Available online at http://ijim.srbiau.ac.ir/
Int. J. Industrial Mathematics (ISSN 2008-5621)
Vol. 8, No. 4, 2016 Article ID IJIM-00771, 5 pages
Research Article

Another Method for Defuzzification Based on Regular Weighted Point

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Abstract

A new method for the defuzzification of fuzzy numbers is developed in this paper. It is well-known, defuzzification methods allow us to find aggregative crisp numbers or crisp set for fuzzy numbers. But different fuzzy numbers are often converted into one crisp number. In this case the loss of essential information is possible. It may result in inadequate final conclusions, for example, expert estimation problems, prediction problems, etc. Accordingly, the necessity to develop a method for the defuzzification of fuzzy numbers, allowing us to save their informative properties has arisen. The purpose of this paper is to develop such a method. The method allows us to find aggregative intervals for fuzzy numbers. These intervals are called the Regular weighted intervals. We start with the definition of regular weighted points for fuzzy numbers. The regular weighted interval for fuzzy number is defined as the set of regular weighted points of all unimodal numbers, that belong to this number. Some propositions and examples about regular weighted point and regular weighted intervals properties are offered.

Keywords : Ranking; Fuzzy number; L-R type; Defuzzification; Regular weighted point.

1 Introduction

As it is well-known, defuzzification methods convert a fuzzy number into a crisp real number [4, 5, 7]. But often different fuzzy numbers are converted into one crisp number. For example, according to the definition of weighted fuzzy arithmetic in [4], two normalized symmetrical triangular numbers with different fuzzy widths are converted into one crisp number. This may not present a problem to solve a number of practical tasks, however, for example, in decision making problems and some other problems the necessity arises to find aggregative indexes that will possibly accumulate different bounds of input fuzzy numbers. Moreover, while making regression models, it is easier to operate with aggregative indexes than with the proper fuzzy numbers. The purpose of this paper is to develop a new method for the defuzzification of fuzzy numbers, that will allow to keep their informative properties. This paper starts with the definitions of weighted points and weighted sets for fuzzy numbers [9]. Then, three propositions about the weighted intervals properties are proved.

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2 Basic Definitions and Notations

In this paper, we assume that the reader is familiar with basics of fuzzy set theory and fuzzy logic in the broad sense.

Definition 2.1 *[3, 6]. Let X be a universe set. A fuzzy set A of X is defined by a membership* *function* $\mu_A(x) \to [0,1]$, *where* $\mu_A(x)$, $\forall x \in X$, *indicates the degree of* x *in* A.

Definition 2.2 *A fuzzy subset* A *of universe set* X *is normal iff* $\sup_{x\in X} \mu_A(x) = 1$, *where* X *is the universe set.*

Definition 2.3 *A fuzzy set* A *is a fuzzy number iff* A *is normal and convex on* X.

Definition 2.4 *For fuzzy set* A *Support function is defined as follows:*

$$supp(A) = \overline{\{x|\mu_A(x) > 0\}},$$

where $\overline{\{x|\mu_A(x) > 0\}}$ *is the closure of set* $\{x|\mu_A(x) > 0\}$.

Definition 2.5 *A L-R fuzzy number* $A = (m,n,\sigma,\beta)_{LR}$, $m \le n$, *is defined as follows:*

$$\mu_A(x) = \begin{cases} L(\frac{m-x}{\sigma}), & -\infty < x < m, \\ 1, & m \le x \le n, \\ R(\frac{x-n}{\beta}) & n < x < +\infty. \end{cases}$$

Where σ *and* β *are the left-hand and right-hand spreads. In the closed interval* $[m,n]$, *the membership function is equal to 1.* $L(\frac{m-x}{\sigma})$ *and* $R(\frac{x-n}{\beta})$ *are non-increasing functions with* $L(0) = 1$ *and* $R(0) = 1$, *respectively. Usually, for convenience, they are, respectively, denotes as* $\mu_{A^L}(x)$ *and* $\mu_{A^R}(x)$. *It needs to point out that when* $L(\frac{m-x}{\sigma})$ *and* $R(\frac{x-n}{\beta})$ *are linear functions and* $m < n$, *fuzzy number* A *denotes trapezoidal fuzzy number. when* $L(\frac{m-x}{\sigma})$ *and* $R(\frac{x-n}{\beta})$ *are linear functions and* $m = n$, *fuzzy number* A *denotes unimodal fuzzy number.*

This definition is very general and allows the quantification of quite different types of information; for instance, if A is supposed to be a real crisp number for $m \in \Re$,

$$A = (m,m,0,0)_{LR},\ \forall L,\ \forall R$$

If A is a crisp interval,

$$A = (a,b,0,0)_{LR},\ \forall L,\ \forall R$$

and if A is a trapezoidal fuzzy number, $L(x) = R(x) = \max(0, 1-x)$ is implied.
Let F denote the space of L-R fuzzy numbers, therefore, in this article it is assumed that, the fuzzy number $A \in F$ is presented by means of the following representation:

$$A = \bigcup_{\alpha\in[0,1]} (\alpha, A_\alpha) \tag{2.1}$$

where

$$\forall\alpha \in [0,1] : A_\alpha = [L_A(\alpha), R_A(\alpha)] \subset (-\infty,\infty) \tag{2.2}$$

Here, $L : [0,1] \to (-\infty,\infty)$ is a monotonically non-decreasing and $R : [0,1] \to (-\infty,\infty)$ is a monotonically non-increasing left-continuous functions. The functions $L(.)$ and $R(.)$ express the left and right sides of a fuzzy number, respectively. In other words ,

$$L(\alpha) = \mu_\uparrow^{-1}(\alpha), \quad R(\alpha) = \mu_\downarrow^{-1}(\alpha), \tag{2.3}$$

where $L(\alpha) = \mu_\uparrow^{-1}(\alpha)$, and $R(\alpha) = \mu_\downarrow^{-1}(\alpha)$, denote quasi-inverse functions of the increasing and decreasing parts of the membership functions $\mu(t)$, respectively. As a result, the decomposition representation of the fuzzy number A, called the L-R representation, has the following form:

$$A = \bigcup_{\alpha\in(0,1]} (\alpha, [L_A(\alpha), R_A(\alpha)]).$$

Definition 2.6 *[7]. A function* $f : [0,1] \to [0,1]$ *symmetric around* $\frac{1}{2}$, *i.e.* $f(\frac{1}{2}-\alpha) = f(\frac{1}{2}+\alpha)$ *for all* $\alpha \in [0,\frac{1}{2}]$, *which reaches its minimum in* $\frac{1}{2}$, *is called the bi-symmetrical weighted function. Moreover, the bi-symmetrical weighted function is called regular if*

(1) $f(\frac{1}{2}) = 0$,

(2) $f(0) = f(1) = 1$,

(3) $\int_0^1 f(\alpha)d\alpha = \frac{1}{2}$.

One can, of course, propose many regular bi-symmetrical weighted functions and hence obtain different bi-symmetrical weighted distances. Further on we consider mainly the following function

$$f(\alpha) = \begin{cases} 1-2\alpha & when\ \alpha \in [0,\frac{1}{2}], \\ 2\alpha - 1 & when\ \alpha \in [\frac{1}{2},1]. \end{cases} \tag{2.4}$$

Definition 2.7 *Let* $A = (m,m,\sigma,\beta)_{LR}$ *be a unimodal L-R fuzzy number and* $A_\alpha =$

$[L_A(\alpha), R_A(\alpha)]$ be its α-cut sets. The regular weighted point for A is as follows:

$$RWP(A) = \frac{\int_0^1 \left(\frac{L_A(\alpha)+R_A(\alpha)}{2}\right) f(\alpha)d\alpha}{\int_0^1 f(\alpha)d\alpha}$$

$$= \int_0^1 (L_A(\alpha) + R_A(\alpha))f(\alpha)d\alpha.$$

Definition 2.8 *The regular weighted set for the L-R fuzzy number $A = (m, n, \sigma, \beta)_{LR}$ is the set of regular weighted points of all unimodal L-R fuzzy numbers $B = (m_B, m_B, \sigma_B, \beta_B)_{LR}$ that belong to the fuzzy number A.*

Proposition 2.1 *The regular weighted set for the L-R fuzzy number $A = (m, n, \sigma, \beta)_{LR}$ is a regular weighted interval $[A_1, A_2]$, such as $A_1 = m - l\sigma$ and $A_2 = n + r\beta$. Where $l = \int_0^1 L_A(\alpha)f(\alpha)d\alpha$ and $r = \int_0^1 R_A(\alpha)f(\alpha)d\alpha$.*

Proof. Let us consider two unimodal L-R fuzzy numbers $B_1 = (m, m, \sigma, 0)_{LR}$ and $B_2 = (n, n, 0, \beta)_{LR}$, that belong to the fuzzy number $A = (m, n, \sigma, \beta)_{LR}$. α-cut sets B_1 and B_2 are designated accordingly as $B_{1\alpha} = [L_{B_1}(\alpha), m]$, $B_{2\alpha} = [n, R_{B_2}(\alpha)]$.

According to the Definition 2.7, we shall define the regular weighted points A_1, A_2 for numbers B_1, B_2 as follows:

$$A_1 = \int_0^1 (L_{B_1}(\alpha) + m)f(\alpha)d\alpha =$$

$$\int_0^1 (2m - L_A(\alpha)\sigma)f(\alpha)d\alpha = m - l\sigma,$$

$$A_2 = \int_0^1 (R_{B_2}(\alpha) + n)f(\alpha)d\alpha =$$

$$\int_0^1 (2n + R_A(\alpha)\beta)f(\alpha)d\alpha = n + r\beta,$$

where

$$l = \int_0^1 L_A(\alpha)f(\alpha)d\alpha$$

and

$$r = \int_0^1 R_A(\alpha)f(\alpha)d\alpha$$

.

Consider the unimodal L-R number $B = (m_B, m_B, \sigma_B, \beta_B)_{LR}$, that belongs to number $A = (m, n, \sigma, \beta)_{LR}$. α-cut set B is designated accordingly as $[L_B(\alpha), R_B(\alpha)]$ and regular weighted point B is designated accordingly as $RWP(B)$.
According to the definition of one fuzzy number belonging to another one in [8, 9], next inequalities can be obtained as follows:

$$L_{B_1}(\alpha) \leq L_B(\alpha),$$

$$m \leq R_B(\alpha),$$

$$n \geq L_B(\alpha),$$

$$R_{B_2}(\alpha) \geq R_B(\alpha).$$

Therefore,

$$\frac{L_{B_1}(\alpha) + m}{2} \leq \frac{L_B(\alpha) + R_B(\alpha)}{2},$$

$$\frac{n + R_{B_2}(\alpha)}{2} \geq \frac{L_B(\alpha) + R_B(\alpha)}{2}.$$

Then $A_1 \leq RWP(B)$ and $A_2 \geq RWP(B)$.

Proposition 2.2 *Let $A = (m_A, n_A, \sigma_A, \beta_A)_{LR}$ and $B = (m_B, n_B, \sigma_B, \beta_B)_{LR}$ br two arbitrary L-R fuzzy numbers. If $[A_1, A_2]$ and $[B_1, B_2]$ are regular weighted intervals for A and B, then regular weighted interval for fuzzy number $A + B$ can be obtained as $[A_1 + B_1, A_2 + B_2]$.*

Proof. We shall designate regular weighted interval for fuzzy number $A + B$ as $[C_1, C_2$. According to the definitions 2.7 and 2.8, the boundaries of regular weighted interval can be obtained as follows:

$$C_1 =$$

$$\int_0^1 [2(m_A + m_B) - L_A(\alpha)\sigma_A - L_B(\alpha)\beta_B]f(\alpha)d\alpha =$$

$$= m_A + m_B - l_A\sigma_A - l_B\sigma_B = A_1 + B_1,$$

$$C_2 =$$

$$\int_0^1 [2(n_A + n_B) - R_A(\alpha)\beta_A + R_B(\alpha)\beta_B]f(\alpha)d\alpha =$$

$$= n_A + n_B - r_A\beta_A + r_B\beta_B = A_2 + B_2,$$

where

$$l_A = \int_0^1 L_A(\alpha)f(\alpha)d\alpha,$$

$$r_A = \int_0^1 R_A(\alpha) f(\alpha) d\alpha,$$

$$l_B = \int_0^1 L_B(\alpha) f(\alpha) d\alpha,$$

$$r_B = \int_0^1 R_B(\alpha) f(\alpha) d\alpha.$$

Example 2.1 *Consider two triangular fuzzy numbers* $A = (2,2,2,2)_{LR}$, *and* $B = (2,2,1,1)$ *in [2]. The regular weighted point for fuzzy numbers* A *and* B, *we shall designated accordingly as* $RWP(A)$ *and* $RWP(B)$ *and the regular weighted intervals for* A *and* B *we shall designate accordingly as* $[A_1, A_2]$ *and* $[B_1, B_2]$. *According to the definition 2.7, we shall define the regular weighted points* A *and* B *as follows:*

$$RWP(A) = \int_0^1 (4-2(1-\alpha)+2(1-\alpha)) f(\alpha) d\alpha = 2,$$

$$RWP(B) = \int_0^1 (4-(1-\alpha)+(1-\alpha)) f(\alpha) d\alpha = 2.$$

According to the definition 2.8, we shall define the regular weighted intervals $[A_1, A_2]$ and $[B_1, B_2]$ for fuzzy numbers A and B as follows:

$$A_1 = \int_0^1 (4-2(1-\alpha) f(\alpha) d\alpha = 1\frac{2}{3},$$

$$A_2 = \int_0^1 (4+2(1-\alpha) f(\alpha) d\alpha = 2\frac{1}{3},$$

$$B_1 = \int_0^1 (4-(1-\alpha) f(\alpha) d\alpha = 1\frac{5}{6},$$

$$B_2 = \int_0^1 (4+2(1-\alpha) f(\alpha) d\alpha = 2\frac{1}{6}.$$

Then $[A_1, A_2] = [1\frac{2}{3}, 2\frac{1}{3}]$ and $[B_1, B_2] = [1\frac{5}{6}, 2\frac{1}{6}]$. Since $[B_1, B_2] \subset [A_1, A_2]$, according to [2], we have $B \prec A$.

It can be observed that regular weighted points (aggregative crisp numbers) are the same for two triangular fuzzy numbers with different fuzzy widths, while the regular weighted intervals for these fuzzy numbers are different. The greater the fuzzy widths, the greater the weighted interval.

3 Conclusion

The fuzzy number defuzzification method with regular weighted intervals was developed in this article. The developed method was suggested to be used in situations where it is necessary to accumulate more information about fuzzy numbers than aggregative point crisp indexes contain, when there is no need to get only aggregative numbers. The numerical example demonstrated that the developed method can be used for the successful accumulation different bounds of fuzzy numbers.

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