \exists f^{(s-1)}(x_0 - h) \ominus f^{(s-1)}(x_0)$$

and the limits (in the metric  $d_\infty$ )

$$\lim_{h \searrow 0} \frac{f^{(s-1)}(x_0 + h) \ominus f^{(s-1)}(x_0)}{h} = \lim_{h \searrow 0} \frac{f^{(s-1)}(x_0 - h) \ominus f^{(s-1)}(x_0)}{-h} = f^{(s)}(x_0)$$

or

(iv) for all  $h > 0$  sufficiently small,

$$\exists f^{(s-1)}(x_0) \ominus f^{(s-1)}(x_0 + h), \quad \exists f^{(s-1)}(x_0) \ominus f^{(s-1)}(x_0 - h)$$

and the limits (in the metric  $d_\infty$ )

$$\lim_{h \searrow 0} \frac{f^{(s-1)}(x_0) \ominus f^{(s-1)}(x_0 + h)}{-h} = \lim_{h \searrow 0} \frac{f^{(s-1)}(x_0) \ominus f^{(s-1)}(x_0 - h)}{h} = f^{(s)}(x_0)$$

( $h$  and  $-h$  at denominators mean  $\frac{1}{h}$  and  $\frac{-1}{h}$ , respectively  $\forall s = 1 \dots n$ )

### 3 Fuzzy Fredholm integro-difference equation

In this section, we are going to introduce Taylor expansion method for solving fuzzy Fredholm integro-difference equation.

**Definition 3.1.** *The Linear equation*

$$\sum_{k=0}^K P_k(x)y(x+k) + \int_a^b K(x,t)y(t)dt = g(x) \quad (3.1)$$

under conditions

$$\sum_{r=0}^R c_{ir}y(c_r) = \tilde{\mu}_i, \quad i = 0, \dots, K \quad (3.2)$$

is called fuzzy Fredholm integro-difference equation where  $P_k(x)$ ,  $g(x)$  and  $K(x,t)$  are crisp functions that have positive derivatives on interval  $[a, b]$  and  $c_{ir}$  and  $c_r$  are crisp constants and  $\tilde{\mu}_i$  is fuzzy constant.

Our idea in this article is approximation of fuzzy valued function  $y(x)$  by Taylor expansion. We suppose the solution is expressed in form

$$y(x) = \sum_{n=0}^N \tilde{a}_n(x-c)^n \quad (3.3)$$

### 3.1 Fundamental matrix relations

Consider the linear difference equation with variable coefficient (3.1). Let us find the truncated Taylor expansion of each term in expression (3.1) at  $x = c$  and their matrix representations.

We first consider  $y(x)$  in the matrix form

$$[y(x)] = XA \tag{3.4}$$

where desired solution  $y(x)$  of Eq. (3.1) is defined by Taylor polynomial in (3.3),

$$X = [ 1 \quad (x - c) \quad (x - c)^2 \quad \dots \quad (x - c)^N ]$$

and

$$A = [ \tilde{a}_0 \quad \tilde{a}_1 \quad \tilde{a}_2 \quad \dots \quad \tilde{a}_N ]^T$$

Now we write the expression  $y(x + k)$  as follows:

$$y(x + k) = \sum_{n=0}^N \tilde{a}_n ((x - c) + k)^n = \sum_{n=0}^N \sum_{i=0}^n C_i^n (x - c)^{n-i} k^i \tilde{a}_n \tag{3.5}$$

where

$$C_i^n = \binom{n}{i} = \frac{n!}{i!(n-i)!}$$

and in the matrix form

$$[y(x + k)] = X X_k A \tag{3.6}$$

where

$$X_k = \begin{bmatrix} C_0^0 & C_1^1 k^1 & C_2^2 k^2 & \dots & C_N^N k^N \\ 0 & C_0^1 & C_1^2 k^1 & \dots & C_{N-1}^N k^{N-1} \\ 0 & 0 & C_0^2 & \dots & C_{N-2}^N k^{N-2} \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \dots & C_0^N \end{bmatrix}$$

The Taylor polynomial expansion of the function  $P_k(x)$  about  $x = c$  can be written as follows:

$$P_k(x) = \sum_{r=0}^N p_{kr} (x - c)^r, \quad p_{kr} = \frac{P_k^{(r)}(c)}{r!} \tag{3.7}$$

If we rename the first part of Eq. (3.1)  $L_1(x)$  i.e.,

$$L_1(x) = \sum_{k=0}^K P_k(x) y(x + k)$$

then we have

$$L_1(x) = \sum_{k=0}^K \sum_{r=0}^N p_{kr} (x - c)^r y(x + k) \tag{3.8}$$

Using Eq. (3.5), the expansion of  $(x - c)^r y(x + k)$  becomes

$$(x - c)^r y(x + k) = \sum_{n=0}^N \sum_{i=0}^n C_i^n (x - c)^{n-i+r} k^i \tilde{a}_n$$

or the matrix form

$$[(x - c)^r y(x + k)] = X I_r X_k A \tag{3.9}$$

where

$$I_r = \begin{bmatrix} 0 & 0 & 0 & \cdot & \cdot & \cdot & 0 & 0 \\ 0 & 0 & 0 & \cdot & \cdot & \cdot & 0 & 0 \\ \dots & \dots & \dots & \cdot & \cdot & \cdot & \dots & \dots \\ 0 & 0 & 0 & \cdot & \cdot & \cdot & 0 & 0 \\ 1 & 0 & 0 & \cdot & \cdot & \cdot & 0 & 0 \\ 0 & 1 & 0 & \cdot & \cdot & \cdot & 0 & 0 \\ 0 & 0 & 1 & \cdot & \cdot & \cdot & 0 & 0 \\ \dots & \dots & \dots & \cdot & \cdot & \cdot & \dots & \dots \\ 0 & 0 & 0 & \dots & 1 & \dots & 0 & 0 \end{bmatrix}_{(N+1) \times (N+1)}$$

Consequently, the matrix form of the Eq. (3.8) is as follows:

$$[L_1(x)] = \sum_{k=0}^K \sum_{r=0}^N p_{kr} X I_r X_k A \tag{3.10}$$

Now, we obtain the matrix form for Fredholm integral part of Eq. (3.1). To this end, we approximate the kernel function  $K(x, t)$  by truncated Taylor series of degree  $N$  about  $x = c$  and  $t = c$  in the form

$$K(x, t) = \sum_{n=0}^N \sum_{m=0}^N k_{nm} (x - c)^n (t - c)^m \tag{3.11}$$

where

$$k_{nm} = \frac{1}{n!m!} \frac{\partial^{n+m} K(c, c)}{\partial x^n \partial t^m}, \quad n, m = 0, 1, \dots, N$$

The Eq. (3.11) can be written in the matrix form

$$[K(x, t)] = X K T^T \tag{3.12}$$

where

$$K = [k_{nm}]_{n,m=0}^N, \quad T = [ 1 \quad (t - c) \quad (t - c)^2 \quad \dots \quad (t - c)^N ]$$

We see from Eq. (3.4)

$$[y(t)] = T A \tag{3.13}$$

Substituting the expressions (3.12) and (3.13) into the Fredholm integral part of Eq. (3.1),

$$[F(x)] = \int_a^b XKT^T TAdt = XKHA \tag{3.14}$$

where

$$F(x) = \int_a^b K(x,t)y(t)dt$$

and

$$H = \left[ \int_a^b T^T T dt \right] = [h_{nm}], \quad h_{nm} = \frac{(b-c)^{n+m+1} - (a-c)^{n+m+1}}{n+m+1}$$

The nonhomogenous part of Eq. (3.1) can be obtained

$$g(x) = \sum_{n=0}^N g_n(x-c)^n, \quad g_n = \frac{g^{(n)}(c)}{n!}$$

or the matrix form

$$[g(x)] = XG \tag{3.15}$$

where

$$G = [ g_0 \quad g_1 \quad g_2 \quad \dots \quad g_N ]^T$$

### 3.2 Method of solution

Now, we are going to construct the matrix form of Eq. (3.1). We first substitute the matrix relations defined (3.10), (3.14) and (3.15) into Eq. (3.1) and then obtain the matrix form equation

$$\sum_{k=0}^K \sum_{r=0}^N p_{kr} I_r X_k A + KHA = G$$

then

$$\left[ \sum_{k=0}^K \sum_{r=0}^N p_{kr} I_r X_k + KH \right] A = G \tag{3.16}$$

which corresponds to a system of  $(N + 1)$  algebraic equations for the  $(N + 1)$  unknown coefficient  $\tilde{a}_0, \tilde{a}_1, \dots, \tilde{a}_N$ . We can write Eq. (3.16) in the form

$$WA = G \tag{3.17}$$

so that

$$W = \sum_{k=0}^K \sum_{r=0}^N p_{kr} I_r X_k + KH = [w_{nh}], \quad n, h = 0, 1, \dots, N$$

We can obtain the matrix form of the conditions (3.2), by means of relation (3.4)

$$\sum_{r=0}^R c_{ir} C_r A = [\tilde{\mu}_i], \quad i = 0, 1, \dots, K, \quad a \leq c_{ir} \leq b \tag{3.18}$$

where

$$C_r = [ 1 \quad (c_r - c) \quad (c_r - c)^2 \quad \dots \quad (c_r - c)^N ]$$

Clearly, the matrix form for (3.2) is

$$U_i A = [\tilde{\mu}_i], \quad i = 0, 1, \dots, K \tag{3.19}$$

where

$$U_i = \sum_{r=0}^R c_{ir} C_r = [ u_{i0} \quad u_{i1} \quad u_{i2} \quad \dots \quad u_{iN} ]$$

To obtain the solution of Eq. (3.1) under the conditions (3.2), replacing the row matrix (3.19) by the last  $K + 1$  rows of the matrix (3.17), we have

$$W' = \begin{bmatrix} w_{00} & w_{01} & \cdot & \cdot & \cdot & w_{0N} \\ w_{10} & w_{11} & \cdot & \cdot & \cdot & w_{1N} \\ \dots & \dots & \cdot & \cdot & \cdot & \dots \\ w_{N-K-1,0} & w_{N-K-1,1} & \cdot & \cdot & \cdot & w_{N-K-1,N} \\ u_{00} & u_{01} & \cdot & \cdot & \cdot & u_{0N} \\ u_{10} & u_{11} & \cdot & \cdot & \cdot & u_{1N} \\ \dots & \dots & \cdot & \cdot & \cdot & \dots \\ u_{K0} & u_{K1} & \cdot & \cdot & \cdot & u_{KN} \end{bmatrix}_{(N+1) \times (N+1)} \tag{3.20}$$

and

$$G' = [ g_0 \quad g_1 \quad \dots \quad g_{N-K-1} \quad \tilde{\mu}_0 \quad \tilde{\mu}_1 \quad \dots \quad \tilde{\mu}_K ]^T \tag{3.21}$$

Now, we solve the linear fuzzy system

$$W' A = G'$$

### 4 Example

**Example 4.1.** Consider fuzzy integro-difference equation

$$y(x + 1) + xy(x) + \int_{-1}^1 (xe^x + t)y(t)dt = (1 - x)e^{x+1} + xe^x - \frac{2}{e}$$

with conditions

$$y(0) = (r, 2 - r), \quad y(1/2) = (r - 1, 1 - r)$$

and approximate the solution  $y(x)$  by the polynomial

$$y(x) = \sum_{n=0}^5 \tilde{a}_n x^n$$

It is clear that  $N = 5, c = 0, P_0(x) = x, P_1(x) = 1$  and  $K = 1$  Using matrix form (3.16),

$$\left[ \sum_{k=0}^1 \sum_{r=0}^5 p_{kr} I_r X_k + KH \right] A = G$$



then

$$I_0 = X_0 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad X_1 = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 & 4 & 5 \\ 0 & 0 & 1 & 3 & 6 & 10 \\ 0 & 0 & 0 & 1 & 4 & 10 \\ 0 & 0 & 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$K = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1/2 & 0 & 0 & 0 & 0 & 0 \\ 1/6 & 0 & 0 & 0 & 0 & 0 \\ 1/24 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad I_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$H = \begin{bmatrix} 2 & 0 & 2/3 & 0 & 2/5 & 0 \\ 0 & 2/3 & 0 & 2/5 & 0 & 2/7 \\ 2/3 & 0 & 2/5 & 0 & 2/7 & 0 \\ 0 & 2/5 & 0 & 2/7 & 0 & 2/9 \\ 2/5 & 0 & 2/7 & 0 & 2/9 & 0 \\ 0 & 2/7 & 0 & 2/9 & 0 & 2/11 \end{bmatrix}, \quad G = \begin{bmatrix} 1.982522 \\ 1.000000 \\ -0.359140 \\ -0.406093 \\ -0.173118 \\ -0.048942 \end{bmatrix}$$

So,

$$W' = \begin{bmatrix} 1 & 5/3 & 1 & 7/5 & 1 & 9/7 \\ 3 & 1 & 8/3 & 3 & 22/5 & 5 \\ 2 & 1 & 5/3 & 3 & 32/3 & 10 \\ 1 & 0 & 4/3 & 1 & 21/5 & 10 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1/2 & 1/4 & 1/8 & 1/16 & 1/32 \end{bmatrix}, \quad G' = \begin{bmatrix} 1.982522 \\ 1.000000 \\ -0.359140 \\ -0.406093 \\ (r, 2-r) \\ (r-1, 1-r) \end{bmatrix}$$

Therefore, we solve  $W'A = G'$  by using Friedman et al. proposed method in [4] and obtain

$$A = \begin{bmatrix} \tilde{a}_0 \\ \tilde{a}_1 \\ \tilde{a}_2 \\ \tilde{a}_3 \\ \tilde{a}_4 \\ \tilde{a}_5 \end{bmatrix} = \begin{bmatrix} (r, 2-r) \\ (-0.7321 + 0.381r, 0.0299 - 0.381r) \\ (-6.14 + 0.2623r, -5.6227 - 0.2623r) \\ (4.7211 + 1.0601r, 6.8414 - 1.0601r) \\ (-1.7056 + 0.1278r, -1.4499 - 0.1278r) \\ (0.7159 + 0.0127r, 0.7413 - 0.0127r) \end{bmatrix}$$

## 5 Conclusion

In this paper, we proposed Taylor polynomial approximation for finding the solution of fuzzy Fredholm integro-difference equation. To do this, the matrix form of this method was introduced and the mentioned equations were transformed into the fuzzy linear system and then by solving the fuzzy linear system the fuzzy coefficients were obtained.

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