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Research Article

Matrix Approach to Robustness Analysis for Strategy Selection

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Abstract

This study aims to discuss the use of robustness analysis in evaluation and selection the strategies of an organization based on a matrix approach. The proposed technique can overcome the weakness of the robustness analysis model related to reviewing a few future scenarios and also, makes it possible to in a short time include the ideas of the decision makers who participate in the strategic planning process. In order to more precisely compare the strategies, especially when their robustness levels are so close and consequently, it is hard to choose the best one, a version of the well–known Dolan–Moré performance profile is employed. To support the theoretical discussions, the proposed approach is applied on a real world problem in the automotive industry of Iran and the results are explained as well.

Keywords : Soft operational research; Strategic programming; Robustness analysis; Matrix data; Performance profile.

1 Introduction

In practice, decision making turns out to be a complex process due to the large number of alternatives, multiple and sometimes conflicting goals, increasing environmental turbulence as well as unavailability of the quantitative data on the future results [19]. This process could be so challenging, especially when it is related to the strategic areas, due to high levels of uncertainty about the future, considering multiple strategic options, cross–linking strategic options, long–term results arising from implementation of the strategies, and needing to use the stakeholders' opinions in discussions related to the strategic decisions [13]. In the past decades, researches focused on improving the classical strategy analysis and selection tools such as SWOT (strengths, weaknesses, opportunities and threats) and QSPM (quantitative strategic planning matrix) to provide reasonable answers for the mentioned challenges. However, their achievements generally have two fundamental failures: (1) inability to formulate probable futures and estimate how and to what extent they can affect the organizational performances

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in the terms of tools, and (2) indetermination of the importance and rank of the criteria identified in the strategic planning process. Responding to the mentioned criticisms led the strategic planning experts to employ the operational research (OR) methods in order to conduct a proper analysis to manage the complex situation surrounding the strategic structures [8]. In this context, researchers applied the multiple–criteria decision making (MCDM) techniques and also, the problem structuring methods (PSM) to manage the mentioned complexity [7, 12, 18, 19]. The main advantage of these tools is their ability of using both of the quantitative and qualitative criteria. However, these methods fail to formulate the probable futures since they use the present information and judgments to collect the data and make predictions in the cases where the future information is required. Now, if we reasonably assume that the future is unpredictable, forecast–based planning won't be reliable anymore [10] and since most of the strategic issues are characterized by the uncertainty quality, it will be tricky to even use the probabilistic estimates [17].

As known, PSMs have been developed since the late 70's in response to the limitations and obstacles ahead of the researchers in using the hard OR [1]. Although being helpful for creating an appropriate strategic plan that requires managing large and complicated qualitative data [8], PSMs can only dominate the complexity of the problem and fail to formulate probable futures through the model. Among all the PSMs, only the strategic choice approach (SCA) and the robustness analysis (RA) are associated with the uncertainty. It is worth noting that RA is simpler and easier than the SCA to understand for the managers and participants [14]. Therefore, it seems that RA is a more suitable tool for the strategy evaluation as well as the strategy selection in contrast to the other methods.

RA is an approach to assess the primary sequential strategic decisions in the sense that robustness and debility of a choice in the future alternatives is examined to make a decision that could provide acceptable and satisfactory results among more futures and simultaneously follows by fewer adverse outcomes [2]. According to the study of Montibeller and Franco [13], the important point in this type of analysis is that there is no consensus on how to calculate the robustness scores [2, 5, 6, 11, 14, 20, 21]. However, the Rosenhead's metric is one of the most widely used methods in RA [16].

Despite the acceptable ability to formulate the probable futures and manage the complexities, RA has some weaknesses. Among them, inability to analyze a large number of scenarios as well as inability to determine more desirable strategies in the specific case of closeness of the robustness and debility levels are of the most important points. Regarding to the first weakness of RA, Ram et al. [15] believed that most of the scenarios used to evaluate strategic options can be introduced as optimistic, pessimistic and most–likely scenarios. Mansson [11] argued that only four scenarios can be essentially used because of the human capacity limitations. In the meantime, we should not forget that soft OR was originally proposed in response to the hard OR weaknesses such as inability to deal with the problem complexities. So, instead of lessening the scenarios, it is better to formulate the problem in a way to cover the environmental complexities.

Here, employing a matrix–based scheme that allows us to define and analyze an arbitrary number of scenarios, we plan to improve effectiveness of the RA. Our approach is explained in details in the second section of this study. The third section describes how to use the Dolan–Moré [4] performance profile as a tool of the RA. In the fourth section, the proposed technique is implemented on a real world problem and the results are reported. The final section is devoted to the conclusions.

2 A matrix–based model for the strategy selection problem

Here, we describe a matrix–based model to be used in the RA of the strategic programming. To start the formulation process, firstly we need to identify the strategies. In this context, the main strategies MS_i, $i = 1, \cdots, r$, are specified as a result of choosing a certain number of predefined sub–strategies S_j, $j = 1, \cdots, m$. More exactly, the dependence relationships are illustrated as follows:

$$MS_i = S_{j_1} \oplus S_{j_2} \oplus \cdots \oplus S_{j_i}, \ i = 1, \cdots, r, \quad (2.1)$$

showing that the main strategy MS_i is a hybridization of the sub–strategies $S_{j_1}, S_{j_2}, \cdots, S_{j_i}$. The second step is to define the future scenarios. As known, common external factors in the strategic literature include the political, economic, social, technological, environmental and legal factors (PESTEL) [9]. Here, the scenarios are defined based on probable situations of the mentioned six factors as ordered in the 6–tuples $Sn_i = (P_i, Ec_i, So_i, T_i, En_i, L_i)$, $i = 1, 2, \cdots, q$. General form of a scenario components is presented in Table 1. Furthermore, the scenarios matrix M which is of the order $6 \times q$ can be defined by setting the 6–tuples Sn_i as its ith column. More precisely,

$$M = [Sn_1, Sn_2, \cdots, Sn_q]. \quad (2.2)$$

Now, we need to determine strategies favorability and non–favorability conditions. In this context, for the strategy S_j two ordered 6–tuples S_j^+ and S_j^- are defined to respectively refer to its favorability and non–favorability conditions. The elements of the two vectors contain some of the states of the indicators displayed in Table 1, determined according to the employer's considerations. Afterwards, the strategic condition matrix A of the order $6 \times 2m$ can be defined as follows:

$$A = [S_1^+, S_1^-, S_2^+, S_2^-, \cdots, S_m^+, S_m^-]. \quad (2.3)$$

Here, based on the available data, we are in a position to define the robustness–debility matrix which is denoted by B, consisting of m rows and q columns where each row corresponds to a sub–strategy S_j, $j = 1, \cdots, m$, and each column corresponds to a scenario Sn_i, $i = 1, \cdots, q$. In order to specify the element (j, i) of B, the ordered 6–tuple Sn_i should be compared with the ordered 6–tuples S_j^+ and S_j^-. For each compliance of Sn_i and S_j^+ a positive score is assigned while for each compliance of Sn_i and S_j^- a negative score is considered; the element B_{ji} of the matrix B is the sum of the mentioned scores. In particular, according to the classical Rosenhead's approach [17], two m–tuple vectors R and F are defined to respectively contain robustness and debility of the sub–strategies. The jth component of R (i.e. R_j) represents the ratio of the number of positive elements of the jth row of B to q (total number of the elements of the jth row) and the jth component of F (i.e. F_j) refers to the ratio of the number of negative elements of the jth row of B to q.

Finally, for each main strategy MS_i, the robustness level is defined as the sum of the elements of R corresponding to the related sub–strategies and also, the debility level is given by summing the elements of F corresponding to the related sub–strategies. The best main strategy is determined by comparing the obtained robustness and debility levels. In the case where the robustness and debility levels are close to each other which makes it hard to ascertain the right strategy, here the performance profile introduced by Dolan and Moré [4] is appropriately employed.

3 Developing the Dolan–Moré performance profile to determine the preferable strategy

As known, Dolan–Moré performance profile [4] is widely and increasingly used to compare performance of the numerical algorithms with respect to the CPU time, the number of iterations, the number of function evaluations, and so on. Benchmark results are usually generated by running a solver on a set P of the problems and recording information of the interest. Here, we deal with the notion of performance profile as a means to evaluate and compare the performance of the set of solvers S on a test set P. Then, we develop it as a tool to compare robustness of the strategies.

We assume that we have n_s solvers and n_p problems. We are interested in using the CPU time as a performance measure; although the ideas below can be used with other measures. For each problem p and solver s, we define

$t_{p,s}$ =computing time required to solve problem p by solver s.

We require a baseline for comparisons. We compare the performance on problem p by solver s with the best performance by any solver on this

Table 1: General form of the scenario components

Factors	Indicators	Situations
Political	P_i	$i = 1, 2, \cdots, p$
Economic	Ec_i	$i = 1, 2, \cdots, c$
Social	So_i	$i = 1, 2, \cdots, s$
Technological	T_i	$i = 1, 2, \cdots, t$
Environmental	En_i	$i = 1, 2, \cdots, n$
Legal	L_i	$i = 1, 2, \cdots, l$

Table 2: Factors affecting the problem and their different states

Factors	Indicators	Situation
Political	Joint Comprehensive Plan of Action (JCPOA)	Continuation of JCPOA (P_1) Rejection of JCPOA (P_2)
Economic	Economic growth	Positive (Ec_1) Negative (Ec_2)
Social	The potential of market size	Improvement (So_1) Stability (So_2) Decline (So_3)
Technological	Technology development	Maintaining technology over the period under review (T_1) Changing the technology to the benefit of the organization (T_2) Changing the technology to the detriment of the organization (T_3)
Environmental	Community's sensitivity to the environmental degradation	Disregarding (En_1) Highly regarding (En_2)
Legal	Supporting the domestic monopoly	Continue to support (L_1) Ending the support (L_2)

Table 3: Robustness and debility levels of the main strategies

Strategy	Robustness	Debility
Aggressive	$\frac{36}{90}$	$\frac{28}{90}$
Competitive	$\frac{25}{60}$	$\frac{14}{60}$
Defensive	$\frac{24}{60}$	$\frac{14}{60}$
Conservative	$\frac{31}{75}$	$\frac{25}{75}$

problem; that is, we use the performance ratio

$$r_{p,s} = \frac{t_{p,s}}{\min\{t_{p,s} : s \in S\}}.$$

The performance of solver s on any given problem may be of interest, but we would like to obtain an overall assessment of the performance of the solver. If we define

$$\rho_s(\omega) = \frac{1}{n_p}\text{size}\{p \in P : r_{p,s} \leq \omega\},$$

then $\rho_s(\omega)$ is the probability for solver $s \in S$ that a performance ratio $r_{p,s}$ is within a factor $\omega \in \mathbb{R}$ of the best possible ratio. The function ρ_s

is the (cumulative) distribution function for the performance ratio. We use the term 'performance profile' for the distribution function of a performance metric. It has been shown that a plot of the performance profile reveals all of the major performance characteristics [4].

To employ the Dolan–Moré performance profile, since in our model a strategy with the maximum robustness is desirable while in the performance profile the minimum value of the outputs is preferable, at first we need to make a decreasing transformation on the matrix B. Here, we use an exponential function and introduce the matrix Q as follows:

$$Q_{ij} = a^{-B_{ij}}, \; i = 1, 2, \cdots, m, \; j = 1, 2, \cdots, q, \tag{3.4}$$

where $a > 1$ is a real constant. Next, we need to define a matrix D, here called the resultant matrix, with r rows and q columns in which each row corresponds to a main strategy MS_i, $i = 1, \cdots, r$, and each column corresponds to a scenario Sn_j, $j = 1, \cdots, q$. The ith row of D is the sum of those rows of the matrix Q corresponding to the sub–strategies that comprise the main strategy MS_i in the sense of (2.1). Now, we can compare the rows of D using the Dolan–Moré performance profile to find the most desirable main strategy.

4 A real–world problem study

Our real–world study is devoted to the automotive industry of Iran. It is worth noting that after the oil industry, the automotive industry is the second most active industry of the country which has faced with a variety of challenges in recent years. Hence, the industry needs to review its strategies to ensure its survival in the future. We should note that decision makers participating in this case study were selected among the strategic planning experts of the automotive industry. Also, we used the MATLAB software to perform computations of our model.

At the first stage, the grand strategy matrix proposed by David [3] was applied in order to define the strategies and their sequences. Hence, the main strategies were classified into the four groups of aggressive (MS_1), competitive (MS_2), defensive (MS_3) and conservative (MS_4). Each of the main strategies consists of a number of sub–strategies demonstrated based on the equation 2.1 as follows:

$$\begin{aligned} MS_1 &= S_1 \oplus S_2 \oplus S_3 \oplus S_8 \oplus S_9 \\ &\quad \oplus S_{10}, \end{aligned} \tag{4.5}$$

$$MS_2 = S_3 \oplus S_4 \oplus S_5 \oplus S_6, \tag{4.6}$$

$$MS_3 = S_3 \oplus S_5 \oplus S_7 \oplus S_8, \tag{4.7}$$

$$MS_4 = S_2 \oplus S_8 \oplus S_9 \oplus S_{10} \oplus S_{11}, \tag{4.8}$$

in which the sub–strategies are classified as the vertical integration (S_1), the horizontal integration (S_2), the concentric diversification (S_3), the horizontal diversification (S_4), the conglomerate diversification (S_5), the joint venture (S_6), the retrenchment (S_7), the divestiture (S_8), the market development (S_9), the market penetration (S_{10}) and the product development (S_{11}). Note that among the sub-strategies of [3] the liquidation strategy was eliminated because the industry does not intend to end its activities.

In order to design the future scenarios, experts identified the most important indicator relevant to the PESTEL, taking into account the Iran's especial circumstances, and then specified their various states as shown in Table 2. Among all the possible scenarios, 15 cases were considered as columns of the matrix M illustrated in Appendix A. In the next step, the strategic condition matrix A indicating favorability or non–favorability conditions of each strategy was determined by the experts (see Appendix A). Then, the matrices M and A are compared to obtain the matrix B, as shown in Appendix A, which contains robustness and debility scores of each strategy in different scenarios. Now, taking into consideration the data presented in the matrix B, the Rosenhead's classical scheme [16] for calculating robustness and debility of the strategies lead to the vectors R and F as follows:

$$\begin{aligned} R &= [\tfrac{6}{15}\; \tfrac{6}{15}\; \tfrac{4}{15}\; \tfrac{7}{15}\; \tfrac{6}{15}\; \tfrac{8}{15}\; \tfrac{7}{15}\; \tfrac{7}{15}\; \tfrac{9}{15} \\ &\quad \tfrac{4}{15}\; \tfrac{5}{15}\,]^T, \\ F &= [\tfrac{8}{15}\; \tfrac{6}{15}\; \tfrac{1}{15}\; \tfrac{4}{15}\; \tfrac{5}{15}\; \tfrac{4}{15}\; \tfrac{4}{15}\; \tfrac{4}{15}\; \tfrac{6}{15} \\ &\quad \tfrac{3}{15}\; \tfrac{6}{15}\,]^T. \end{aligned}$$

Finally, according to the relations (4.5)–(4.8), the strategies robustness and debility levels are depicted in Table 3 and Figure 1.

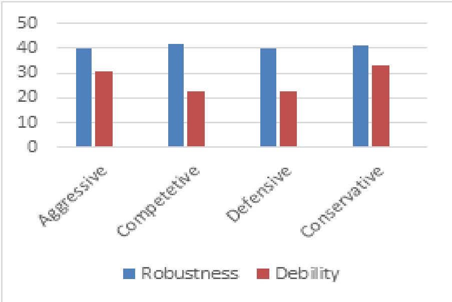

Figure 1: Comparing robustness and debility of the main strategies

The results of comparisons show that the conservative and aggressive strategies are not favorable enough. Besides, among the defensive and competitive strategies the best one cannot be easily determined since their robustness and debility levels are close to each other. As discussed in Section 3, in such circumstances the Dolan–Moré performance profile [4] may be helpful. So, here we need to compute the resultant matrix D based on the rows of the matrix B. As described in Section 3, using the equation (3.4) with $a = 1.2$, we compute the matrix D as given in Appendix A, being necessary to draw the performance profile figure.

Figure 2 shows the results of comparisons. As seen, the competitive strategy turns out to be more desirable in contrast to the defensive strategy in the sense of the Dolan–Moré performance profile. Now, according to the vectors R and F, it can be stated that among the sub–strategies corresponding to the competitive strategy the joint venture is more robust while the concentric diversification possesses the less debility.

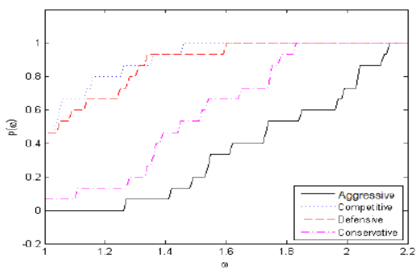

Figure 2: Dolan–Moré performance profile for the main strategies

5 Conclusions

This study unveiled that an appropriate matrix model can allow us to include intricacies and complexities of a strategic planning problem and to review arbitrary number of future scenarios quickly as well as accurately. Furthermore, based on the well–known Dolan–Moré performance profile, an approach has been developed to analyze the strategies robustness which can be quite successful in selecting the right strategy, especially when the classical Rosenhead measure fails. Finally, a real world problem in the automotive industry of Iran has been studied using the proposed techniques and the results have been reported.

According to the results of comparisons, although the defensive and competitive strategies are more robust than the other two strategies, it is hard to select one of them as the best. In this situation, the Dolan–Moré performance profile helped us to see that the aggressive strategy is the robust one. To achieve more reasonable results, it seems that assigning proper weights to the indicators of Table 2 or considering the experts' verbal judgments with fuzzy numbers can be helpful.

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Appendix A: Matrix Data of the real–world problem study of Section 4

$$M = \begin{pmatrix} 1 & 2 & 1 & 1 & 1 & 2 & 2 & 2 & 1 & 2 & 2 & 1 & 2 & 1 & 2 \\ 1 & 1 & 2 & 1 & 2 & 1 & 2 & 2 & 1 & 1 & 1 & 2 & 2 & 2 & 2 \\ 1 & 3 & 2 & 1 & 2 & 2 & 1 & 3 & 2 & 1 & 2 & 3 & 2 & 3 & 3 \\ 1 & 2 & 1 & 3 & 2 & 1 & 1 & 1 & 3 & 2 & 1 & 1 & 2 & 1 & 3 \\ 1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 & 2 & 1 & 1 & 2 & 1 & 2 \\ 1 & 1 & 1 & 2 & 1 & 2 & 2 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 \end{pmatrix},$$

$$A = \begin{pmatrix} 0 & 0 & 1 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 1 & 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 1 & 0 & 0 & 2 & 1 & 2 & 1 & 1 & 2 & 1 & 2 & 1 & 2 \\ 1 & 3 & 1 & 3 & 0 & 0 & 2 & 3 & 0 & 0 & 2 & 3 & 3 & 1 & 3 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 3 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 3 & 1 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 3 \\ 0 & 0 & 0 & 0 & 1 & 2 & 1 & 2 & 1 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 & 1 & 2 & 0 & 0 & 0 & 0 & 1 & 2 & 0 & 0 & 0 & 0 & 1 & 2 & 1 & 2 & 0 & 0 \end{pmatrix},$$

$$B = \begin{pmatrix} 2 & -1 & -1 & 3 & -2 & 1 & 0 & -2 & 2 & 1 & 1 & -2 & -2 & -2 & -1 \\ 3 & -1 & 2 & 1 & 2 & -2 & -1 & -1 & 0 & 1 & 0 & -1 & 0 & 1 & -3 \\ 2 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 2 & -2 \\ 1 & -2 & 2 & 1 & 0 & 2 & 1 & -2 & 2 & -1 & 2 & 0 & 0 & 0 & -2 \\ 0 & -1 & 2 & -1 & 1 & 0 & 2 & 0 & -1 & -1 & 0 & 2 & 1 & 2 & -1 \\ 2 & -1 & 3 & -1 & 1 & 1 & 0 & 1 & 0 & 0 & 3 & -1 & 1 & 1 & -2 \\ -3 & 1 & 0 & -3 & 0 & 0 & 1 & 3 & -2 & -1 & 0 & 1 & 2 & 1 & 3 \\ -3 & 1 & 0 & -3 & 0 & 0 & 1 & 3 & -2 & -1 & 0 & 1 & 2 & 1 & 3 \\ 3 & 1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & 3 & -1 & -1 & 1 & -3 \\ 2 & 2 & 0 & 0 & 0 & 0 & -2 & 0 & 0 & 2 & 2 & -2 & 0 & 0 & -2 \\ 1 & 2 & -1 & 0 & 0 & 1 & -1 & -1 & 0 & 2 & 1 & -1 & 0 & -1 & -2 \end{pmatrix},$$

$$D = \begin{pmatrix} 4.97 & 5.76 & 5.42 & 5.97 & 6.33 & 6.11 & 6.67 & 6.42 & 5.97 & 5.39 & 4.80 & 7.11 & 6.33 & 5.63 & 8.11 \\ 3.22 & 4.84 & 2.66 & 4.23 & 3.67 & 3.53 & 3.53 & 4.27 & 3.89 & 4.40 & 2.97 & 3.89 & 3.67 & 3.22 & 5.52 \\ 5.15 & 3.87 & 3.39 & 5.66 & 3.83 & 4.00 & 3.36 & 3.16 & 5.08 & 4.60 & 3.69 & 3.36 & 3.22 & 3.06 & 3.80 \\ 4.41 & 4.26 & 4.73 & 5.39 & 4.89 & 5.11 & 5.87 & 5.18 & 5.27 & 4.26 & 4.11 & 5.87 & 4.89 & 4.70 & 6.91 \end{pmatrix}.$$