



A Fast Strategy to Find Solution for Survivable Multicommodity Network

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Abstract

This paper proposes an immediately efficient method, based on Benders Decomposition (BD), for solving the survivable multicommodity network. This problem involves selecting a set of arcs for building a survivable network at a minimum cost and within a satisfied flow. The system is subject to failure and capacity restriction. To solve this problem, the BD was initially proposed with far outperformed than mixed integer programming. The employed method in this paper is the modified BD i.e. proposing a new strategy on the basis of s-t cut theorem to identifies the initial failure scenario which are to be solved by BD. For a comparison between the BD method and our modified BD method, four network sets including origins, destinations, failure scenarios, supply nodes and demand nodes have been randomly produced. Next, by using GAMS software, the two methods are compared with respect to their required CPU time where it is shown that the proposed method vitally saves time. This strategy reduces iterations more greatly as compared with the BD approach. While other methods concentrate on valid inequalities for reducing iterations of BD, this study does not use such valid inequalities. In addition, the new method is easily and quickly implementable.

Keywords : Survivable; Network Design; Benders Decomposition; Quickly implementable method; New strategy.

1 Introduction

Today, with the rapid development of technology, high-capacity transmission equipment has been increasingly used in communication networks. As a result, network design has turned to one of the most fundamental issues in engineering fields such as transportation, communications, power distribution, and so on [15, 1, 8]. Billions of dollars are spent on increasing the existing facilities or building communication networks with higher bandwidth every year. Consequently,

sparse communication networks (in which a high amount of flow passes through every link) are available. Due to such a structure (being sparse), networks have low survivability. Failure of one component of the network leads to the loss of a large amount of flow (traffic). At present, one of the issues considered in networks is the design of flexible and survivable networks. The property of survivability in networks refers to the state in which the network done operating tasks in the failure of one or more components. Since failure in communication networks can break down the whole network (like what happens in the real state), survivability in communication networks is very difficult. Adding extra components to the network increases its survivability and also

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the related costs. Therefore, when the network survivability is considered, it is difficult to design a cost-effective network. In the previous works [17, 20, 19], researchers have proposed different methods for establishing survivability conditions. One of the powerful methods compared with mixed integer programming is the use of BD, in which the problem is divided into multiple smaller problems, "master problem" and "sub-problem" as a linear programming problems. Among the advantages of BD are easy solution of the sub-problem and solution of large-scale problems [6]. A large number of studies have covered this area, but here we will mention only a few. Zhao et. al. [21] proposed a formal analysis method for survivable network design problem (SNDP) based on stochastic process algebra, which incorporates formal modeling into performance analysis perfectly. Saharidis et. al. [16] introduce a novel method, which reinitializes the BD master problem by applying a set of valid inequalities. Gendron [9] concentrates on three methods (cutting plane, BD and Lagrangian relaxation), which are used for untangling large-scale multicommodity capacitated fixed-charge SNDP. Costa [7] reviews the BD approach in cases of dissolved fixed-charge network design problems. Castet and Saleh [4] brought considerations of survivability to bear on space systems. They developed a conceptual framework and quantitative analyses based on stochastic Petri nets (SPN) to characterize and compare the survivability of different space architectures. Botton et. al. [3] investigated the hop-constrained SNDP with reliable edges. They considered a subset of reliable edges that are not subject to failure and also studies a static problem where the reliability of edges is given and an upgrading problem where edges can be upgraded to the reliable status at a given cost. Bley et. al. [2] introduced a branch-and-cut algorithm based on BD method to obtain an optimal solution by considering a location problem with survivability. Ljubi et. al. [13] by using a two-stage branch-and-cut algorithm obtain optimal solution for stochastic network design problem. Konak and Bartolacci [10] proposed a methodology for the difficult estimation of traffic efficiency (TE), a measure of network resilience, and a hybrid genetic algorithm to design networks using this measure. Lin [11] defines an

algorithm based on properties of minimal cuts for evaluating performance. Lin [12] introduces a simple algorithm for a stochastic-flow network to estimate the system reliability. Terblanche et. al. [18] expressed algorithmic ideas for improving tractability in solving the SNDP by taking into account uncertainty in the traffic requirements. Chen et. al. [5] constructed the survivable network that has a feasible multicommodity flow even after any k -edges failures by using a cutting plane and a column-and-cut algorithm. Ljubic et. al. [14] perused SNDP with edge-connectivity requirements under a two-stage stochastic model with recourse and finitely many scenarios. They introduced stronger cut-based formulations and suggested a two-stage branch and cut algorithm to solve this problem.

The rest of the paper is organized as follows. Section 2 introduces a mathematical model for designing survivable multicommodity networks and proposes the BD method in order to solve the model. In Section 3, we present a procedure for scenario selection which uses the BD method in order to faster obtain the optimal solution to the original problem. Computational results are given in Section 4. The paper ends with a summary of the main conclusions and suggestions future research in section 5.

2 Model Construction

This study examines the direct network including sets of nodes and capacitated arcs. This network had known origins, destinations, and demands, which was the amount of flow that had to be sent from origins to destinations. Associated with each arc was the construction and failure cost, both of which were assumed to be non-negative. A set of arcs that could simultaneously fail and lose a part of their capacity was also considered. Each set of failing arcs was called a failure scenario. The survivable multicommodity network flow attempted to design a network that minimized the total cost of construction and failure in a way that a feasible multicommodity flow existed in the event of any failure scenario. To formulate this problem we introduce the following notations and definitions:

Sets and Parameters:

N : Set of nodes.

A : Set of arcs(arc that emanate from node i and terminate at node j denoted by (i, j)).

c_{ij} : Construction cost of arc (i, j) .

c_{ij}^f : Failure cost of arc (i, j) .

u_{ij} : Capacity of arc (i, j) .

$N^+(i)$: Set of arcs with positive capacity that emanate from node i .

$N^-(i)$: Set of arcs with positive capacity that terminate at node i .

F : Set of failure scenarios.

k ($k = 1, 2, \dots, K$): commodity of network.

d_k : Demand of commodity k .

s_k : Origin of commodity k .

t_k : Destination of commodity k .

p_e ($e = 1, 2, \dots, E$): E fraction numbers in interval $[0, 1]$.

f_{p_e} : a failure scenario. f_{p_e} is a set of arcs that could simultaneously fail and the failed arcs lose $(1 - p_e)$ th of their capacity (the capacity of other arcs is not touched).

$$w_{ij}^{p_e}: \begin{cases} p_e & (i, j) \in f_{p_e} \\ 1 & (i, j) \notin f_{p_e} \end{cases}$$

Variables:

$x_{ijk}^{p_e}$: The amount of k th commodity on arc (i, j) when the scenario f_{p_e} happens.

$$y_{ij}: \begin{cases} 1 & \text{If arc } (i,j) \text{ is constructed} \\ 0 & \text{o.w} \end{cases}$$

The model (Original Problem) can be stated as follows:

$$\min \sum_{(i,j) \in A} (c_{ij}^f + c_{ij})y_{ij} \tag{2.1}$$

s.t

$$\sum_{k \in K} x_{ijk}^{p_e} \leq y_{ij}w_{ij}^{p_e}u_{ij} \tag{2.2}$$

$$\forall f_{p_e} \in F, \forall (i, j) \in A$$

$$\sum_{(i,j) \in N^+(s_k)} x_{ijk}^{p_e} - \sum_{(i,j) \in N^-(s_k)} x_{ijk}^{p_e} = d_k \tag{2.3}$$

$$\forall f_{p_e} \in F, \forall k \in K$$

$$\sum_{(i,j) \in N^+(t_k)} x_{ijk}^{p_e} - \sum_{(i,j) \in N^-(t_k)} x_{ijk}^{p_e} = -d_k \tag{2.4}$$

$$\forall f_{p_e} \in F, \forall k \in K$$

$$\sum_{(i,j) \in N^+(v)} x_{ijk}^{p_e} - \sum_{(i,j) \in N^-(v)} x_{ijk}^{p_e} = 0 \tag{2.5}$$

$$\forall f_{p_e} \in F, \forall k \in K, \forall v \in N - \{s_k, t_k\}$$

$$x_{ijk}^{p_e} \geq 0, y_{ij} \in \{0, 1\} \tag{2.6}$$

$$\forall k \in K, \forall f_{p_e} \in F, \forall (i, j) \in A$$

The objective function (2.1) represents the total cost of arc construction and failure which has to be minimized. Constraints (2.2) imply that flow exists on an arc, if the arc has been constructed, and limit the flow on the arc to be no more than its capacity in each failure scenario. Constraints (2.3) and (2.4) guarantee that the demand between all origin-destination pairs and is satisfied in each failure scenario. Constraints (2.5) show the flow balance for other nodes (except the origin-destination nodes). Given a failure scenario $f_{p_e} \in F$, Problem (2.1)-(2.6) may be written as:

$$\min \sum_{(i,j) \in A} (c_{ij}^f + c_{ij})y_{ij} + 0x_{ijk}^{p_e} \tag{2.7}$$

s.t

$$\sum_{k \in K} x_{ijk}^{p_e} \leq y_{ij}w_{ij}^{p_e}u_{ij} \quad \forall (i, j) \in A \tag{2.8}$$

$$\sum_{(i,j) \in N^+(s_k)} x_{ijk}^{p_e} - \sum_{(i,j) \in N^-(s_k)} x_{ijk}^{p_e} = d_k \tag{2.9}$$

$$\forall k \in K$$

$$\sum_{(i,j) \in N^+(t_k)} x_{ijk}^{p_e} - \sum_{(i,j) \in N^-(t_k)} x_{ijk}^{p_e} = -d_k \tag{2.10}$$

$$\forall k \in K$$

$$\sum_{(i,j) \in N^+(v)} x_{ijk}^{pe} - \sum_{(i,j) \in N^-(v)} x_{ijk}^{pe} = 0 \quad (2.11)$$

$$\begin{aligned} &\forall k \in K, \forall v \in N - \{s_k, t_k\} \\ &x_{ijk}^{pe} \geq 0, y_{ij} \in \{0, 1\} \\ &\forall k \in K, \forall (i, j) \in A \end{aligned} \quad (2.12)$$

Now let us define

$$Y = \{y_{ij} | (i, j) \in A, y_{ij} \in \{0, 1\}\}$$

and suppose that $y_{ij}, (i, j) \in A$ are fixed. Thus problem (2.7)-(2.12) can be expressed as:

$$\begin{aligned} &\min_{y_{ij} \in Y} \left\{ \sum_{(i,j) \in A} (c_{ij}^f + c_{ij}) y_{ij} + \min_{x_{ijk}^{pe} \geq 0} \left\{ \right. \right. \\ &\sum_{k \in K} x_{ijk}^{pe} \leq y_{ij} w_{ij}^{pe} u_{ij} \\ &\sum_{(i,j) \in N^+(s_k)} x_{ijk}^{pe} - \sum_{(i,j) \in N^-(s_k)} x_{ijk}^{pe} = d_k \\ &\sum_{(i,j) \in N^+(t_k)} x_{ijk}^{pe} - \sum_{(i,j) \in N^-(t_k)} x_{ijk}^{pe} = -d_k \\ &\sum_{(i,j) \in N^+(v)} x_{ijk}^{pe} - \sum_{(i,j) \in N^-(v)} x_{ijk}^{pe} = 0 \\ &\left. \left. \forall k \in K \right\} \right\} \quad (2.13) \end{aligned}$$

The dual variables β_{ij}, π_{ik} can be introduced for the inner minimization problem (2.13). Because the objective function is zero, "max" instead of "min" is used in the objective function of the inner problem (2.13) to obtain dual problem of it. Drawing upon the duality theory, Problem (2.13) can be re-written as:

$$\begin{aligned} &\min_{y_{ij} \in Y} \left\{ \sum_{(i,j) \in A} (c_{ij}^f + c_{ij}) y_{ij} + \min_{\beta_{ij} \geq 0} \left\{ \right. \right. \\ &\sum_{(i,j) \in f_{pe}} \beta_{ij} u_{ij} w_{ij}^{pe} y_{ij} + \sum_{k \in K} d_k (\pi_{s_k, k} - \pi_{t_k, k}) \\ &\text{s.t} \\ &\left. \left. \begin{aligned} &\beta_{ij} + \pi_{ik} - \pi_{jk} \geq 0 \\ &\forall k \in K, \forall (i, j) \in A \end{aligned} \right\} \right\} \quad (2.14) \end{aligned}$$

As a result, we can re-write original problem in the following two separate problems:

$$\begin{aligned} &\min \sum_{(i,j) \in A} (c_{ij}^f + c_{ij}) y_{ij} \\ &\text{s.t} \\ &y_{ij} \in Y \end{aligned} \quad (2.15)$$

and

$$\begin{aligned} &\min \sum_{(i,j) \in f_{pe}} \beta_{ij} u_{ij} w_{ij}^{pe} y_{ij} + \sum_{k \in K} d_k (\pi_{s_k, k} - \pi_{t_k, k}) \\ &\text{s.t} \\ &\beta_{ij} + \pi_{ik} - \pi_{jk} \geq 0 \quad \forall k \in K, \forall (i, j) \\ &\beta_{ij} \geq 0 \end{aligned} \quad (2.16)$$

Problem (2.16) is called a sub-problem (SP) of BD.

If the feasible space of SP be empty, then primal problem is unbounded or infeasible. Suppose that $(h_\beta^p, h_\pi^p)^t, P = 1, 2, \dots, P$ are extreme points and $(h_\beta^r, h_\pi^r)^t, r = 1, 2, \dots, R$ are the extreme directions of the feasible space of SP.

The SP can take either bounded or unbounded values. When SP is bounded, there is a optimal extreme point $(h_\beta^p, h_\pi^p)^t$ and optimal value of SP equal to zero. thus unboundedness of SP is considered. In the unbounded case, there is a extreme direction $(h_\beta^r, h_\pi^r)^t$ such that $(u_{ij} w_{ij}^{pe} y_{ij}, d_k) (h_\beta^r, h_\pi^r)^t < 0$. The unboundedness of SP makes the original problem infeasible. To prevent the unbounded case, the following restrictions are added:

$$(u_{ij} w_{ij}^{pe} y_{ij}, d_k) (h_\beta^r, h_\pi^r)^t \geq 0.$$

If extreme directions of SP under scenarios represented by

$$(h_{\beta_{f_{pe}}}^r, h_{\pi_{f_{pe}}}^r)^t \quad r = 1, 2, \dots, R, \forall f_{pe} \in F.$$

thus we can write Problem (2.14) as:

$$\min_{y \in Y} \sum_{(i,j) \in A} (c_{ij}^f + c_{ij}) y_{ij} \quad (2.17)$$

s.t

$$\begin{aligned} &(u_{ij} w_{ij}^{pe} y_{ij}, d_k) (h_{\beta_{f_{pe}}}^r, h_{\pi_{f_{pe}}}^r)^t \geq 0 \\ &r=1,2,\dots,R, \forall f_{pe} \in F \end{aligned} \quad (2.18)$$

Table 1: ($|K|= 2, |F|= 4$) (in seconds)

	$ N =5$	$ N =10$	$ N =15$
BD	7	66	325
f_m & BD	4	55	274

Table 2: ($|F|= 4, |N|= 10$) (in seconds)

	$ K =2$	$ K =4$	$ K =10$
BD	66	70	72
f_m & BD	39	42	56

Table 3: ($|K|= 2, |N|= 10$) (in seconds)

	$ F =4$	$ F =6$	$ F =10$
BD	80	97	395
f_m & BD	47	56	201

$$y_{ij} \in Y \tag{2.19}$$

The above problem (2.17)-(2.19) is Master Problem (MP) of BD. In practical situations, there are typically too many extreme rays, which is a disadvantage of the above formulation. To overcome this weakness, generating Constraints (2.18) are postponed. Initially, only the last constraint (2.19) is considered. Thus, the first relaxed MP is as follows:

$$\begin{aligned} \min_{y \in Y} \sum_{(i,j) \in A} (c_{ij}^f + c_{ij})y_{ij} \\ s.t \\ y_{ij} \in Y \end{aligned} \tag{2.20}$$

Once Problem (2.20) is solved, SP of all scenarios would be solved using the trial configuration of y_{ij} . If SP is unbounded, a constraint of type (2.18) is added to the relaxed MP. This process is iteratively performed until obtaining an optimal solution for the original problem.

3 Optimality conditions for a certain scenario and modified BD

At the first iteration of BD, the relaxed MP is (2.20) and we obtain $y_{ij} = 0, \forall (i, j) \in A$. As Constraints (2.18) are added to the model.

Indeed, the choice of scenario also affects on reducing the number of iterations of relaxed MP. We focus on finding a scenario that start with this, finding optimal solution of original problem or reduce the number of iterations of BD approach.

The capacity of an s-t cut (s, \bar{s}) under f_{p_e} can be defined as:

$$U_{p_e}(s, \bar{s}) = \min \left\{ \sum_{(i,j) \in (s, \bar{s})} w_{ij}^{p_e} u_{ij} \right\} \tag{3.21}$$

And also define $Z_{f_{p_e}}$ as the capacity of the minimum cut:

$$Z_{f_{p_e}} = \min_{(s, \bar{s})} \{U_{p_e}(s, \bar{s})\} \tag{3.22}$$

Theorem 3.1 *The optimal solution under f_{p_e} which is $Q_{p_e}^*$ ($Q_{p_e}^* \subseteq A$) is optimal for original problem if and only if the capacity of every $s_k - t_k$ cut of $Q_{p_e}^*$ under other scenarios is greater than or equal to d_k ($\forall k \in K$).*

Proof. Without loss of generality, suppose that $Q_{p_1}^*$ ($Q_{p_1}^* \subseteq A$) is the optimal solution under scenario f_{p_1} such that the capacity of every $s_k - t_k$ cut of $Q_{p_1}^*$ under other scenarios is greater than or equal to the d_k . Suppose that Q^* ($Q^* \subseteq A$) is the optimal solution to the original problem. Since

$Q_{p_1}^*$ is the optimal solution under f_{p_1} , it follows that

$$\sum_{(i,j) \in Q_{p_1}^*} (c_{ij} + c_{ij}^f) \leq \sum_{(i,j) \in Q^*} (c_{ij} + c_{ij}^f) \quad (3.23)$$

Now, we consider $Q_{p_1}^*$ under another scenario say f_{p_2} . $Q_{p_1}^*$ is not the optimal solution under f_{p_2} . Since the capacity of every $s_k - t_k$ cut of $Q_{p_1}^*$ under f_{p_2} is greater than or equal to the d_k the destination demands is satisfied. Therefore

$$\sum_{(i,j) \in Q^*} (c_{ij} + c_{ij}^f) \leq \sum_{(i,j) \in Q_{p_1}^*} (c_{ij} + c_{ij}^f) \quad (3.24)$$

then

$$\sum_{(i,j) \in Q^*} (c_{ij} + c_{ij}^f) = \sum_{(i,j) \in Q_{p_1}^*} (c_{ij} + c_{ij}^f) \quad (3.25)$$

Thus $Q_{p_1}^*$ is also optimal solution for original problem.

Now, suppose that the optimal solution under scenario f_{p_1} (which is $Q_{p_1}^*$) is the optimal solution to the original problem. Without loss of generality, we consider $Q_{p_1}^*$ under scenario f_{p_2} and suppose that there is an $s_k - t_k$ cut corresponding to f_{p_2} such that the capacity of $Q_{p_1}^*$ on this cut is less than the d_k . Thus, $Q_{p_1}^*$ under f_{p_2} cannot satisfy the destination demands, indicating that $Q_{p_1}^*$ is not the optimal solution to the original problem.

Finally, note that this scenario may be difficult or even impossible to be identified. We will denote the number of nodes, the number of commodities and the number of scenarios by $|N|$, $|K|$ and $|F|$ respectively. In order to find a scenario which would allow us to obtain the optimal solution to the original problem at fewer iterations, we suggest to start the BD approach with a failure scenario that has least $s_k - t_k$ cut of all scenarios. We nominate this scenario as f_m . If the optimal solution to this scenario is optimal to the original problem (i.e. it satisfies the conditions of Theorem 1), the original problem is solved. Otherwise, we put the optimal solution obtained under f_m into others scenarios.

4 Computational results

In this section, in order to demonstrate computational efficiency of the suggested approach in

terms of cpu time (to obtain the optimal solution), some sets of random network design examples with different problem sizes are generated. The first approach only uses the BD method and the second approach, as suggested above, starts the BD method with f_m . For comparison purposes, three sets of random network designs are created. Size of each problem and the total demands pertinent to each problem are so chosen that it is possible to show the impact of problem size on computational time of the two above approaches. The number of nodes, number of commodity, and number of failure scenario are changed respectively in the first, second, and third sets of random networks. The target arc density is set to 0.5, which is the ratio of the total number of network arcs to those arcs that can exist in the network. Capacity, construction, and failure costs of arcs are randomly set between 0 and 100 according to a uniform probability distribution. Demands are randomly chosen between 45% and 65% of f_m . In each failure scenario, the number of failed arcs is equal. All the results are obtained by an intel core i7, 2.2GHz cpu with 8 GB RAM. The results are summarized in the tables 1, 2, 3.

5 Conclusion

This research aimed to develop an effective algorithm for solving survivable network design problems. To solve this problem, the BD was initially proposed with far outperformed than mixed integer programming. To reduce the number of iterations of using the BD approach, we introduced a new strategy on basis of s-t cut theorem to find the particular initial failure scenario. Further research in this area could develop a branch-and-price-and-cut algorithm for SNDP. The obtained results can be useful for those actual instances of network arc failure.

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