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Ranking Efficient DMUs with Stochastic Data by Considering Inefficient Frontier

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Abstract

Data Envelopment Analysis (DEA) models which evaluate the efficiency of a set of decision making units (DMUs) are unable to discriminate between efficient DMUs. The problem of discriminating between these efficient DMUs is an interesting subject. A large number of methods for fully ranking both efficient and inefficient DMUs have been proposed. Through real world applications, analysis may encounter data that are not deterministic or on have a stochastic essence but whose distribution can be defined by collecting data in successive periods and by statistical methods. In this paper, a method for ranking stochastic efficient DMUs is suggested which is based on the full inefficient frontier method. Using a numerical example, we will demonstrate how to use the result.

Keywords: Data envelopment analysis, Quadratic programming; Ranking, Standard normal distribution.

1 Introduction

Data Envelopment Analysis (DEA) was originated by Charnes et al. [2] and then it extended to an approach for evaluating the relative efficiency of DMUs. In real application, we know that usually plural DMUs are efficient. The problem of discriminating between these efficient DMUs is an interesting subject [5]. Sexton et al. [7] were pioneers in the ranking field. They introduced a ranking method based on cross-efficiency. Then, the ranking of DEA-efficient DMUs based on benchmarking, was an approach initially developed by Torgersen et al. [10]. In this method, a DMU is highly ranked if it is chosen as a reference by many other inefficient DMUs. The most popular research stream in ranking DMUs is called super-efficiency. This stream was first developed by Andersen

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and Petersen [1]. Thrall [8] pointed out that the model developed by Andersen and Petersen may result in instability when some inputs are close to zero. Then, to avoid this problem, MAJ [6] and SBM [9] models were proposed. All of these methods rank DMUs by comparing DMUs with the efficient frontier. One of the disadvantages of DEA is assessing DMUs in the best conditions. Jahanshahloo and Afzalinejad [4] have introduced the full inefficient frontier and they proposed a method for assessing DMUs in the worst conditions and a ranking method by using this factor. All of the proposed ranking models consider different data such as: deterministic, interval, fuzzy, etc.data In different real world applications, analysis may encounter stochastic data. In this paper, on the basis of Cooper's method [3], the stochastic efficient DMUs have been distinguished and, with the contribution of full inefficient stochastic frontier, a model for ranking DMUs with the stochastic data has been presented. The paper is organized as follows: First, the inefficient frontier and then the stochastic DEA models are introduced. After that, ranking DMUs with the stochastic inefficient frontier is discussed. Using a numerical example, we will demonstrate how to use the result.

2 Inefficient frontier

Jahanshahloo and Afzalinejad [4] have defined the full inefficient frontier. By the contribution of the full inefficient frontier, they have also proposed a model for identifying the worst score of efficiency.
DMU_j is full inefficient if it can not be dominated by other dummy DMUs. That is, DMU_j is full inefficient if it belongs to $F(S)$ which is defined as follows:

$$F(S) = \left\{ (x,y)|\ \forall (x',y') \in R^{m+s}((-x',y') \underset{\neq}{\leq} (-x,y) \Rightarrow (x',y') \notin S) \right\} \subseteq S,$$

where S is the convex hull of observed DMUs. Thus the full inefficient frontier in radial input orientation is defined as $F_I(S)$ where

$$F_I(S) = \{(x,y)|(x,y) \in S \quad \& \quad \forall \psi (\psi > 1 \Rightarrow (\psi x, y) \notin S)\}.$$

Thus, DMU_o is located on the full inefficient frontier with variable returns to scale, if in the following model we have $\psi_o^* = 1$, and it is not located on the full inefficient frontier if $\psi_o^* > 1$.

$$\begin{aligned} \psi_o^* = \max \quad & \psi \\ s.t. \quad & \sum_{j=1}^{n} \lambda_j x_{ij} \geq \psi x_{io}, \quad i = 1,...,m, \\ & \sum_{j=1}^{n} \lambda_j y_{rj} \leq y_{ro}, \quad r = 1,...,s, \\ & \sum_{j=1}^{n} \lambda_j = 1, \\ & \lambda_j \geq 0, \quad j = 1,...,n. \end{aligned} \tag{2.1}$$

ψ_o^* indicates the distance of DMU_o from the inefficient frontier. Therefore, the greater ψ_o^* the better the ranking score. In Fig. 1, the convex hull of DMUs is schematically portrayed. The piecewise linear frontier AGFE is the efficient frontier and ABCD is the inefficient frontier.