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Int. J. Industrial Mathematics Vol. 1, No. 4 (2009) 333-342

Ranking of DMUs on Interval Data by Imposing a Minimum Weight Restriction in DEA

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Received 22 September 2009; accepted 20 November 2009.

Abstract

In this paper, we present a method for ranking decision making units (DMUs) with interval data in data envelopment analysis that is different from the other methods. In this method we use a new pair of interval DEA models that are constructed on the basis of interval arithmetic, which differ from the existing DEA models that work with interval data in fact it is a linear CCR model without a need for extra variable alternations and uses a fixed and unified production frontier (i.e. the same constraint set) to measure the efficiencies of decision making units (DMUs) with interval input and output data. This methodology exerts an appropriate minimum weight restriction on all input and output weights which is called maxmin weight. New linear programming (LP) model constructs proportion with each efficient unit for determining maximin weight. Choosing a maxmin weight, all efficient units put in order as fully or partially. One numerical example is inspected using the proposed ranking methodology to illustrate its ability in discriminating between DEA efficient unit with interval data.

Keywords : Data envelopment analysis ; DEA ranking ; Interval data ; Maximin weights ; Minimum weight restriction.

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1 Introduction

Data envelopment analysis (DEA) was originally developed to measure the relative efficiency of peer decision making units (DMUs) in multiple input-multiple output settings [1, 2]. The standard DEA models assume that all data are known exactly without any variation. However, this assumption may not be true. Due to the existence of uncertainty, DEA sometimes faces the situation of imprecise data, especially when a set of decision-making units (DMUs) contains missing data, judgment data, forecasting data or ordinal

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preference information, Generally speaking, uncertain information or imprecise data can be expressed in interval or fuzzy numbers.
Cooper et al. [3, 4, 5] were the first, to the best of our knowledge, to study how to deal with imprecise data such as bounded data, ordinal data and ratio bounded data in DEA. The resulting DEA model was called imprecise DEA (IDEA), which transformed a non-linear programming problem into a linear programming problem (LP) equivalent through a series of scale transformations and variable alternations. The final efficiency score for each DMU was derived as a deterministic numerical value less than or equal to unity.
In the begining Wang et al. [7], proposed a procedure for ranking efficient units with precise data by imposing a minimum weight restriction on all input and output weights in DEA. In this paper, we present a new method for ranking decision making units with interval data, also use a new pair of interval DEA models that recommended by Wang et al. [8], to measure the efficiencies of decision-making units (DMUs) with interval data.The methodology puts in order DMUs by imposing an appropriate minimum weight restriction on all inputs and outputs. For determining minimum weight, restrictions of new linear programming models construct proportionate with each efficient unit. Through exerting a minimum weight restriction on all inputs and outputs, all efficient units can be fully or partially distinguished from the other one.
The paper is organized as follows. In Section 2, we present a new pair of interval DEA models [8]. In Section 3, we develop a maximin weight model for each efficient unit with interval data to determine a maximin weight for each of them. Section 4 presents a new efficiency model with a minimum weight restriction for all efficient units with interval data to reevaluate their efficiencies. Numerical example is provided in Section 5 to illustrate the ability of the proposed ranking methodology in distinguishing between DEA efficient units. The paper concludes in Section 6.

2 Interval models of DEA based on interval arithmetic

Suppose that there are n DMUs to be evaluated. Each $DMU_j (j = 1, ..., n)$ produces s different outputs $y_{rj} (r = 1, ..., s)$ utilizing m different inputs $x_{ij} (i = 1, ..., m)$. Without loss of generality, we assume that all the input and output data y_{rj} and $x_{ij} (i = 1, ..., m; r = 1, ..., s; j = 1, ..., n)$ cannot be exactly obtained due to the existence of uncertainty. They are known and lie within the upper and lower bounds represented by the intervals $[x^L_{ij}, x^U_{ij}]$ and $[y^L_{rj}, y^U_{rj}]$, where lower and upper bounds are known exactly, positive and finite.

New interval models of DEA are made as follows [8]:

Let

$$\theta_j = \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}}$$

be the efficiency of DMU_j. According to the operation rules on interval data, we have

$$\theta_j = \frac{\sum_{r=1}^{s} u_r [y^L_{rj}, y^U_{rj}]}{\sum_{i=1}^{m} v_i [x^L_{ij}, x^U_{ij}]} = \frac{\left[\sum_{r=1}^{s} u_r y^L_{rj} \ , \ \sum_{r=1}^{s} u_r y^U_{rj}\right]}{\left[\sum_{i=1}^{m} v_i x^L_{ij} \ , \ \sum_{i=1}^{m} v_i x^U_{ij}\right]} = \left[\frac{\sum_{r=1}^{s} u_r y^L_{rj}}{\sum_{i=1}^{m} v_i x^U_{ij}} \ , \ \frac{\sum_{r=1}^{s} u_r y^U_{rj}}{\sum_{i=1}^{m} v_i x^L_{ij}}\right]$$
$$j = 1, ..., n$$

It is obvious that θ_j should also be an interval number, which we denote by $[\theta^L_j, \theta^U_j] \quad (j = 1, ...n)$.

Let

$$\theta_j = [\theta_j^L, \theta_j^U] = \left[\frac{\sum_{r=1}^{s} u_r y_{rj}^L}{\sum_{i=1}^{m} v_i x_{ij}^U} \ , \ \frac{\sum_{r=1}^{s} u_r y_{rj}^U}{\sum_{i=1}^{m} v_i x_{ij}^L}\right] \subseteq (0,1] \qquad j = 1, ..., n$$

Then

$$\theta_j^U = \left[\frac{\sum_{r=1}^{s} u_r y_{rj}^U}{\sum_{i=1}^{m} v_i x_{ij}^L}\right] \leq 1 \qquad j = 1, ..., n$$

$$\theta_j^L = \left[\frac{\sum_{r=1}^{s} u_r y_{rj}^L}{\sum_{i=1}^{m} v_i x_{ij}^U}\right] > 0 \ , \qquad j = 1, ..., n$$

In order to assess the upper and lower bounds of the efficiency of DMU_o, we develop the following pair of fractional programming models for DMU_o:

$$\begin{aligned} \max \quad & \theta_o^U = \frac{\sum_{r=1}^{s} u_r y_{ro}^U}{\sum_{i=1}^{m} v_i x_{io}^L} \\ s.t. \quad & \frac{\sum_{r=1}^{s} u_r y_{rj}^U}{\sum_{i=1}^{m} v_i x_{ij}^L} \leq 1 && j = 1, ..., n \\ & u_r \geq 0 && r = 1, ..., s \\ & v_i \geq 0 && i = 1, ..., m \end{aligned} \tag{2.1}$$

$$\begin{aligned} \max \quad & \theta_o^L = \frac{\sum_{r=1}^{s} u_r y_{ro}^L}{\sum_{i=1}^{m} v_i x_{io}^U} \\ s.t. \quad & \frac{\sum_{r=1}^{s} u_r y_{rj}^U}{\sum_{i=1}^{m} v_i x_{ij}^L} \leq 1 && j = 1, ..., n \\ & u_r \geq 0 && r = 1, ..., s \\ & v_i \geq 0 && i = 1, ..., m \end{aligned} \tag{2.2}$$

Using Charnes - Cooper transformation, the above pair of fractional programming models can be made as the following equivalent LP models:

$$\begin{aligned} \max \quad & \theta_o^U = \textstyle\sum_{r=1}^{s} u_r y_{ro}^U \\ s.t. \quad & \textstyle\sum_{i=1}^{m} v_i x_{io}^L = 1 \\ & \textstyle\sum_{r=1}^{s} u_r y_{rj}^U - \sum_{i=1}^{m} v_i x_{ij}^L \leq 0 && j = 1, ..., n \\ & u_r \geq 0 && r = 1, ..., s \\ & v_i \geq 0 && i = 1, ..., m \end{aligned} \tag{2.3}$$

$$
\begin{array}{lll}
\max & \theta_o^L = \sum_{r=1}^{s} u_r y_{ro}^L & \\
s.t. & \sum_{i=1}^{m} v_i x_{io}^U = 1 & \\
& \sum_{r=1}^{s} u_r y_{rj}^U - \sum_{i=1}^{m} v_i x_{ij}^L \leq 0 & j = 1, ..., n \\
& u_r \geq 0 & r = 1, ..., s \\
& v_i \geq 0 & i = 1, ..., m
\end{array} \tag{2.4}
$$

Where θ_o^U stands for the best possible relative efficiency achieved by DMU_o when all the DMUs are in the state of the best production activity. Also θ_o^L stands for the lower bound of the best possible relative efficiency of DMU_o. The possible best relative efficiency interval $[\theta_o^L, \theta_o^U]$ is constructed by θ_o^L and θ_o^U.

Definition 2.1. *DMU_o, is DEA efficient if $\theta_o^{U^*} = 1$ and it is DEA inefficient if $\theta_o^{U^*} < 1$.*

Totally the LP model (2.3) is solved n times for each DMU. As a result, at least one DMU is evaluated as DEA efficient, but very often more than one DMU is assessed as DEA efficient. Therefore we cannot distinguish the difference between their performance. In the following sections, we will construct a maximin weight model for each DEA efficient unit and a new efficiency model with a minimum weight restriction to reassess the efficiencies of DEA efficient units.

3 Maximin weight model for DEA efficient units

Assume that DMU_o is a DEA efficient unit identified by model (2.3). There are the following equations and inequalities:

$$
\begin{array}{ll}
\sum_{r=1}^{s} u_r y_{ro}^U - \sum_{i=1}^{m} v_i x_{io}^L = 0 & \\
\sum_{r=1}^{s} u_r y_{rj}^U - \sum_{i=1}^{m} v_i x_{ij}^L \leq 0 & j = 1, ..., n \\
u_r \geq 0 & r = 1, ..., s \\
v_i \geq 0 & i = 1, ..., m
\end{array} \tag{3.5}
$$

There are an infinite number of input and output weights that can satisfy the above conditions because ku_r and kv_i for any $k > 0$ will be solutions of (3.5) as much as $u_r \quad (r = 1, ..., s)$ and $v_i \quad (i = 1, ..., m)$. To avoid this arbitrariness, we need an equation such as $\sum_{i=1}^{m} v_i x_{io}^L = 1$ or $\sum_{i=1}^{m} v_i (\sum_{j=1}^{n} x_{ij}^L) = 1$ to be added to (3.5) to form a benchmark for comparison of input and output weights of different DMUs. Since the equation $\sum_{i=1}^{m} v_i x_{io}^L = 1$ changes from one DEA efficient unit to another, this causes that the input and output weights of different DMUs somewhat be incomparable. Then the equation $\sum_{i=1}^{m} v_i (\sum_{j=1}^{n} x_{ij}^L) = 1$ must be added.

Let ϵ^* be the maximin weight proportionate with DMU_o, i.e.

$$
\epsilon^* = max_{u_r, v_i} \left\{ min \left(min_r (u_r (\sum_{j=1}^{n} y_{rj}^U)), min_i (v_i (\sum_{j=1}^{n} x_{ij}^L)) \right) \right\}
$$

The following LP model is constructed to determine the value of ϵ^* for each DEA efficient unit:

$$\begin{array}{lll} \max & \epsilon & \\ s.t. & \sum_{i=1}^{m} v_i(\sum_{j=1}^{n} x_{ij}^L) = 1 & \\ & \sum_{r=1}^{s} u_r y_{ro}^U - \sum_{i=1}^{m} v_i x_{io}^L = 0 & \\ & \sum_{r=1}^{s} u_r y_{rj}^U - \sum_{i=1}^{m} v_i x_{ij}^L \leq 0 & j = 1, ..., n \\ & u_r(\sum_{j=1}^{n} y_{rj}^U) \geq \epsilon & r = 1, ..., s \\ & v_i(\sum_{j=1}^{n} x_{ij}^L) \geq \epsilon & i = 1, ..., m \end{array} \tag{3.6}$$

The ϵ^* in the above LP model is a decision variable rather than a constant and is not necessarily very small. It also is the maximin weight that can keep the DMU_o as efficient. However, the maximin weight determined by the above LP model is independent of the units of measurement of lower and upper bounds of inputs and outputs intervals. By solving LP model (3.6) for each DEA efficient unit, we can obtain a set of maximin weights, $\epsilon_{i_1}^*, ..., \epsilon_{i_k}^*$ for all DEA efficient units, respectively, where $i_1, ..., i_k$ are the labels of k DEA efficient units.

4 New efficiency model for ranking efficient units

For making new efficiency model, we are emposing a minimum weight restriction ω on all inputs and outputs weights so that efficient units can be distinguished by adjusting its magnitud and as shown below :

$$\begin{array}{lll} \max & \frac{\sum_{r=1}^{s} u_r y_{ro}^U}{\sum_{i=1}^{m} v_i x_{io}^L} & \\ s.t. & \sum_{i=1}^{m} v_i(\sum_{j=1}^{n} x_{ij}^L) = 1 & \\ & \frac{\sum_{r=1}^{s} u_r y_{rj}^U}{\sum_{i=1}^{m} v_i x_{ij}^L} \leq 0 & j = 1, ..., n \\ & u_r(\sum_{j=1}^{n} y_{rj}^U) \geq \omega & r = 1, ..., s \\ & v_i(\sum_{j=1}^{n} x_{ij}^L) \geq \omega & i = 1, ..., m \end{array} \tag{4.7}$$

This efficiency model is different from CCR model (2.1). An essential difference between them is the minimum weight restriction ω, which is not required to CCR model (2.1).

Now consider the following transformation:

$$\begin{array}{ll} z = \frac{1}{\sum_{i=1}^{m} v_i x_{io}^L} & \\ \tilde{u}_r = u_r \cdot z, & (r = 1, ...s) \\ \tilde{v}_i = v_i \cdot z, & (i = 1, ..., m) \end{array} \tag{4.8}$$

With the mentioned transformations, the model (4.7) can be transformed into an equivalent model, which as shown below:

$$
\begin{array}{lll}
\max & \sum_{r=1}^{s} \tilde{u}_r y_{ro}^U & \\
s.t. & \sum_{i=1}^{m} \tilde{v}_i x_{io}^L = 1 & \\
& \sum_{i=1}^{m} \tilde{v}_i (\sum_{j=1}^{n} x_{ij}^L) = z & \\
& \sum_{r=1}^{s} \tilde{u}_r y_{rj}^U - \sum_{i=1}^{m} \tilde{v}_i x_{ij}^L \le 0 & j = 1, ..., n \\
& \tilde{u}_r (\sum_{j=1}^{n} y_{rj}^U) \ge \omega \cdot z & r = 1, ..., s \\
& \tilde{v}_i (\sum_{j=1}^{n} x_{ij}^L) \ge \omega \cdot z & i = 1, ..., m
\end{array}
\tag{4.9}
$$

For any efficient unit the LP model (4.9) is solved for k times. By setting an appropriate value for the minimum weight restriction ω, all efficient units can be expected to be fully or partially ranked in terms of their new efficiency scores.
We first arrange the maximin weights of all DEA efficient units. Suppose that the maximin weights $\epsilon_{i_1}^*, ..., \epsilon_{i_k}^*$ of k DEA efficient units have already been ordered from the smallest to the largest, i.e. $\epsilon_{i_1}^* \le \epsilon_{i_2}^* \le ... \le \epsilon_{i_k}^*$.

If we put the minimum weight restriction of ω equal to $\epsilon_{i_k}^*$ and apply the model (4.9) for efficient units, the new efficiency score of DMU_{i_k} will be obtained equal to 1 and for the other DMUs , it will be less than one. The reason is that the equation $\sum_{r=1}^{s} u_r y_{rk}^U - \sum_{i=1}^{m} v_i x_{ik}^L = 0$ in model (3.6) and the constraints of model (4.7) are hold at the same time, for $\omega = \epsilon_{i_k}^*$ proportionate with DMU_{i_k} . As we decrease the value of ω in model (4.7) from $\omega = \epsilon_{i_k}^*$ to $\omega = \epsilon_{i_{k-1}}^*$, the new efficiency score of $DMU_{i_{k-1}}$ will be equal to one. In other words, $DMU_{i_{k-1}}$ add to new set of DEA efficient units. This reduce process can be continued same as the value of ω. Finally , by setting $\omega = \epsilon_{i_1}^*$, the new efficiency score for each k DEA efficient unit will be one.Therefore, the efficient units rank their new efficiency score, which are obtained by model (4.9). In other words, in model (4.7) by choosing the largest maximin weigh, i.e. $\epsilon_{i_k}^*$, for minimum weight restriction ω, all DEA efficient units can be ranked.

Remark 1. In some extreme cases, the largest maximin weight may occasionally be achieved by two or more DEA efficient units. In such cases, these DEA efficient units should be considered as good as each other.

In summary, the proposed ranking methodology for efficient units can be performed by the following steps:
Step 1. Perform model (2.3) for each DMU to identify DEA efficient units.
Step 2. Solving maximin weight model (3.6) for each DEA efficient unit to find its maximin weight.
Step 3. Set an appropriate value as a minimum weight restriction on input and output weights and perform new efficiency model (4.9) to reevaluate the efficiencies of efficient units .
Step 4. Rank efficient units by their new efficiency scores.

5 Numerical example

Example 5.1. *We now apply this new ranking approach to some commercial bank branches in Iran. There are 10 branches in this district. Each branch uses 3 inputs to produce 5 outputs. Table 1 shows the kind of these inputs and outputs.*

Table 1

Inputs	Output
Payable interest	The total sum of four main deposits
Personnel	Other deposits
Non-Performing loans	Loans granted
	Received interest
	Fee

Tables 2 , 3 and 4 records the interval inputs and interval outputs of the ten commercial bank branches taken from Hosseinzadeh Lotfi et al. [6].

Table 2
Input-data for the 10 bank branches.

DMU_j	x_{1j}^L	x_{1j}^U	x_{2j}^L	x_{2j}^U	x_{3j}^L	x_{3j}^U
1	45007.37	9613.37	36.29	36.86	87243	87243
2	2926.81	5961.55	18.8	2016	9945	12120
3	8732.7	17752.5	25.74	25.74	47575	50013
4	945.93	1966.39	20.81	22.54	19292	19753
5	8487.07	17521.66	14.16	14.8	3428	3911
6	13759.35	27359.36	19.46	19.46	13929	15657
7	587.69	1205.47	27.29	27.48	27827	29005
8	4646.39	9559.61	24.52	25.07	9070	9983
9	1554.29	3427.89	20.47	21.59	412036	413902
10	17528.31	36297.54	14.84	15.05	8638	10229

Table 3
Output-data for the 10 bank branches.

DMU_j	y_{1j}^L	y_{1j}^U	y_{2j}^L	y_{2j}^U	y_{3j}^L	y_{3j}^U
1	2696995	3126798	263643	382545	1675519	1853365
2	340377	440355	95978	117659	377309	390203
3	1027546	1061260	37911	503089	468520	1822028
4	1145235	1213541	229646	268460	19292	542101
5	390902	395241	4924	12136	129751	142873
6	988115	1087392	74133	111324	507502	574355
7	144906	165818	180530	180617	288513	323721
8	408163	416416	405396	486431	1044221	1071812
9	335070	410427	337971	449336	1584722	1802942
10	700842	768593	14378	15192	2290745	2573512

Table 4
Output-data for the 10 bank branches.

DMU_j	y_{4j}^L	y_{4j}^U	y_{5j}^L	y_{5j}^U
1	108634.76	125740.28	965.97	6957.33
2	32396.65	37836.56	304.67	749.4
3	96842.33	108080.01	2285.03	3174
4	32362.8	39273.37	207.98	510.93
5	12662.71	14165.44	63.32	92.3
6	53591.3	72257.28	480.16	869.52
7	40507.97	45847.48	176.58	370.81
8	56260.09	73948.09	4654.71	5882.53
9	176436.81	189006.12	560.26	2506.67
10	662725.21	791463.08	58.89	86.86

Table 5
Upper bounds efficiency.

DMU_j	1	2	3	4	5	6	7	8	9	10
θ_j^{U*}	1	0.6959	1	1	1	1	1	1	1	1

Table 6
Maximin weights for DEA efficient units.

DMU_j	1	3	4	5	6	7	8	9	10
ϵ_j^*	0.1292	0.0727	0.01010	0.0046	0.0070	0.0289	0.1278	0.0722	0.1292

We first solve LP model (2.3) for each DMU, which the results are presented in Table 5. It is observed that DMUs 1 , 3 , 4 , 5 , 6 , 7 , 8 , 9 and 10 are DEA efficient and cannot distinguish between them any further. Then the results of solving model (3.6) to find maximin weights are given in Table 6, it is seen that DMU_1 and DMU_{10} have the biggest maximin weight therefore they are the bests.

Table 7
Efficiencies under different minimum weight restrictions.

DMU_j	Minimum weight restriction ω							
	0.0046	0.0070	0.0289	0.0722	0.0727	0.1010	0.1278	0.1292
1	1	1	1	1	1	1	1	1
3	1	1	1	1	1	0.9064	0.8011	0.7860
4	1	1	1	1	1	1	0.6862	0.6632
5	1	0.9121	0.4785	0.2737	0.2718	0.1875	0.1351	0.1328
6	1	1	0.7889	0.5483	0.5459	0.4351	0.3611	0.3575
7	1	1	1	0.5886	0.5852	0.4219	0.2638	0.2550
8	1	1	1	1	1	1	1	0.9656
9	1	1	1	1	0.9950	0.7327	0.5660	0.5543
10	1	1	1	1	1	1	1	1

Then we solve the model (4.9) for each DEA efficient unit by specifying a value for the minimum weight restriction ω to generate a partial or full ranking of the nine efficient *units 1 , 3 , 4 , 5 , 6 , 7 , 8 , 9 and 10. Table 7 shows the efficiency score of the nine efficient units measured by model (4.9) with different minimum weight restriction. If we impose a minimum weight restriction* $w = 0.0046$ *on input and output weights, then* DMU_1 *,* DMU_3 *to* DMU_{10} *will be DEA efficient. If a minimum weight restriction* $w = 0.0070$ *is exerted on inputs and output, then* DMU_5 *will not be efficient any more. If a minimum weight restriction* $w = 0.0289$ *is applied on inputs and output weights, then* DMU_5 *and* DMU_6 *will not be efficient any more. This removal process can be continued by constantly increasing the value of* ω. *If a minimum weight restriction* $w = 0.1292$ *is imposed on input and output weights,* DMU_1 *and* DMU_{10} *will be the only units to remain efficient and therefore a full ranking will be achieved. Apparently, by choosing an appropriate value for the minimum weight restriction on input and output weights, the DM or assessor can give a partial or full ranking of the nine DMUs.*

6 Conclusions

In this paper, we presented a ranking methodology for DEA efficient units with interval data, also used a new pair of interval DEA models that recommended by Wang et al. [8], to measure the efficiencies of decision-making units (DMUs) with interval data. The methodology ranks DMUs by imposing an appropriate minimum weight restriction on input and output weights. All efficient units can be partially or fully put in order by exerting an appropriate minimum weight restriction on input and output weights. We also developed an LP model for DEA efficient units to find their maximin weights that can keep them efficient to the best possible extent. The maximin weights provide very useful information for the DM or assessor to decide that a minimum weight restriction should be imposed on input and output weights. Also in this paper, new efficiency model was developed for reevaluating of the efficiencies of the efficient units with interval data. And one numerical example was tested and inspected using the proposed ranking methodology. It was shown that the proposed ranking methodology as successfully distinguish the best units among all efficient units with interval data.

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