The Combined Reproducing Kernel Method and Taylor Series for Handling Fractional Differential Equations

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Abstract

This paper presents the numerical solution for a class of fractional differential equations. The fractional derivatives are described in the Caputo [1] sense. We developed a reproducing kernel method (RKM) to solve fractional differential equations in reproducing kernel Hilbert space. This method cannot be used directly to solve these equations, so an equivalent transformation is made by using Taylor series. Some numerical examples are studied to demonstrate the accuracy of the given method.

Keywords : Kernel; Reproducing kernel; Fractional differential equation; Fractional derivative.

1 Introduction

Fractional calculus has various applications in the nonlinear oscillation of earthquakes [2], fluid-dynamic traffic model [3], the transient analysis of viscoelastically damped structures [4, 5], continuum and statistical mechanics [6], colored noise [7], solid mechanics [8], economics [9], bio-engineering [10, 11], anomalous transport [12], and dynamics of interfaces between nanoparticles and substrates [13]. The approximate solutions to the fractional differential equations were given during the last decades, such as method of Collocation [14], a new operational matrix method [15], Laplace transform method [16], Adomian decomposition method [17], variational iteration method [18, 19], homotopy perturbation method [20], and other methods occurred in [21, 22, 23, 24].

Reproducing kernel method is practical to solve singular or nonsingular integral equations in the reproducing kernel Hilbert space which was presented by Zaremba in 1908. For example, singular integral equations with cosecant kernel [25], Fredholm integral equation of the first kind [25], singular Fredholm integro-differential equations with weakly singularity [27], a class of third-order differential equations [28], nonlinear Fredholm-Volterra integro-differential equations [29], integro-differential equations of fractional order [30], singularly perturbed turning point problems [31], Wiener-Hopf equations of the second kind [32], nonlinear Abel’s integral equations with weakly singular kernel [33]. For further see [34, 35, 36, 37].

In this paper, we introduce the use of RKM to solve fractional differential equations of the form:

\[ a_1(x)D^{\alpha_1}u(x) + a_2(x)D^{\alpha_2}u(x) + \cdots + a_q(x)D^{\alpha_q}u(x) + a_{q+1}u(x) = f(x) \]  

(1.1)
with initial conditions
\[ u^{(i)}(0) = d_i, \quad i = 0, 1, \cdots, \gamma, \quad (1.2) \]
where \( a_j(x) \), for \( j = 1, 2, \cdots, q + 1 \) are continuous real valued functions and also \( 0 < \alpha < \alpha_1 < \cdots < \alpha_1, \) \( D^{\alpha_1}, i = 1, 2, \cdots, q. \)

denote the Caputo fractional derivative of order \( \alpha_i \), \( f(x) \in C[0, 1] \) is a given function defined and \( u(x) \) is an unknown function to be determined.

This paper is organized in six sections including the introduction. In the next section, Some preliminaries in fractional calculus are presented. Reproducing kernel space is defined in Section 3. Transformation of Eq. (1.1) is introduced in Section 4. We report our numerical findings and demonstrate the accuracy of the new numerical scheme by considering some examples in Section 5. The last section is a brief conclusion.

2 Some preliminaries in fractional calculus

In this section, we give some basic definitions and properties of fractional calculus theory which are further used in this article.

**Definition 2.1** A real function \( f(t), t > 0, \) is said to be in the space \( C_\mu, \mu \in R, \) if there exists a real number \( p > \mu \) such that \( f(t) = t^p f_1(t), \) where \( f_1(t) \in C[0, +\infty) \) and it is said to be in the space \( C_\mu \) iff \( f^{(r)} \in C_\mu \) for \( r \in N. \)

**Definition 2.2** The Riemann-Liouville fractional integral operator of order \( \alpha \geq 0 \) of a function \( f \in C_\mu, \mu \geq -1 \) is defined as:
\[
\begin{align*}
\alpha > 0, t > 0, \\
\alpha I_0^\alpha f(t) &= \frac{1}{\Gamma(\alpha)} \int_0^t (t - \tau)^{\alpha - 1} f(t) d\tau, \\
\alpha I_0^0 f(t) &= f(t),
\end{align*}
\]
(2.3)

**Definition 2.3** The fractional derivative of \( f(t) \) in the Caputo sense is defined as follows:
\[
\begin{align*}
\frac{D^\alpha}{\partial t^\alpha} f(t) &= \frac{1}{\Gamma(\gamma - \alpha)} \frac{d}{d\tau} \int_0^\tau (\tau - \sigma)^{\gamma - \alpha - 1} f(\sigma) d\sigma, \\
\gamma - 1 < \alpha < \gamma, \gamma \in N, f \in C_\mu^{\gamma - 1},
\end{align*}
\]
(2.4)

where \( \alpha > 0 \) is the order of the derivative and \( \gamma \) is the smallest integer greater than \( \alpha. \) For the Caputo derivative, we have
\[
D^\alpha C = 0 \quad (C \text{ is a constant}) \quad (2.5)
\]
\[
D^\alpha x^\beta = \begin{cases} 
0, & \text{for } \beta \in N \cup \{0\} \text{ and } \beta < [\alpha], \\
\frac{\Gamma(\beta + 1)}{\Gamma(\beta + 1 - \alpha)} x^{\beta - \alpha}, & \text{for } \beta \in N \cup \{0\} \text{ and } \beta \geq [\alpha] \text{ or } \beta \notin N \text{ and } \beta > [\alpha].
\end{cases} \quad (2.6)
\]

We use the ceiling function \([\alpha]\) to denote the smallest integer greater than or equal to \( \alpha, \) and the floor function \([\alpha]\) to denote the largest integer less than or equal to \( \alpha. \) Recall that for \( \alpha \in N, \) the Caputo differential operator coincides with the usual differential operator of an integer order.

In [1], the following properties for \( f \in C_\mu, \mu \geq -1 \) have been proved
\[
\begin{align*}
1) \quad &\alpha I_0^\alpha t^k = \frac{\Gamma(k + 1)}{\Gamma(k + 1 + \alpha)} t^{\alpha + k}, k \in N \cup 0, t > 0, \\
2) \quad &\frac{\partial}{\partial t} (D^\alpha f(t)) = f(t), \quad \alpha > 0, f(t) \in C[0, 1], \\
3) \quad &\frac{\partial}{\partial t} D^\beta f(t) = \alpha I_0^{\alpha - \beta} D^\alpha f(t), \quad \alpha, \beta > 0, \\
4) \quad &\alpha I_0^\alpha D^\beta f(t) = f(t) - \sum_{j=0}^{\gamma - 1} \frac{\Gamma(\gamma)}{\Gamma(\gamma + j)} I_0^{\gamma - j} (D^\beta f(0)), \\
& \quad f(t) \in C[0, 1], \gamma - 1 < \alpha \leq \gamma, \\
& \quad (2.7)
\end{align*}
\]

if \( 0 < \alpha \leq 1 \) and \( f(t) \in C^1[0, 1] \) then
\[
\alpha I_0^\alpha D^\beta f(t) = f(t) - f(0).
\]

3 Reproducing kernel space

First, we construct the closet subspace \( {}^aW^m[0, 1] \) of the reproducing kernel space \( W^m[0, 1] \) by imposing condition \( u(0) = 0, u'(0) = 0, \cdots, u^{(\gamma)}(0) = 0. \)

**Definition 3.1** \( {}^aW^m[0, 1] = \{ u(x) | u^{(m-1)}(x) \text{ is an absolutely continuous real value function,} \}

\[
\begin{align*}
& u^{(m)}(x) \in L^2[0, 1], \\
& u(0) = 0, u'(0) = 0, \cdots, u^{(\gamma)}(0) = 0.
\end{align*}
\]

The inner product and norm in \( {}^aW^m[0, 1] \) are given respectively by \( \langle u, v \rangle = \langle u, v \rangle, \langle u, v \rangle = \sum_{i=0}^{n} \int_0^1 u(t) v(t) dt. \)
\[ \sum_{i=0}^{m-1} u^{(i)}(0)v^{(i)}(0) + \int_0^1 u^{(m)}(x)v^{(m)}(x) \, dx \]
and
\[ \|u\|_m = \sqrt{\langle u, u \rangle_m}, \quad u, v \in \mathcal{O}W^m[0,1]. \]

According to [37], the space \( \mathcal{O}W^m[0,1] \) is a reproducing kernel Hilbert space. There exists \( R_y(x) \in \mathcal{O}W^m[0,1] \), for any \( u(y) \in \mathcal{O}W^m[0,1] \) and each fixed \( x \in [0,1], y \in [0,1] \), such that \( \langle u(y), R_x(y) \rangle = u(x) \). The reproducing kernel \( R_y(x) \) can be denoted by
\[
R_y(x) = \begin{cases} 
R_1(x,y) = \sum_{i=1}^{2m} c_i (y)x^{i-1}, y \leq x, \\
R_2(x,y) = \sum_{i=1}^{2m} d_i (y)x^{i-1}, y > x, 
\end{cases}
\]
where coefficients \( c_i(y), d_i(y), (i = 1, 2, \ldots, 2m) \), could be obtained by solving the following equations
\[
\frac{\partial R_y(x)}{\partial x^i} \bigg|_{x=y+0} = \frac{\partial R_x(y)}{\partial x^i} \bigg|_{x=y-0}, \quad i = 0, 1, 2, \ldots, 2m-2, \tag{3.8}
\]
\[
(-1)^m \left( \frac{\partial^{2m-1} R_y(x)}{\partial x^{2m-1}} \right) \bigg|_{x=y+0} = 1,
\]
\[
\frac{\partial^i R_y(0)}{\partial x^i} - (-1)^{m-i-1} \frac{\partial^{2m-i-1} R_y(0)}{\partial x^{2m-i-1}} = 0, \quad i = \gamma + 1, \gamma + 2, \ldots, m-1,
\]
\[
\frac{\partial^{2m-i-1} R_y(0)}{\partial x^{2m-i-1}} = 0, \quad i = 0, 1, \ldots, m-1,
\]
\[
R_y^{(i)}(0) = 0, \quad i = 0, 1, \ldots, \gamma. \tag{3.10}
\]

4 The analytical solution

4.1 Transformation of Eq. (1.1)

In this section, we convert Eq. (1.1) into an equivalent equation, which is easily solved by using RKM.

With the Taylor series expansion of \( u^{(\gamma)}(t) \) based on expanding about the given point \( x \) belonging to the interval \([0,1]\), we have the Taylor series approximation of \( u^{(\gamma)}(t) \) in the following form
\[
u^{(\gamma)}(t) = u^{(\gamma)}(x) + (t-x)u^{(\gamma+1)}(x) + \frac{(t-x)^2}{2!}u^{(\gamma+2)}(x) + \cdots + \frac{(t-x)^n}{n!}u^{(\gamma+n)}(x) + \frac{(t-x)^{n+1}}{(n+1)!}u^{(\gamma+n+1)}(\zeta_x,t),
\]
where \( \zeta_x,t \) is between \( x \) and \( t \). We use the truncated Taylor series and substitute in fractional derivative of \( u(x) \),
\[
C_0^D_x u(x) = \frac{1}{\Gamma(\gamma-\alpha)} \int_0^x (x-t)^{\gamma-\alpha-1} \frac{\partial^\gamma}{\partial x^\gamma} u(t)dt, \tag{4.11}
\]
Then, we have
\[
C_0^D_x u(x) = \frac{1}{\Gamma(\gamma-\alpha)} \int_0^x (x-t)^{\gamma-\alpha-1} \frac{\partial^\gamma}{\partial x^\gamma} u(t)dt,
\]
\[
\sum_{K=0}^n \frac{(-1)^K u^{(\gamma+K)}(x)}{K!} \int_0^x (x-t)^{\gamma+K-\alpha-1}dt,
\]
where \( u^{(0)}(x) = u(x) \), and for the two cases \( K = 0 \) and \( K > 0 \), \( \int_0^x (x-t)^{\gamma+K-\alpha-1}dt \) is computable.

\[\text{Figure 1: The comparisons between numerical and exact solution for } m = 8 \text{ and } \gamma = 1 \text{ for Example 5.1.}\]

4.2 Solution of Eq. (1.1)

In this section, we shall give the exact and approximate solution of Eq. (1.1) in a reproducing kernel space \( \mathcal{O}W^m[0,1] \). We assume that Eq. (1.1) has a unique solution. To deal with the system, we consider Eq. (1.1) as
\[
L(u) = a_1(x)D^\alpha_1 u(x) + a_2(x)D^\alpha_2 u(x) + \cdots + a_q(x)D^\alpha_q u(x) + a_{q+1}u(x), \tag{4.13}
\]
Table 1: Numerical results of Example 5.1.

| Node | $|u_N(x) - u(x)|_{W^6[0,1]}$ | $|u_N(x) - u(x)|_{W^7[0,1]}$ | $|u_N(x) - u(x)|_{W^8[0,1]}$ |
|------|-----------------------------|-----------------------------|-----------------------------|
| 0    | 0                           | 0                           | 0                           |
| 0.1  | 2.39620E-10                 | 3.40281E-12                 | 2.31398E-13                 |
| 0.2  | 6.10843E-9                  | 5.88242E-11                 | 1.35855E-12                 |
| 0.3  | 3.14653E-8                  | 2.11194E-10                 | 3.47779E-12                 |
| 0.4  | 7.64493E-8                  | 3.03003E-10                 | 4.19513E-12                 |
| 0.5  | 1.14036E-7                  | 7.10780E-11                 | 2.38995E-12                 |
| 0.6  | 8.82634E-8                  | 1.51490E-9                  | 2.31165E-11                 |
| 0.7  | 8.20796E-8                  | 4.75600E-9                  | 6.68909E-11                 |
| 0.8  | 4.97484E-7                  | 1.05654E-8                  | 1.40545E-10                 |
| 0.9  | 1.27101E-6                  | 1.96875E-8                  | 2.52866E-10                 |
| 1    | 2.52153E-6                  | 3.27755E-8                  | 4.08365E-10                 |

Table 2: Numerical results of Ex. 5.2.

| Node | $|u_N(x) - u(x)|_{\alpha = 0.5}$ | $|u_N(x) - u(x)|_{\alpha = 0.75}$ | $|u_N(x) - u(x)|_{\alpha = 0.95}$ |
|------|---------------------------------|---------------------------------|---------------------------------|
| 0    | 0                               | 0                               | 0                               |
| 0.1  | 2.52709E-12                    | 1.19622E-12                    | 1.86295E-13                    |
| 0.2  | 1.05300E-11                    | 1.39666E-13                    | 4.22162E-14                    |
| 0.3  | 9.06302E-11                    | 1.29017E-11                    | 1.89376E-13                    |
| 0.4  | 3.68997E-10                    | 1.24541E-11                    | 7.11486E-13                    |
| 0.5  | 7.35732E-10                    | 3.60521E-11                    | 7.77822E-13                    |
| 0.6  | 9.80372E-10                    | 1.35855E-10                    | 1.47532E-12                    |
| 0.7  | 8.42491E-10                    | 2.48874E-10                    | 2.80775E-13                    |
| 0.8  | 6.77430E-11                    | 3.13591E-10                    | 4.75972E-12                    |
| 0.9  | 1.54901E-9                     | 2.64031E-10                    | 9.08093E-12                    |
| 1    | 4.13576E-9                     | 4.48061E-11                    | 6.62046E-12                    |

Figure 2: The absolute errors in spaces $W^6[0,1]$ for Example 5.1.

Figure 3: The absolute errors in spaces $W^7[0,1]$ for Example 5.1.

Then equation (4.13) can be written as $\mathbb{L}(u) = f(x)$. It is clear that $\mathbb{L} : W^m[0,1] \rightarrow W^1[0,1]$ is a bounded linear operator and $\mathbb{L}^*$ is the adjoint operator of $\mathbb{L}$. We shall give the representation of analytical solution of Eq. (1.1) in the space $W^1[0,1]$.

Now, we choose a countable dense subset $\{x_i\}_{i=1}^\infty$ in $[0,1]$, and define

$$\varphi_i(x) = R_y(x_i),$$
$$\psi_i(x) = [\mathbb{L}_yR_y(x)](x_i) = \mathbb{L}^*R_y(x_i), \quad i = 1, 2, \ldots,$$

(4.14)

where the subscript $y$ in the operator $\mathbb{L}$ indicates that the operator $\mathbb{L}$ applies to the function of $y$. 


The orthonormal system \( \{ \tilde{\psi}_i(x) \}_{i=1}^{\infty} \) of \( \mathcal{O}W^m[0,1] \) is constructed from \( \{ \psi_i(x) \}_{i=1}^{\infty} \) by using the Gram-Schmidt algorithm, and then the approximate solution will be obtained by calculating a truncated series based on these functions, such that

\[
\tilde{\psi}_i(x) = \sum_{k=1}^{i} \beta_{ik} \psi_k(x), \quad (\beta_{ii} > 0, \ i = 1, 2, \ldots), \tag{4.15}
\]

where \( \beta_{ik} \) are orthogonal coefficients. In order to obtain \( \beta_{ik} \), take

\[
\psi_i(x) = \sum_{k=1}^{i} B_{ik} \tilde{\psi}_k(x). \tag{4.16}
\]

\[
\langle \psi_i(x), \psi_i(x) \rangle = \sum_{k=1}^{i-1} B_{ik}^2 + B_{ii}^2, \tag{4.17}
\]

\[
B_{ii}^2 = \sqrt{\langle \psi_i(x), \psi_i(x) \rangle - \sum_{k=1}^{i-1} B_{ik}^2}. \tag{4.18}
\]

\[
\beta_{ii} = \frac{1}{\sqrt{\langle \psi_i(x), \psi_i(x) \rangle}}, \tag{4.19}
\]

\[
\beta_{ij} = \beta_{ii} \left( - \sum_{k=j}^{i-1} B_{ik} \beta_{kj} \right). \tag{4.20}
\]

**Theorem 4.1** If \( \{ x_i \}_{i=1}^{\infty} \) is dense in \([0,1]\), then \( \{ \psi_i(x) \}_{i=1}^{\infty} \) is a complete system in \( \mathcal{O}W^m[0,1] \).

**Theorem 4.2** If \( \{ x_i \}_{i=1}^{\infty} \) is dense on \([0,1]\) and...
Proof. \( u(x) \) can be expanded to Fourier series in terms of the normal orthogonal basis \( \tilde{v}_i(x) \) in \( ^0W^m[0, 1] \),

\[
    u(x) = \sum_{i=1}^{\infty} \langle u(x), \tilde{v}_i(x) \rangle \tilde{v}_i(x)
\]

\[
    = \sum_{i=1}^{\infty} \sum_{k=1}^{i} \beta_{ik} \langle u(x), \psi_k(x) \rangle \tilde{v}_i(x)
    = \sum_{i=1}^{\infty} \sum_{k=1}^{i} \beta_{ik} \langle u(x), L^s \varphi_k(x) \rangle \tilde{v}_i(x)
    = \sum_{i=1}^{\infty} \sum_{k=1}^{i} \beta_{ik} \langle f(x), \varphi_k(x) \rangle \tilde{v}_i(x)
    = \sum_{i=1}^{\infty} \sum_{k=1}^{i} \beta_{ik} f(x_k) \tilde{v}_i(x)
\]

The proof is complete. \( \Box \)

By truncating the series of the left-hand side of (4.21), we obtain the approximate solution of Eq. (1.1)

\[
    u_N(x) = \sum_{i=1}^{N} \sum_{k=1}^{i} \beta_{ik} f(x_k) \tilde{v}_i(x).
\]  

(4.22)

\( u_N(x) \) in (4.22) is the N-term intercept of \( u(x) \) in (4.21), so \( u_N(x) \rightarrow u(x) \) in \( W^m[0, 1] \) as \( N \rightarrow \infty \).

Lemma 4.1 If \( u(x) \in ^0W^m[0, 1] \), then there exists a constant \( c \) such that \( |u(x)| \leq c \| u \|_m \).
It will be proved that \( u(x) \) is the solution of Eq. (1.1).

From (4.23), it follows

\[
(Lu)(x_j) = \sum_{i=1}^{\infty} B_i \langle L \tilde{\psi}_i(x), \varphi_j(x) \rangle
= \sum_{i=1}^{\infty} B_i \langle \tilde{\psi}_i(x), \mathbb{L}^* \varphi_j(x) \rangle
= \sum_{i=1}^{\infty} B_i \langle \tilde{\psi}_i(x), \psi_j(x) \rangle.
\]

Therefore,

\[
\sum_{j=1}^{N} \beta_N \langle L \varphi \rangle(x_j) = \sum_{i=1}^{\infty} B_i \langle \tilde{\psi}_i(x), \sum_{j=1}^{N} \beta_N \psi_j(x) \rangle_{W^m}
= \sum_{i=1}^{\infty} B_i \langle \tilde{\psi}_i(x), \tilde{\psi}_N(x) \rangle_{W^m}
= B_N.
\]

If \( n = 1 \), then \((Lu)(x_1) = f(x_1)\).

If \( n = 2 \) then \( \beta_21(Lu)(x_1) + \beta_22(Lu)(x_2) = \beta_21f(x_1) + \beta_22f(x_2) \).

It is clear that \((Lu)(x_2) = f(x_2)\).

Moreover, it is easy to see by induction that

\[
(Lu)(x_j) = f(x_j), j = 1, 2, \ldots. \quad (4.28)
\]

Since \( \{x_i\}_{i=1}^{\infty} \) is dense on \([0, 1]\), for any \( x \in [0, 1] \), we have

\[
(Lu)(x) = f(x). \quad (4.29)
\]

That is, \( u(x) \) is the solution of Equation (1.1) and

\[
u(x) = \sum_{i=1}^{\infty} B_i \tilde{\psi}_i(x). \quad (4.30)
\]

The proof is complete. \( \square \)

5 Numerical experiments

To test the accuracy of the present method, some numerical examples are presented in this section. All examples are done by taking parameter \( N = 20 \) where \( N \) is the number of terms of the Fourier series of the unknown function \( u(x) \). Parameter \( n \) is the number of terms of the Taylor series and we choose \( m > n \) to solve these examples. The examples are computed using Mathematica 8.0. Results obtained by the method are compared with the exact solution of each example and found to be in a good agreement.

Example 5.1 As the first example, we consider the following initial value problem in the case of the inhomogeneous Lane-Emden equation [14]:

\[
\begin{cases}
D^{\alpha_1}u(x) + \frac{c}{x^{\alpha_2}}D^{\alpha_2}u(x) + \frac{1}{x^{\alpha_1-1}}u(x) = f(x), \\
u(0) = 0, \quad u'(0) = 0.
\end{cases}
\]

The exact solution of this problem is \( u(x) = x^3 - x^2 \). In this problem \( \alpha_1 = \frac{3}{2}, \alpha_2 = \frac{1}{2} \).

We find the approximate solution by the proposed method for \( n = 2 \). The absolute errors obtained by reproducing Kernel in space \( W^8[0, 1], W^7[0, 1], W^5[0, 1] \) are shown in Table 1. This is an indication of stability on the reproducing Kernel. However, by increasing \( m \), the behavior improves.

The comparisons between the numerical solutions and the exact solution for \( m = 8 \) and \( \gamma = 2 \) are given in Figure 1. We can clearly see that the numerical solutions and exact solution are coincided completely. Figures 2 - 4 reveals the absolute errors in spaces \( W^6[0, 1], W^7[0, 1], W^8[0, 1] \), respectively.

Comparing between the numerical solutions presented in [14] with these numerical solutions, we find that the reproducing Kernel method can reach a higher degree of accuracy when solving the same equation.

Example 5.2 Consider the following semi differential equation [16, 21]:

\[
\begin{cases}
D^{\alpha}u(x) + u(x) = \frac{2x^{2-\alpha}}{\Gamma(3-\alpha)} - \frac{x^{1-\alpha}}{\Gamma(2-\alpha)} + x^2 - x, \\
u(0) = 0, \quad 0 < \alpha \leq 1.
\end{cases}
\]

The exact solution is \( u(x) = x^2 - x \).

We find the approximate solution by the proposed method for \( n = 2 \) and \( \gamma = 1 \). The absolute errors obtained by reproducing Kernel in space \( W^6[0, 1] \) and \( \alpha = 0.5, \alpha = 0.75, \alpha = 0.95 \) are shown in Table 2. Figure 6 reveals the absolute errors in space.
and $W^6[0,1]$ and $\alpha = .5, \alpha = .75, \alpha = .95$, respectively. Note that as $\alpha$ approaches 1, the numerical solution converges to the exact solution. i.e. in the limit, the solution of the fractional differential equations approaches to that of the integer-order differential equations. The comparisons between the numerical solutions and the exact solution for $m = 6, \gamma = 1$ and $\alpha = 0.95$ are given in Figures 6 - 8, respectively. We can clearly see that the numerical solutions and exact solution are coincide completely.

6 Concluding

In this study, we developed an efficient and computationally attractive method to solve the fractional differential equations. Using the definition of Caputo-type fractional derivative, Taylor series and the properties of proposed method; we transform the initial problem into an equivalent equation. The error analysis of reproducing kernel method is introduced. The comparisons between the numerical solutions and the exact solution show the present method is accurate.

References


357


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