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On End and Coupled Endpoints of θ-F-Contractive Set-Valued Mappings

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Abstract

In this paper, we introduce a new concept in set-valued mappings which we have called condition (UHS). Then, adding this condition to a new type of contractive set-valued mappings, recently has been introduced by Amini-Harandi [Fixed and coupled fixed points of a new type contractive set-valued mapping in complete metric spaces, Fixed point theory and applications, 215 (2012)], we prove that this mapping have a unique end point. Then, we state and prove a result about existence of coupled fixed point of this type of contractive set-valued mappings defined on $M \times M$, where M is a complete metric space (Recently, Amini-Harandi proved existence of coupled fixed point only for self mappings). Finally, we introduce one another new concept, which we have called condition $(UHS)^*$. Then, adding this condition we state and prove existence of coupled endpoint for such contractive set-valued mappings. Some examples are given to illustrate the results.

Keywords : Endpoint; Coupled fixed point; Coupled endpoint; θ-F-Contractive; Set-valued mappings.

1 Introduction

There are many extentions of the Banach contraction principle in literature. Let (X, d) be a metric space and let $CB(X)$ denote the set of all nonempty closed bounded subsets of X. Let H be the Hausdorff metric on $CB(X)$ with respect to metric d, that is, $H(A, B) = max\{\sup_{x\in A} d(x, B), \sup_{y\in B} d(y, A)\}$ for all $A, B \in CB(X)$, where $d(y, A) = inf_{x\in A} d(y, x)$. Let $T : X \to 2^X$ is a set-valued mapping. It is called that x is a fixed point of T if $x \in Tx$. In 1969, Nadler extended the Banach contraction principle to set-valued mappings as follows: (Nadler [10]) Let (X, d) be a complete metric space and let $T : X \to CB(X)$ be a set-valued mapping such that

$$H(Tx, Ty) \le kd(x, y),$$

for all $x, y \in X$.Then T has a fixed point. In 1989, Mizoguchi and takahashi extended Nadler's result as follows: (Mizoguchi and takahashi [8]) Let (X, d) be a complete metric space and let $T : X \to CB(X)$ be a set-valued mapping such that

$$H(Tx, Ty) \le \alpha(d(x, y))d(x, y),$$

for all $x, y \in X$, where $\alpha : [0, +\infty) \to [0, 1)$ satisfies $\limsup_{t\to r^+} \alpha(t) < 1$, for all $r \in [0, +\infty)$. Then T has a fixed point.

Let $F : (0, +\infty) \to \mathbb{R}$ and $\theta : (0, +\infty) \to (0, +\infty)$ be two maps. Througout this paper let Δ be the set of all pairs of (F, θ) satisfying the following conditions:

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(δ_1) For each strictly decreasing sequence $\{t_n\}$ in $(0,+\infty)$, $\theta(t_n) \nrightarrow 0$.

(δ_2) F is strictly increasing.

(δ_3) For each sequence $\{\alpha_n\}$ in $(0,+\infty)$, $\lim_{n\to\infty} \alpha_n = 0$ if and only if $\lim_{n\to\infty} F(\alpha_n) = -\infty$.

(δ_4) If $t_n \downarrow 0$ and $\theta(t_n) \leq F(t_n) - F(t_{n+1})$ for all $n \in \mathbb{N}$, then $\Sigma_{n=1}^{\infty} t_n < \infty$.

For example , let $\theta(t) = \tau$, for some $\tau > 0$ and $F(t) = \ln(t) + t$. It is easy to see that $(F, \theta) \in \Delta$ (for details see [4]). Another example is $\theta(t) = -ln(\alpha(t))$, where $\alpha : [0, \infty) \to [0, 1)$ and $\limsup_{t\to r^+} \alpha(t) < 1$, for all $r \in (0, \infty)$ and $F(t) = \ln(t)$ (see [4]). Recently, Amini-Harandi introduced the following generalization of Theorem 1 and the theorem of Wardowski (see Wardowski's [14]). (Amini Harandi [4]) Let (X, d) be a metric space and let $T : X \to CB(X)$ be a set-valued mapping and $(F, \frac{\theta}{2}) \in \Delta$ such that

$$\theta(d(x, y)) + F(H(Tx, Ty)) \leq F(d(x, y)), \tag{1.1}$$

for all $x, y \in X$ with $Tx \neq Ty$. If T be compact valued or F be continuous from the right, Then T has a fixed point.

2 Main Results

Let (X, d) be a complete metric space and let $T : X \to CB(X)$ be a set-valued mapping. It is called that T has the approximate endpoint property if $\inf_{x\in X} \sup_{y\in Tx} d(x, y) = 0$. In 2010, Amini Harandi proved that if $H(Tx, Ty \leq \psi(d(x, y))$, for all $x, y \in X$, where $\psi : [0, +\infty) \to [0, +\infty)$ is a mapping with some properties, then T has a unique endpoint $x \in X$, that is, $Tx = \{x\}$ if and only if T has the approximate endpoint property ([2]). We say that T satisfies condition (UHS) if for any $x \in X$ there exists $y \in Tx$ such that $H(Tx, Ty) \geq \sup_{b\in Ty} d(y, b)$. Also, we say that T is θ-F-contractive if (1.1) holds for all $x, y \in X$ with $Tx \neq Ty$.

Now, we state and prove the main result of this paper. Let (X, d) be a complete metric space and $(F, \frac{\theta}{2}) \in \Delta$. Let $T : X \to CB(X)$ be a θ-F-contractive set-valued mapping satisfying condition (UHS). Then T has a unique endpoint. Let $x_0 \in X$. Since T satisfies condition (UHS), hence there exists $x_1 \in Tx_0$ such that $H(Tx_0, Tx_1) \geq \sup_{b\in Tx_1} d(x_1, b)$. If $Tx_0 = Tx_1$, then $x_1 \in Tx_0 = Tx_1$ and so $H(\{x_1\}, Tx_1) = \sup_{b\in Tx_1} d(x_1, b) \leq H(Tx_0, Tx_1) = 0$. Hence $Tx_1 = \{x_1\}$ and so x_1 is an endpoint of T. So, we may assume that $Tx_0 \neq Tx_1$. Now since T is θ-F-contractive, hence $\theta(d(x_0, x_1)) + F(H(Tx_0, Tx_1)) \leq F(d(x_0, x_1))$. By continuing this process, we obtain a sequence $\{x_n\}$ such that $x_{n+1} \in Tx_n$, $H(Tx_n, Tx_{n+1}) \geq \sup_{b\in Tx_{n+1}} d(x_{n+1}, b)$, $Tx_n \neq Tx_{n+1}$ and

$$\begin{aligned} \theta(d(x_n, x_{n+1})) &+ F(H(Tx_n, Tx_{n+1})) \\ &\leq F(d(x_n, x_{n+1})), \end{aligned} \tag{2.2}$$

for all $n \in \mathbb{N}$. Now we have

$$\begin{aligned} d(x_{n+1}, x_{n+2}) &\leq \sup_{b\in Tx_{n+1}} d(x_{n+1}, b) \\ &\leq H(Tx_n, Tx_{n+1}), \end{aligned} \tag{2.3}$$

for all $n \in \mathbb{N}$. Since F is increasing and $x_n \neq x_{n+1}$ (since $Tx_n \neq Tx_{n+1}$), so

$$\begin{aligned} F(d(x_{n+1}, x_{n+2})) &< F(H(Tx_n, Tx_{n+1})) \\ &+ \frac{\theta(d(x_n, x_{n+1}))}{2}. \end{aligned} \tag{2.4}$$

Now,

$$\begin{aligned} &\frac{\theta(d(x_n, x_{n+1}))}{2} + F(d(x_{n+1}, x_{n+2})) \\ &< F(H(Tx_n, Tx_{n+1})) + \theta(d(x_n, x_{n+1})) \\ &\leq F(d(x_n, x_{n+1})). \end{aligned} \tag{2.5}$$

Put $t_n = d(x_n, x_{n+1})$. Then, from (2.4) we have $\frac{\theta(t_n)}{2} + F(t_{n+1}) \leq F(t_n)$ and so

$$\frac{\theta(t_n)}{2} \leq F(t_n) - F(t_{n+1}) \quad \text{for all } n \in \mathbb{N}. \tag{2.6}$$

Since $\theta(t_n) > 0$, then we have $F(t_{n+1}) < F(t_n)$. Since F is strictly increasing, hence $t_{n+1} < t_n$ and so $\{t_n\}$ is a strictly decreasing sequence of positive real numbers and so converges to some $r \geq 0$. Now we show that $r = 0$. By (δ_1) we have $\theta(t_n) \nrightarrow 0$ and hence $\Sigma_{n=1}^{\infty}\theta(t_n) = \infty$. Now, from (2.5), we have $\frac{1}{2}\Sigma_{i=1}^{n}\theta(t_i) \leq F(t_1) - F(t_{n+1})$. Therefore $\infty = \frac{1}{2}\Sigma_{i=1}^{\infty}\theta(t_i) \leq F(t_1) - \lim_{n\to\infty} F(t_{n+1})$. Hence $\lim_{n\to\infty} F(t_n) = -\infty$ and so $\lim_{n\to\infty} t_n = 0$. From ($\delta_4$), we have $\Sigma_{n=1}^{\infty} t_n < \infty$. Hence

$\Sigma_{n=1}^{\infty} d(x_n, x_{n+1}) < \infty$. Therefore, from the triangle inequality $\{x_n\}$ is a Cauchy sequence in X. Since (X, d) is complete, there exists $x \in X$ such that $x_n \to x$. Now we show that x is an endpoint of T. To show this, we get two cases:

(i) There exists $N \in \mathbb{N}$ such that $Tx_n \neq Tx$ for all $n \geq N$.

(ii) There exists a subsequence $\{x_{n_i}\}$ of $\{x_n\}$ such that $Tx_{n_i} = Tx$ for all $i \in \mathbb{N}$.

In the case (i), we have

$$\theta(d(x_n, x)) + F(H(Tx_n, Tx)) \leq F(d(x_n, x)), \tag{2.7}$$

for all $n \in \mathbb{N}$. Now since $\lim_{n\to\infty} d(x_n, x) = 0$, hence from (δ_3), we get $\lim_{n\to\infty} F(d(x_n, x)) = -\infty$. From (2.6) we result $\lim_{n\to\infty} F(H(Tx_n, Tx)) = -\infty$ and so $\lim_{n\to\infty} H(Tx_n, Tx) = 0$. On the other hand,

$$\begin{aligned} & H(\{x_n\}, Tx_n) \\ = & \max\{d(x_n, Tx_n), \sup_{b\in Tx_n} d(x_n, b)\} \\ \leq & H(Tx_{n-1}, Tx_n). \end{aligned} \tag{2.8}$$

Now since F is increasing, from (2.7) we obtain

$$\begin{aligned} & \theta(d(x_{n-1}, x_n)) + F(H(\{x_n\}, Tx_n)) \\ \leq & \ \theta(d(x_{n-1}, x_n)) + F(H(Tx_{n-1}, Tx_n)) \\ \leq & \ F(d(x_{n-1}, x_n)). \end{aligned} \tag{2.9}$$

Since $d(x_{n-1}, x_n)) \to 0$, hence $F(d(x_{n-1}, x_n))) \to -\infty$. Hence, from (2.8), $F(H(\{x_n\}, Tx_n)) \to -\infty$ and so $H(\{x_n\}, Tx_n) \to 0$. Now

$$H(\{x\}, Tx) \leq d(x, x_n) + H(\{x_n\}, Tx_n)$$

$$+H(Tx_n, Tx) \to 0.$$

Hence, $H(\{x\}, Tx) = 0$ and so $\{x\} = Tx$. Therefore, x is an endpoint of T.
In the case (ii),

$$\begin{aligned} H(\{x\}, Tx) & \leq d(x, x_{n_i}) + H(\{x_{n_i}\}, Tx_{n_i}) \\ & \leq d(x, x_{n_i}) + H(Tx_{n_i-1}, Tx_{ni}). \end{aligned} \tag{2.10}$$

But since $d(x_n, x_{n+1}) \to 0$, from (2.2) we can conclude that $H(Tx_n, Tx_{n+1}) \to 0$. Hence $H(Tx_{n_i-1}, Tx_{n_i}) \to 0$. Now the right side of inequality (2.10) tends to zero and hence H({x},Tx)=0. So, we have shown that x is an endpoint of T.

For the uniquness of endpoint let x, y are two endpoints of T such that $x \neq y$. Then $Tx = \{x\} \neq \{y\} = Ty$. Now we have $\theta(d(x, y)) + F(H(Tx, Ty)) \leq F(d(x, y))$ and $H(Tx, Ty) = d(x, y)$. Hence $\theta(d(x, y)) \leq 0$. Which is a contradiction.

Example 2.1 *Let $X = \{0, 1, 2, ...\}$ and define the metric d on X by*

$$d(x, y) = \begin{cases} 0 & x = y \\ x + y & x \neq y. \end{cases}$$

Let $T : X \to CB(X)$ is defined by

$$Tx = \begin{cases} \{0\} & x = 0 \\ \{0, 1, 2, ..., x-1\} & x \neq 0. \end{cases}$$

If $Tx \neq Ty$, then $x \neq y$. In the case where $x, y \in \{1, 2, ...\}$, then $H(Tx, Ty) = x + y - 2$. If $x = 0$ and $y \in \{1, 2, ...\}$, then $H(Tx, Ty) = y - 1$. In any case $H(Tx, Ty) - d(x, y) \leq -1$. Hence

$$\frac{H(Tx, Ty)}{d(x, y)} e^{H(Tx,Ty)-d(x,y)} \leq e^{-1}.$$

Therefore

$$1 + \ln(H(Tx, Ty)) + H(Tx, Ty)$$
$$\leq \ln(d(x, y)) + d(x, y)).$$

Now put $\theta(t) = 1$ and $F(t) = \ln t + t$. Then $(F, \frac{\theta}{2}) \in \Delta$ and F is θ-F-contractive set-valued mapping. Now we show that T satisfies condition (UHS). For this let $x \in X$. If $x = 0$ or $x = 1$, then $Tx = \{0\}$. Now put $y = 0$. Then $y \in Tx$ and $H(Tx, Ty) = 0 = \sup_{b\in Ty} d(y, b)$. In the case where $x \in \{2, ...\}$, we have $Tx = \{0, 1, 2, ..., x-1\}$ and since $x \geq 2$, hence $x-1 \geq 1$. Now put $y = 1$. Then $y \in Tx$ and $Ty = \{0\}$. Hence $H(Tx, Ty) = x - 1 \geq 1 = \sup_{b\in Ty} d(y, b)$. Therefore T satisfies condition (UHS). Then by Theorem 2, T has a unique endpoint. Here 0 is the only endpoint of T.

In 2012, Amini-Harandi proved the following result about coupled fixed point of θ-F-contractive mappings.

Theorem 2.1 *(Amini Harandi [4]) Let (M, ρ) be a complete metric space and let $(F, \frac{\theta}{2}) \in \Delta$. let $f : M \times M \to M$ be a mapping such that*

$$\theta(\rho(x, u) + \rho(y, v)) + F(\rho(f(x, y), f(u, v))$$

$$+\rho(f(y,x),f(v,u)))\le F(\rho(x,u)+\rho(y,v)),$$

for all $x,y,u,v\in M$, *with* $f(x,y)\neq f(u,v)$ *or* $f(y,x)\neq f(v,u)$. *Then* f *has a coupled fixed point* $(x,y)\in M\times M$. *That is* $f(x,y)=x$ *and* $f(y,x)=y$. In the following theorem we extend Theorem 2.1 to set-valued mappings. Let (M,ρ) be a complete metric space and let $(F,\frac{\theta}{2})\in\Delta$. Let $\hbar: M\times M\to CB(M)$ be a set-valued mapping such that

$$\theta(\rho(x,u)+\rho(y,v))+F(H(\hbar(x,y),\hbar(u,v))$$
$$+H(\hbar(y,x),\hbar(v,u))\le F(\rho(x,u)+\rho(y,v))), \tag{2.11}$$

for all $x,y,u,v\in M$, with $\hbar(x,y)\neq\hbar(u,v)$ or $\hbar(y,x)\neq\hbar(v,u)$. If $\hbar$ be compact valued or F be continuous from the right, then $\hbar$ has a coupled fixed point (x,y) in $M\times M$. That is $x\in\hbar(x,y)$ and $y\in\hbar(y,x)$.
Let $X=M\times M$ and define the metric d on X by $d((x,y),(u,v))=\rho(x,u)+\rho(y,v)$, for all $(x,y),(u,v)\in X$. It is easy to show that (X,d) is a complete metric space. Define $T: X\to X$ by $T(x,y)=\hbar(x,y)\times\hbar(y,x)$. Using (2.11), we shall show that

$$\theta(d((x,y),(u,v)))+F(H_d(T(x,y),T(u,v))) \le F(d((x,y),(u,v))) \tag{2.12}$$

for all $(x,y),(u,v)\in X$ with $T(x,y)\neq T(u,v)$, where H_d is the Hausdorff metric on $CB(X)$ with respect to the metric d on X. At first, note that

$$\begin{aligned}
&H_d(T(x,y),T(u,v))\\
=\;&H_d(\hbar(x,y)\times\hbar(y,x),\hbar(u,v)\times\hbar(v,u))\\
=\;&\max\{\sup_{(\xi_1,\xi_2)\in\hbar(x,y)\times\hbar(y,x)} d((\xi_1,\xi_2),\hbar(u,v)\times\hbar(v,u))\\
,\;&\sup_{(\eta_1,\eta_2)\in\hbar(u,v)\times\hbar(v,u)} d((\eta_1,\eta_2),\hbar(x,y)\times\hbar(y,x))\}\\
=\;&\max\{\sup_{\xi_1\in\hbar(x,y)}\rho(\xi_1,\hbar(u,v))\\
+\;&\sup_{\xi_2\in\hbar(y,x)}\rho(\xi_2,\hbar(u,v)),\\
&\sup_{\eta_1\in\hbar(u,v)}\rho(\eta_1,\hbar(x,y))\\
+\;&\sup_{\eta_2\in\hbar(v,u)}\rho(\eta_2,\hbar(y,x))\}\\
\le\;&H(\hbar(x,y),\hbar(u,v))+H(\hbar(y,x),\hbar(v,u)).
\end{aligned} \tag{2.13}$$

Now since F is strictly increasing, from (2.11) and (2.13) we have (2.12) holds. Now if $\hbar$ be compact valued then T is compact valued. Now all of the conditions of Theorem 1 holds. Hence by the theorem T has a fixed point (x,y) in $X=M\times M$, that is, $(x,y)\in T(x,y)=\hbar(x,y)\times\hbar(y,x)$. Hence $x\in\hbar(x,y)$ and $y\in\hbar(y,x)$. So (x,y) is a coupled fixed point of $\hbar$.

Definition 2.1 *Let* (M,ρ) *be a metric space and let* $\hbar: M\times M\to CB(M)$ *be a set-valued mapping. We say that* $\hbar$ *satisfies condition* $(UHS)^*$, *if for any* $x,y\in M$, *there exist* $u\in\hbar(x,y)$ *and* $v\in\hbar(y,x)$ *such that*

$$\max\{\sup_{\xi_1\in\hbar(x,y)}\rho(\xi_1,\hbar(u,v))+\sup_{\xi_2\in\hbar(y,x)}\rho(\xi_2,\hbar(u,v)),$$
$$\sup_{\eta_1\in\hbar(u,v)}\rho(\eta_1,\hbar(x,y))+\sup_{\eta_2\in\hbar(v,u)}\rho(\eta_2,\hbar(y,x))\}$$
$$\ge\sup_{a\in\hbar(u,v)}\rho(u,a)+\sup_{b\in\hbar(v,u)}\rho(v,b). \tag{2.14}$$

In following theorem we introduce and prove a result about coupled endpoints of set-valued mappings that satisfies condition $(UHS)^*$. Let (M,ρ) be a complete metric space and let $(F,\frac{\theta}{2})\in\Delta$. Let $\hbar: M\times M\to CB(M)$ be a set-valued mapping satisfying condition $(UHS)^*$ such that

$$\begin{aligned}
&\theta(\rho(x,u)+\rho(y,v))\\
+\;&F(H(\hbar(x,y),\hbar(u,v))+H(\hbar(y,x),\hbar(v,u)))\\
\le\;&F(\rho(x,u)+\rho(y,v)),
\end{aligned} \tag{2.15}$$

for all $x,y,u,v\in M$ with $\hbar(x,y)\neq\hbar(u,v)$ or $\hbar(y,x)\neq\hbar(v,u)$. Then $\hbar$ has a unique coupled endpoint (x,y) in $M\times M$, that is, $\{x\}=\hbar(x,y)$ and $\{y\}=\hbar(y,x)$. Let (X,d) and $T: X\to X$ be as in the proof of Theorem 2. We want to show that T has the condition (UHS). Let $(x,y)\in X$. Then, $x,y\in M$. Since $\hbar$ has the condition $(UHS)^*$, then there exist $u\in\hbar(x,y)$ and $v\in\hbar(y,x)$ such that (2.13) holds. From (2.13) and (2.14), we have

$$\begin{aligned}
&H_d(T(x,y),T(u,v))\\
\ge\;&\sup_{a\in\hbar(u,v)}\rho(u,a)+\sup_{b\in\hbar(v,u)}\rho(v,b)\\
=\;&\sup_{(a,b)\in\hbar(u,v)\times\hbar(v,u)} d((u,v),(a,b))\\
=\;&\sup_{(a,b)\in T(u,v)} d((u,v),(a,b)).
\end{aligned} \tag{2.16}$$

Now since $(u,v)\in T(x,y)$ and (2.16) holds, hence T has condition (UHS). From (2.15)

and as in the proof of Theorem (2), we have
$$\theta(d((x,y),(u,v))) + F(H_d(T(x,y),T(u,v))) \leq F(d((x,y),(u,v))),$$
for all $(x,y),(u,v) \in X$ with $T(x,y) \neq T(u,v)$. Hence by Theorem 2, we can say that T has a unique end point (x,y) in X. That is $\{(x,y)\} = T(x,y)$. Hence $\{x\} = \hbar(x,y)$ and $\{y\} = \hbar(y,x)$.

Example 2.2 *Let $M = [0,\infty)$ and define the metric ρ on X by $\rho(x,y) = |x-y|$. Let $\hbar : M \times M \to CB(M)$ is defined by $\hbar(x,y) = [0, \frac{|x-y|}{4}]$. Then we have*
$H(\hbar(x,y),\hbar(u,v)) + H(\hbar(y,x),\hbar(v,u))$
$$= 2(\frac{|x-y|}{4} - \frac{|u-v|}{4})$$
$$\leq \frac{1}{2}(|x-u|+|y-v|) = \frac{1}{2}(\rho(x,u)+\rho(y,v)),$$
for all $x,y,u,v \in M$. Then we will have
$$\ln 2 + \ln(H(\hbar(x,y),\hbar(u,v)) + H(\hbar(y,x),\hbar(v,u)))$$
$$\leq \ln(\rho(x,u)+\rho(y,v)),$$
for all $x,y,u,v \in M$ with $\hbar(x,y) \neq \hbar(u,v)$ or $\hbar(y,x) \neq \hbar(v,u)$. If we put $\theta(t) = \ln 2$ and $F(t) = \ln t$, then (2.15) holds. Also we show that $\hbar$ satisfies condition $(UHS)^*$. To see this, let $x,y \in M$. Put $u = v = 0$, then obviously $u \in \hbar(x,y)$, $v \in \hbar(y,x)$ and $\hbar(u,v) = \hbar(v,u) = \{0\}$. Hence $\sup_{a\in\hbar(u,v)} \rho(u,a) + \sup_{b\in\hbar(v,u)} \rho(v,b) = 0$. So the inequality (2.14) holds. Now we have shown that $\hbar$ has the condition $(UHS)^*$. Now by Theorem 2 we can say that $\hbar$ has a unique coupled endpoint (x,y) in $M \times M$. Here $(0,0)$ is the only endpoint of $\hbar$.

Example 2.3 *Let $M = [0,\infty)$ and define the metric ρ on X by $\rho(x,y) = |x-y|$. Define $\hbar : M \times M \to CB(M)$ by $\hbar(x,y) = [0, \frac{r}{2}(x+y)]$, where $r < 1$. Then we have*
$H(\hbar(x,y),\hbar(u,v)) + H(\hbar(y,x),\hbar(v,u))$
$$= 2|\frac{r}{2}(x+y) - \frac{r}{2}(u+v)|$$
$$\leq r(|x-u|+|y-v|) = r(\rho(x,u)+\rho(y,v)),$$
for all $x,y,u,v \in M$. Then we will have
$$-\ln r + \ln(H(\hbar(x,y),\hbar(u,v)) + H(\hbar(y,x),\hbar(v,u)))$$
$$\leq \ln(\rho(x,u)+\rho(y,v)),$$
for all $x,y,u,v \in M$ with $\hbar(x,y) \neq \hbar(u,v)$ or $\hbar(y,x) \neq \hbar(v,u)$. If we put $\theta(t) = -\ln r$ and $F(t) = \ln t$, then (2.15) holds. It is easy to show that $\hbar$ satisfies condition $(UHS)^*$. Now by Theorem 2 we can say that $\hbar$ has a unique coupled endpoint (x,y) in $M \times M$. Here $(0,0)$ is the only endpoint of $\hbar$.

3 Conclusion

In this research, existence of endpoint, coupled fixed point and coupled endpoint are proved for θ-F-contractive set-valued mappings. For further research, existence of endpoint, coupled fixed point and coupled endpoint are recommended for θ-F-quasicontractive set-valued mappings.

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4 Conclusion

In this paper, we have presented a new approach for ranking of fuzzy numbers. First, we present a new method for ranking fuzzy numbers based on the γ-cuts, the belief features and the signal/noise ratios of fuzzy numbers. The proposed method calculates the signal/noise ratio of each γ-cut of a fuzzy number to evaluate the quantity and the quality of a fuzzy number, where the signal and the noise are defined as the middle-point and the spread of each γ-cut of a fuzzy number, respectively. We use the value of a as the weight of the signal/noise ratio of each γ-cut of a fuzzy number to calculate the ranking index of each fuzzy number. The proposed fuzzy ranking method can rank any kinds of fuzzy numbers with different kinds of membership functions.

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